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Yoshiyuki Tanaka · Toshio Tsuji · Makoto Kaneko

Dynamic control of redundant manipulators using the artificial potential field approach with time scaling

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Abstract This paper proposes a new method for the dynamic control of redundant manipulators via the artificial potential field approach. In the artificial potential field approach, the goal is represented by an artificial attractive potential field and the obstacles by corresponding repulsive field, so that the trajectory to the target can be associated with the unique flow-line of the gradient field through the initial position and can be generated via a flow-line tracking process. This approach is suitable for real-time motion planning because of its simplicity and smaller computational time than other methods based on global information about the task space. However, little attention has been paid to the control of the dynamic behavior of the trajectories generated. The proposed method is based on the artificial potential field approach with a combination of a time-scale transformation and a time-base generator which works as a time-scale compressor and can control the dynamic behavior of the robot without any change in the form of the designed controller itself. It can also be applied to the dynamic motion planning problem of redundant manipulators. The effectiveness of the proposed method is verified by computer simulations for a three-joint planar manipulator.

Key words Artificial potential field approach · Dynamic control · Redundant manipulator · time-base generator · Time-scale transformation

Introduction

In the artificial potential field approach (APFA),¹⁻⁴ the goal is represented by an artificial attractive potential field and

the obstacles by corresponding repulsive fields, so that the trajectory to the target can be generated via a flow-line tracking process taking obstacle avoidance into consideration. This method is often used for the trajectory generation problem of vehicles and manipulators because of its simplicity and smaller computational time than other methods that are based on global information about the task space. However, little attention has been paid to the control of the dynamic behavior of the trajectories generated such as movement time from the initial position to the goal, and the velocity profile of the trajectory generated. Although one of the most crucial winning features of the APFA is real-time applicability, it is difficult to use the generated trajectory for the control of robots in real time.

To meet the disadvantage mentioned above, Hashimoto et al.⁴ proposed a method using an electrostatic potential field and a sliding mode for a manipulator that can regulate the movement time but not the dynamic behavior of a robot. Recently, Tsuji et al.^{5,6} proposed a method introducing a time base generator (TBG) into the APFA which can regulate the movement time and also the velocity profile of the robot, but cannot be applied to the dynamic control.

Generally, it is harder to develop dynamic control of the robot than kinematic control because of the existence of a drift component in the dynamic system. In fact, without considering the holonomy or nonholonomy of the system, most previous studies have dealt with the kinematic model for the trajectory generation problem. However, a few studies⁷ have taken account of the dynamics of the robot in the trajectory path-following problem.

Hollerbach⁷ developed the trajectory time-scaling method for the torque-limited path-following problem. This method can lead the end-effector to the goal along the given path by modifying the movement speed. Sampei and Furuta⁸ showed that the stability of a system is preserved for any time-scale transformation as long as the defined new time never goes backward against the actual time. They then proposed time-scale transformation for a linearized nonlinear system. More recently, Tanaka et al.⁹ have developed the trajectory generation method for the dynamic control of robots based on the APFA, with a combination of

Y. Tanaka · T. Tsuji (✉) · M. Kaneko
Faculty of Engineering, Hiroshima University, Higashi-Hiroshima
739-8527, Japan
Tel. +81-824-24-7692; Fax +81-824-22-7158
e-mail: tsuji@huis.hiroshima-u.ac.jp

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time-scale transformation and a time base generator, and applied it to the dynamic control of a holonomic mobile robot.

In this paper, we propose a new trajectory generation method for the dynamic control of redundant manipulators using Tanaka's method.⁹ The redundant manipulator has desirable features that may lead to more dexterity and versatility in the robot's motions, for instance, avoiding obstacles or singular configurations when performing a given task.¹⁰⁻¹² The present method can control the spatiotemporal trajectories of the end-effector with significant advantages of redundancy.

This paper starts with a section formulating the dynamics of a redundant manipulator. The next section describes the general problems of the APFA. Then, the new trajectory generation method based on the APFA is explained in detail, and finally, the effectiveness of the proposed method is shown by computer simulations with a dynamic model of a redundant manipulator with three joints.

Dynamics of redundant manipulators

The joint space motion equation of an n degree-of-freedom manipulator whose end-effector is operating in the m dimensional task space can be expressed as

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{h}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{g}(\mathbf{q}) = \boldsymbol{\tau} \quad (1)$$

where $\mathbf{q} \in \mathfrak{R}^n$ is the joint angle vector, $\mathbf{M}(\mathbf{q}) \in \mathfrak{R}^{n \times n}$ is the nonsingular inertia matrix, $\mathbf{h}(\mathbf{q}, \dot{\mathbf{q}}) \in \mathfrak{R}^n$ is the nonlinear term including the joint torque due to the Coriolis and centrifugal force, $\mathbf{g}(\mathbf{q}) \in \mathfrak{R}^n$ is the joint torque due to gravity, and $\boldsymbol{\tau} \in \mathfrak{R}^n$ is the joint torque vector. The dynamics of the end-effector can also be written in the operational space as¹²

$$\mathbf{M}_x(\mathbf{q})\ddot{\mathbf{x}} + \mathbf{h}_x(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{g}_x(\mathbf{q}) = \mathbf{F} \quad (2)$$

where $\mathbf{x} \in \mathfrak{R}^m$ is the current end-effector position, $\mathbf{F} \in \mathfrak{R}^m$ is the end-effector force vector, $\mathbf{M}_x(\mathbf{q}) = (\mathbf{J}\mathbf{M}^{-1}(\mathbf{q})\mathbf{J}^T)^{-1} \in \mathfrak{R}^{m \times m}$ is the operational space kinetic energy matrix, $\mathbf{J} \in \mathfrak{R}^{m \times n}$ is the Jacobian matrix, and

$$\mathbf{h}_x(\mathbf{q}, \dot{\mathbf{q}}) = \bar{\mathbf{J}}^T \mathbf{h}(\mathbf{q}, \dot{\mathbf{q}}) - \mathbf{M}_x(\mathbf{q})\dot{\bar{\mathbf{J}}}\dot{\mathbf{q}}$$

$$\mathbf{g}_x(\mathbf{q}) = \bar{\mathbf{J}}^T \mathbf{g}(\mathbf{q})$$

$$\bar{\mathbf{J}} = (\mathbf{M}_x(\mathbf{q})\mathbf{J}\mathbf{M}^{-1}(\mathbf{q}))^T$$

When a manipulator possesses an extra degree of freedom to execute a given task, i.e., $m < n$, the joint torque of redundant manipulators can be decomposed into two elements: the joint torque $\boldsymbol{\tau}_{\text{effector}} \in \mathfrak{R}^n$ to operate the end-effector, and the joint torque $\boldsymbol{\tau}_{\text{joint}} \in \mathfrak{R}^n$ to control the additional freedom of joint motion with the redundancy of a manipulator. In this case, there exists the following force/torque relationship between the joint torque $\boldsymbol{\tau}_{\text{effector}}$ and the operational force \mathbf{F} :

$$\boldsymbol{\tau}_{\text{effector}} = \mathbf{J}^T \mathbf{F} \quad (3)$$

However, the joint torque $\boldsymbol{\tau}_{\text{joint}}$ always satisfies the condition¹² given by

$$\bar{\mathbf{J}}^T \boldsymbol{\tau}_{\text{joint}} = \mathbf{0} \quad (4)$$

This equation implies that the joint torque $\boldsymbol{\tau}_{\text{joint}}$ must lie in the null space associated with the matrix $\bar{\mathbf{J}}^T$ so as not to produce any acceleration at the end-effector. The general solution $\boldsymbol{\tau}_{\text{joint}}$ for this condition is given by

$$\boldsymbol{\tau}_{\text{joint}} = \mathbf{G}\boldsymbol{\tau}^* \quad (5)$$

where $\boldsymbol{\tau}^*$ is an arbitrary n dimensional vector, and $\mathbf{G} = \mathbf{I} - \mathbf{J}^T \bar{\mathbf{J}}^T \in \mathfrak{R}^{n \times n}$ defines the mapping to the null space associated with $\bar{\mathbf{J}}^T$. Consequently, the total joint torque $\boldsymbol{\tau}$ for a redundant manipulator can be recomposed of Eqs. 3 and 5 as follows:

$$\begin{aligned} \boldsymbol{\tau} &= \boldsymbol{\tau}_{\text{effector}} + \boldsymbol{\tau}_{\text{joint}} \\ &= \mathbf{J}^T \mathbf{F} + \mathbf{G}\boldsymbol{\tau}^* \end{aligned} \quad (6)$$

In this paper, we explain the feedback control law \mathbf{F} for operating the end-effector and $\boldsymbol{\tau}^*$ for controlling an additional freedom of motion of a manipulator. The total joint torque composed of these designed controllers allows a redundant manipulator to perform a given task by utilizing arm redundancy efficiently.

Artificial potential field approach

In this section, we attempt to design the feedback control laws \mathbf{F} in order to lead the end-effector to the target position and $\boldsymbol{\tau}^*$ in order to control the extra joint motion of redundant manipulators.

Here, we define the potential function with quadratic form V_{effector} to derive the feedback controller \mathbf{F} as

$$V_{\text{effector}} = \frac{1}{2}(\mathbf{x}^* - \mathbf{x})^T \mathbf{K}_1(\mathbf{x}^* - \mathbf{x}) + \frac{1}{2}\dot{\mathbf{x}}^T \mathbf{K}_2 \dot{\mathbf{x}} \quad (7)$$

where \mathbf{x}^* denotes the target position of the end-effector, and $\mathbf{K}_i = \text{diag}(k_1^i, k_2^i, \dots, k_m^i)$ under $k_m^i > 0$ ($i = 1, 2$). When we design the feedback control law \mathbf{F} based on the potential function V_{effector} as

$$\mathbf{F} = -\mathbf{M}_x(\mathbf{q})\mathbf{K}_2^{-1}\{\mathbf{K}_1(\mathbf{x} - \mathbf{x}^*) + \dot{\mathbf{x}}\} + \mathbf{h}_x(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{g}_x(\mathbf{q}) \quad (8)$$

the time-derivative of the potential function V_{effector} yields

$$\dot{V}_{\text{effector}} = -\|\dot{\mathbf{x}}\|^2 \leq 0 \quad (9)$$

with the operational space dynamic Eq. 2. $\dot{V}_{\text{effector}}$ is always nonincreasing except at the equilibrium point. It follows that the end-effector can reach the target position by means of the joint torque $\boldsymbol{\tau}_{\text{effector}}$ in Eq. 3, which is equivalent to the derived control law \mathbf{F} given in Eq. 8. For a redundant manipulator, however, the joints may continue to move although the end-effector arrives at the target position, since designed controller \mathbf{F} cannot control the extra freedom of joint motion directly. For this problem, we utilize the null

space on the force/torque transformation to control the internal motion.

Here, we define the potential function V_{joint} in order to design the feedback controller τ^* which can utilize the internal motion of a redundant manipulator as

$$V_{joint} = \frac{1}{2} \dot{q}^T M(q) \dot{q} + \kappa(t) Q(q) \quad (10)$$

where $\kappa(t)$ is a positive nonincreasing continuous function under $\kappa(t_f) = 0.0$ at the specified time t_f , and $Q(q)$ is a differentiable potential function. The first term on the right-hand side of Eq. 10 is used in order to dampen the redundant joint motion when the end-effector arrives at the goal, and the second one is used to realize the desired posture of the manipulator q^* corresponding to the minimum of the potential function $Q(q)$. It should be noted that the potential function $Q(q)$ can be maximized under the negative nondecreasing coefficient function $\kappa(t)$. If we design the feedback control law τ based on the potential function V_{joint} as

$$\tau = -\dot{q} + g(q) - \kappa(t) \frac{\partial Q}{\partial q} \quad (11)$$

the time-derivative of the potential function V_{joint} yields

$$\dot{V}_{joint} = -\|\dot{q}\|^2 + \dot{\kappa}(t) Q(q) \leq 0 \quad (12)$$

using the joint space motion Eq. 1 and the definition that $\kappa(t)$ is nonincreasing in the actual time scale t . Selecting the designed joint torque τ in Eq. 11 as τ^* , we can obtain the new feedback controller τ_{joint} to control the internal motion without altering the generating trajectory of the end-effector.

By means of the total feedback control law τ given in Eq. 6, composed of the designed controller F (Eq. 8) and τ^* (Eq. 11), the end-effector can reach the target position, and also the desired posture may be realized through an optimization procedure of the potential function $Q(q)$. Figure 1 shows the block diagram of this feedback system of the redundant manipulator.

Moreover, substituting Eq. 8 into Eq. 2, we can derive the following linear damped system:

$$\ddot{x} + K_2^{-1} \dot{x} + K_2^{-1} K_1^{-1} (x - x^*) = 0 \quad (13)$$

Obviously, the system of the manipulator in the operational space in Eq. 2 is asymptotically stable to the equilibrium

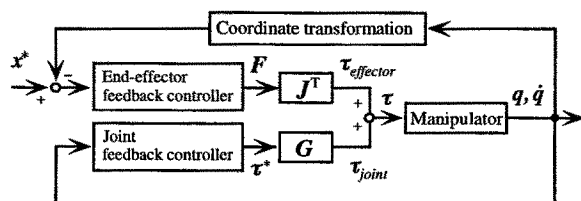


Fig. 1 Block diagram of the feedback control system for a redundant manipulator

point x^* by means of the designed feedback controller F given in Eq. 8. Following the above discussion, we can conclude that it is impossible to regulate the convergence time and the dynamic behavior of the end-effector as hoped.⁹

APFA with time scaling

Generally, the stability and dynamic property of a system do not change in any time scale that is a strictly monotone increasing function with respect to the actual time.⁸ This indicates that we can design the feedback control law to make the original system converge to the equilibrium point at finite time t_f as long as the asymptotic stabilizer for the system in the new time scale, where infinite time corresponds to t_f in actual time, is found.

In this section, we present details of the proposed method based on the APFA combined with the time-scale transformation.

Virtual time s and TBG

The relationship between actual time, t , and virtual time, s , is given by

$$\frac{ds}{dt} = a(t) \quad (14)$$

where the continuous function $a(t)$, called the time-scale function,⁸ is defined as follows:

$$a(t) = -p \frac{\dot{\xi}}{\xi} \quad (15)$$

where p is a positive constant and $\xi(t)$ is a nonincreasing function called the time base generator (TBG)^{5,6} which generates a bell-shaped velocity profile satisfying $\xi(0) = 1$ and $\xi(t_f) = 0$ with the convergence time t_f . The dynamics of ξ is defined as

$$\dot{\xi} = -\gamma(\xi(1 - \xi))^\beta \quad (16)$$

where γ is a positive constant that can control the convergence time t_f , and β is also a positive constant, $0 < \beta < 1.0$, which determines the behavior of ξ . The convergence time t_f can be calculated with the gamma function $\Gamma(\cdot)$ as

$$t_f = \int_0^{t_f} dt = \int_1^0 \frac{d\xi}{\dot{\xi}} = \frac{\Gamma^2(1 - \beta)}{\gamma \Gamma(2 - 2\beta)} \quad (17)$$

Thus, the system converges to the equilibrium point $\xi = 0$ in the finite time t_f if the parameter γ is chosen as

$$\gamma = \frac{\Gamma^2(1 - \beta)}{t_f \Gamma(2 - 2\beta)} \quad (18)$$

Figure 2 show the time histories of ξ and $\dot{\xi}$ depending on convergence time $t_f = 1.0, 3.0, 5.0$ (s) under the parameter $\beta = 0.5$.

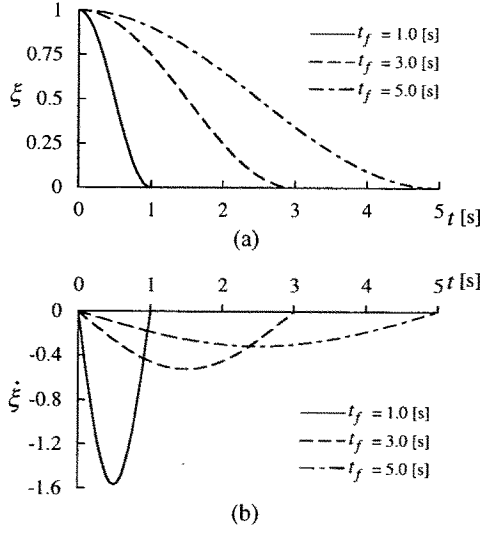


Fig. 2 Time history of ξ and $\dot{\xi}$ for different prespecified times

From Eqs. 14 and 15, the virtual time s can be represented with respect to ξ as

$$s = \int_0^t a(t) dt = -p \ln \xi(t) \quad (19)$$

It is obvious that the virtual time s given in Eq. 19 never goes backward against the actual time t . We take this virtual time s as a new time scale in time-scale transformation.

Time-scaling of the system

We can rewrite the two dynamic equations in the joint space (Eq. 1) and in the operational space (Eq. 2) into the following linear system with the state variable $\mathbf{Z} = (\mathbf{x}, \mathbf{q}, \dot{\mathbf{x}}, \dot{\mathbf{q}})^T$ as

$$\frac{d}{dt} \mathbf{Z} = \begin{pmatrix} \mathbf{0} & \mathbf{I} \\ \mathbf{0} & \mathbf{0} \end{pmatrix} \mathbf{Z} = \begin{pmatrix} \mathbf{0} \\ \mathbf{I} \end{pmatrix} \begin{pmatrix} \mathbf{F}_t \\ \boldsymbol{\tau}_t \end{pmatrix} \quad (20)$$

$$\mathbf{F}_t = \mathbf{M}_x^{-1}(\mathbf{q}) \{ \mathbf{F} - (\mathbf{h}_x(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{g}_x(\mathbf{q})) \} \quad (21)$$

$$\boldsymbol{\tau}_t = \mathbf{M}^{-1}(\mathbf{q}) \{ \boldsymbol{\tau} - (\mathbf{h}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{g}(\mathbf{q})) \} \quad (22)$$

where $\mathbf{0} \in \mathfrak{R}^{(m+n) \times (m+n)}$ is the zero matrix and $\mathbf{I} \in \mathfrak{R}^{(m+n) \times (m+n)}$ is the unit matrix.

The system given in Eq. 20 can be rewritten in the virtual time scale s as

$$\frac{d}{ds} \boldsymbol{\Psi} = \begin{pmatrix} \mathbf{0} & \mathbf{I} \\ \mathbf{0} & \mathbf{0} \end{pmatrix} \boldsymbol{\Psi} + \begin{pmatrix} \mathbf{0} \\ \mathbf{I} \end{pmatrix} \begin{pmatrix} \mathbf{F}_s \\ \boldsymbol{\tau}_s \end{pmatrix} \quad (23)$$

where

$$\boldsymbol{\Psi} = (\psi_1, \psi_2, \psi_3, \psi_4) = \left(\mathbf{x}, \mathbf{q}, \frac{\dot{\mathbf{x}}}{a(t)}, \frac{\dot{\mathbf{q}}}{a(t)} \right)^T \quad (24)$$

$$\mathbf{F}_s = \frac{d}{dt} \left(\frac{1}{a(t)} \right) \dot{\mathbf{x}} + \frac{1}{a^2(t)} \mathbf{F}_t \quad (25)$$

$$\boldsymbol{\tau}_s = \frac{d}{dt} \left(\frac{1}{a(t)} \right) \dot{\mathbf{q}} + \frac{1}{a^2(t)} \boldsymbol{\tau}_t \quad (26)$$

As previously defined in the relationship between actual time and virtual time, the stability of the new system given in Eq. 23 is the same as that of the original system in actual time.⁸ Hence, there exists a feedback control law to stabilize the new system asymptotically.

Design of the feedback control law

In this subsection, we design the feedback control law with the APFA to stabilize the new system given in Eq. 23 in the virtual time scale.

We can define the potential function with the quadratic form $V_{effector}^\psi$ for the control of the end-effector to the target position in the virtual time scale as

$$V_{effector}^\psi = \frac{1}{2} (\psi_1^* - \psi_1)^T \mathbf{K}_1 (\psi_1^* - \psi_1) + \frac{1}{2} \psi_3^T \mathbf{K}_2 \psi_3 \quad (27)$$

If we design the feedback control law \mathbf{F}_s based on $V_{effector}^\psi$ as

$$\mathbf{F}_s = -\mathbf{K}_2^{-1} \{ \mathbf{K}_1 (\psi_1 - \psi_1^*) + \psi_3 \} \quad (28)$$

the time-derivative of the potential function $V_{effector}^\psi$ in the new time scale yields

$$\begin{aligned} \frac{d}{ds} V_{effector}^\psi &= \psi_3^T \left\{ \mathbf{K}_1 (\psi_1 - \psi_1^*) + \mathbf{K}_2 \frac{d\psi_3}{ds} \right\} \\ &= -\|\psi_3\|^2 \leq 0 \end{aligned} \quad (29)$$

By inverse transformation of time scale from the virtual time s to the actual time t for the controller \mathbf{F}_s with Eqs. 23 and 25, the controller \mathbf{F}_t in actual time is derived as

$$\mathbf{F}_t = -a^2(t) \mathbf{K}_2^{-1} \mathbf{K}_1 (\mathbf{x} - \mathbf{x}^*) - \left\{ a(t) \mathbf{K}_2^{-1} - \frac{\dot{a}(t)}{a(t)} \right\} \dot{\mathbf{x}} \quad (30)$$

From Eqs. 21 and 30, we can obtain the feedback control law \mathbf{F}^ψ for control of the dynamic behavior of the end-effector as

$$\mathbf{F}^\psi = \mathbf{M}_x(\mathbf{q}) \mathbf{F}_t + \mathbf{h}_x(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{g}_x(\mathbf{q}) \quad (31)$$

The end-effector of the manipulator is controlled to the target position at the convergence time t_f by means of the joint torque $\boldsymbol{\tau}_{effector}$, which is equivalent to the feedback control law \mathbf{F}^ψ given in Eq. 31.

On the other hand, we can define the potential function V_{joint}^ψ to derive the feedback controller $\boldsymbol{\tau}_s$ in virtual time s as

$$V_{joint}^\psi = \frac{1}{2} \psi_4^T \mathbf{K}_3 \psi_4 + \zeta(s) Q_s(\psi_2) \quad (32)$$

where $\mathbf{K}_3 = \text{diag.}(k_1^3, k_2^3, \dots, k_n^3)$ under $k_n^3 \geq 0$, $Q_s(\psi_2)$ is the differentiable potential function in the virtual time scale, and $\zeta(s)$ is a positive nonincreasing scalar function. It should be noted that the potential function $Q_s(\psi_2)$ can be maximized under the negative nondecreasing coefficient function $\zeta(s)$. Differentiating the potential function $Q_s(\psi_2)$ with respect to the virtual time s , we get

$$\frac{\partial Q_s}{\partial s} = \left(\frac{\partial \psi_2}{\partial s} \right)^T \frac{\partial Q_s}{\partial \psi_2} = \psi_4^T \frac{\partial Q_s}{\partial \psi_2} \quad (33)$$

With the new system Eq. 23 and Eq. 33, the time-derivative of the potential function V_{joint}^ψ in virtual time s yields

$$\frac{d}{ds} V_{joint}^\psi = \psi_4^T \left\{ \mathbf{K}_3 \tau_s + \zeta(s) \frac{\partial Q_s}{\partial \psi_2} \right\} + \frac{d\zeta(s)}{ds} Q_s(\psi_2) \quad (34)$$

If we define the feedback controller τ_s with the non-increasing scalar function $\zeta(s)$ in the new time scale as

$$\tau_s = -\mathbf{K}_3^{-1} \left\{ \psi_4 + \zeta(s) \frac{\partial Q_s}{\partial \psi_2} \right\} \quad (35)$$

Eq. 34 can be calculated as

$$\frac{d}{ds} V_{joint}^\psi = -\|\psi_4\|^2 + \frac{d\zeta(s)}{ds} Q_s(\psi_2) \leq 0 \quad (36)$$

This indicates that the potential function V_{joint}^ψ is stabilized to the equilibrium point by means of the feedback controller τ_s in the virtual time scale.

Here, we define the nonincreasing function $\zeta(s)$ in the new time scale as

$$\zeta(s) = ae^{-\frac{2s}{p}} \quad (37)$$

Through the inverse time-scale transformation from virtual time to actual time for the controller τ_s with Eqs. 23 and 26, the feedback control law τ_t in actual time is derived as

$$\tau_t = - \left\{ a(t) \mathbf{K}_3^{-1} - \frac{\dot{a}(t)}{a(t)} \right\} \dot{q} - \alpha \xi^2(t) a^2(t) \mathbf{K}_3^{-1} \frac{\partial Q}{\partial q} \quad (38)$$

where α is a positive constant. From Eqs. 22 and 38, we can derive the feedback controller τ^ψ as

$$\tau^\psi = \mathbf{M}(q) \tau_t + \mathbf{h}(q, \dot{q}) + \mathbf{g}(q) \quad (39)$$

When the joint torque τ^ψ (Eq. 39) is selected as τ^* , we can obtain the joint torque τ_{joint} (Eq. 5) to control the internal motion of the redundant manipulator.

The total feedback control law τ (Eq. 6), composed of the designed controller given in Eqs. 31 and 39, can lead the end-effector to the target position at the specified time t_f and may attain the desired posture by utilizing the redundancy of the manipulator effectively without altering the configuration of the end-effector.

Dynamic behavior of the end effector

In this section, the dynamic behavior of the end-effector controlled by the total joint torque feedback controller τ (above) is analyzed. To simplify the discussion, the target position for the end-effector is set at the origin in the operational space. Substituting the feedback control law \mathbf{F}^ψ for the end-effector (Eq. 31) into the original linear system equation given in Eq. 20, we have the second-order differential equation

$$\ddot{x} = -p^2 \left(\frac{\xi}{\xi} \right)^2 \mathbf{K}_2^{-1} \mathbf{K}_1 x + \left\{ (p-1) \frac{\xi}{\xi} + \frac{\xi}{\xi} \right\} \dot{x} \quad (40)$$

Here, we first analyze the behavior of the end-effector on the x coordinate. From Eq. 40, the following Euler's equation with respect to x and ξ can be derived:

$$\xi^2 \frac{d^2 x}{d\xi^2} - (p-1) \xi \frac{dx}{d\xi} + \frac{k_1^1}{k_1^2} p^2 x = 0 \quad (41)$$

Since the nonincreasing function ξ converges to zero at finite time t_f , the necessary and sufficient condition to converge x , \dot{x} , and \ddot{x} to zero at the specified time t_f is given as follows according to the discriminant of the characteristic

polynomial $D_x = 4 \frac{k_1^1}{k_1^2} - 1$ of Eq. 41:

- (1) if $D_x \geq 0$ then $p > 4(1 - \beta)$
- (2) if $D_x < 0$ then $p > \frac{4(1 - \beta)}{1 - \sqrt{-D_x}}$

The dynamic behavior of the other state variables in the operational space can be analyzed in the same manner.

It can be proven that the feedback control law \mathbf{F}^ψ (Eq. 31) of the proposed method can regulate the dynamic behavior of the end-effector and the convergence time to reach the goal.

Computer simulations

The proposed trajectory generation method was applied to a redundant manipulator. Figure 3 shows the simulation results with a three-joint planar manipulator. The initial posture of the manipulator is $q(0) = \left(\pi, -\frac{11\pi}{12}, -\frac{\pi}{3} \right)^T$ (rad), and the target position of the end-effector is $x^* = (0.0, 1.5)^T$ (m) with convergence time $t_f = 5.0$ (s) under $p = 8.0$, $\alpha = 1.0$.

The gain matrices \mathbf{K}_i ($i = 1, 2, 3$) in the potential functions are set at $\mathbf{K}_1 = \text{diag.}(0.25, 0.25)$ (N/m), $\mathbf{K}_2 = \text{diag.}(1.0, 1.0)$ (Ns/m), and $\mathbf{K}_3 = \text{diag.}(1.0, 1.0, 1.0)$ (Nm/(rad/s)), respectively. We used the Appel method for the manipulator dynamics¹⁵ and the link parameters of the manipulator, as shown in Table 1.

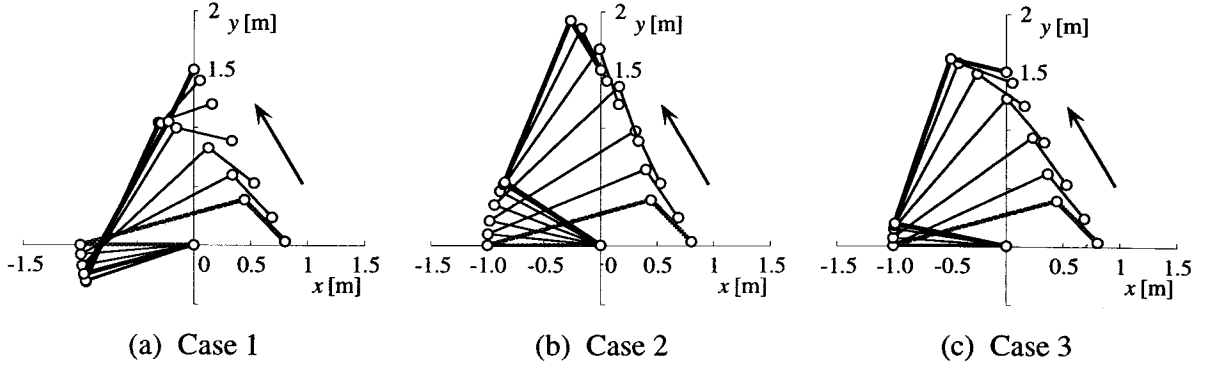


Fig. 3 Changes in the generated end-effector trajectories with different positive functions $Q_i(q)$ ($i = 1, 2, 3$)

Table 1 Link parameters of a three-joint planar manipulator

	Link 1	Link 2	Link 3
Length (m)	1.0	1.5	0.5
Mass (kg)	0.8	1.2	0.4
Center of mass (m)	0.4	0.6	0.25
Moment of inertia (kgm^2)	0.06666	0.22500	0.00833

Figure 3a shows the generated trajectory with the potential function $Q(q)$ set at

$$Q_1(q) = 0 \quad (42)$$

which means that arm redundancy is not utilized. On the other hand, the joint angle control of the first joint and maximization of the manipulability¹¹ is considered as a subtask in Fig. 3b,c. In these cases, the potential functions $Q(q)$ are given as

$$Q_2(q) = \frac{1}{2}(q_1^* - q_1(t))^2 \quad (43)$$

$$Q_3(q) = \sqrt{\det \mathbf{J}\mathbf{J}^T} \quad (44)$$

where the target angle of the first joint q_1^* is specified as

$$q_1^* = \frac{5\pi}{6} \text{ (rad)}. \text{ For the maximization of } Q_3(q), \text{ we use the}$$

negative nondecreasing function $\zeta(s)$:

$$\zeta(s) = -ae^{-\frac{2s}{p}} \quad (45)$$

instead of Eq. 37.

It can be seen that the generated trajectories are influenced by the corresponding potential function $Q(q)$ defined above. In Fig. 3a, it can be seen that the third joint of the manipulator is overstretched while the end-effector reaches the target position.

In contrast, the end-effector reaches the target position without any singular configurations by utilizing the redundancy control of the manipulator corresponding to local optimization of the potential functions $Q(q)$ in Fig. 3b,c.

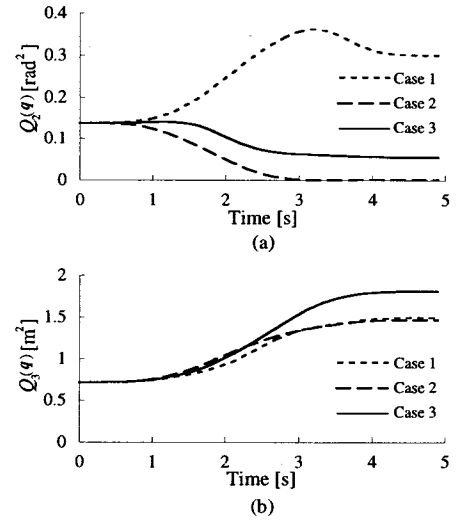


Fig. 4 Time histories of the positive function $Q_i(q)$ ($i = 1, 2, 3$)

Figure 4 shows the time histories of $Q_2(q)$ and $Q_3(q)$. It can be seen that each potential function in Fig. 3b,c is optimized much better than in other cases.

Figure 5 shows the time history of the end-effector position \mathbf{x} , velocity $\dot{\mathbf{x}}$, and the squared sum of the joint angular velocity. It should be noted that all generated trajectories of the end-effector in Fig. 3 completely coincide with the one trajectory depicted in Fig. 5a. It can also be seen that the end-effector reached the target position via the smooth trajectory, and that the joints of the manipulator do not move after the specified time $t_f = 5.0$ (s) in all cases.

Conclusions

In this paper, a new trajectory generation method for a dynamic model of redundant manipulators using the concept of the APFA and the time-scale transformation has been presented. We have developed a control strategy for the redundant manipulators that allows the achievement of performance with each redundancy. We also analyzed the

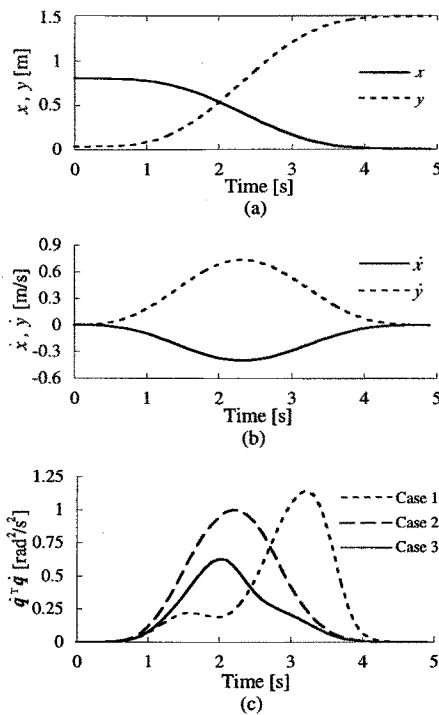


Fig. 5 Time histories of the end-effector position, velocity, and squared sum of joint velocities

dynamic behavior of the end-effector mathematically, and derived the necessary and sufficient conditions to reach the target point at the specified time under the proposed control law. In simulation results with a three-joint planar manipulator, the effectiveness of the proposed method was ascertained. Since the proposed method can specify the necessary time for the robots to reach the goal, it may be useful for time-scheduling problems for a robot or the synchronous control of multiple robots.

Future research will be directed to developing a method that can be applied to a torque-limited trajectory generation

problem, and can also regulate the dynamic behavior of joints.

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