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Paper:

Trajectory Generation of a Multi-Arm Robot Utilizing Kinematic Redundancy

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This paper proposes a method of generating trajectories of multiple manipulators by using virtual arms. First, it is shown that a closed link structure composed by multiple manipulators can be resolved into closed link structures composed by a pair of manipulators and then each closed link structure is modelled as a connected arm. A connected arm expresses a single manipulator as a pair of manipulators that are connected at the end-points. By defining a connected arm in this way, it becomes possible to make use of redundant degrees of freedom of manipulators by means of virtual arms. A virtual arm is a virtually supposed arm whose end-points are placed on the joints or links of the connected arms. The virtual arms makes it possible to treat the postures of multiple manipulators as a set of virtual end-points. This paper not only carries out the modeling of the kinematics of connected arms and virtual arms in general but also derives inverse kinematic solutions of connected arms by the use of virtual arms. In addition, the paper shows that it is possible to realize a variety of postures of manipulators while controlling the object being grasped.

Keywords: Multi-arm robot, Kinematic redundancy, Trajectory generation, Virtual arm

1. Introduction

If multiple robot arms can carry out tasks while coordinating with each other, then it becomes possible to carry out a complicated task that could not have been accomplished by a single robot, and a possibility exists for improving to a drastic degree the work capability of a robotic system as a whole. This paper is to deal with the problem of grasping and moving a common object by means of multiple arms.

Such problem of trajectory generation by a closed link structure is a very important one in realizing a coordinated task among multiple robots[1,2]. In order to realize such a coordinated task, it is necessary that the following two problems be solved: (1) the closed link structure formed by multiple arms should be maintained during movements; and (2) to generate not only the trajectory of the object being grasped but also the trajectory of the whole robotic system. For example, let us consider a case where an obstacle exists in the task space. In such a case, it is necessary to generate a trajectory such that not only the object being grasped and the end-point of each arm but also the system as a whole would not interfere with the obstacle. Needless to say, the end-point of each arm must always form a closed link structure during movements in order to keep grasping the object. In particular, when the number of manipulators comprising the system increases, the closed link structure becomes very complicated and, moreover, comes to possess many redundant degrees of freedom.

In regard to research on such a closed link structure formed by multiple arms, emphasis has been on the grasping and moving of objects from the point of view of statics, and few investigations have been made on methods of regulating a posture as a whole of multiple arms by utilizing the redundant degree of freedom.

The author has proposed the concept of a virtual arm in order to utilize the redundant degree of freedom of manipulators, and shown methods for carrying out trajectory generation by expressing the postures of the multi-joint arms in the task space[3-5]. A virtual arm is defined as an imaginary arm which has an end-point on each joint or link of the manipulator and whose parameters such as link lengths and joint angles are the same as those of the original manipulator. If multiple such virtual arms are defined, then it is possible to express the posture of the original manipulator as a set of end-points of the virtual arms and also to consider any interference between the environment and the entire arms in the task space.

Previously, the author has used these virtual arms to propose various methods such as a method of trajectory generation for a redundant manipulator[3], a method for obstacle avoidance[4], and a method for distributed control[5]. However, all these methods are based on a single manipulator, so if they are to be applied to a coordinated task by multiple robots, the concept of virtual arms must be expanded for closed link structures of multiple arms. For this reason, this paper shows a method of formulating a closed link structure composed of multiple arms by means of virtual arms and also makes clear a method of generating the trajectory of the multiple arm as a whole by making positive use of the redundant degree of freedom of the arms. Then, in Chapter 2, the modeling of closed link structures and the definition of virtual arms will be explained, and in Chapter 3, a method of trajectory generation for the multiple arms by using the virtual arms will be proposed. Moreover, in Chapter 4, the effectiveness of this method will be made clear by a simulation experiment.

2. Virtual Arms of Closed Link Structures

2.1. Definition of Connected Arms

Now, let us consider a closed link structure by multiple

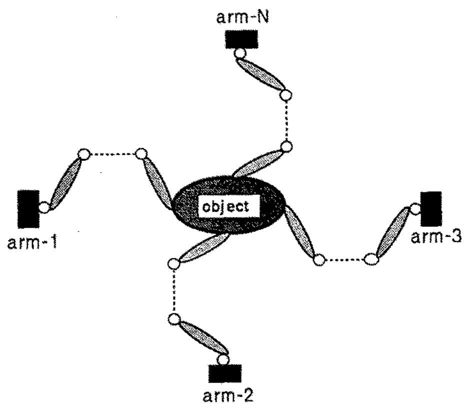


Fig. 1. A closed-chain formed by multiple arms.

manipulators as shown **Fig.1**. N different manipulators are coordinated to grasp a common object. Here, the number of joints for each manipulator is denoted by m_i (i=1, 2, ..., N), and the total number of joints is given by $M\ (=\sum_{i=1}^{N} m_i)$.

In order to formalize a closed link structure by multiple manipulators, a closed link composed of an arm 1 and arm 2 is first considered. In this paper, the segment from the base of the arm 1 to the base of the arm 2 is regarded as a single manipulator, and this is called a connected arm. In this case, the base of the arm 1 is considered as the base of this connected arm and the base of the arm 2 is regarded as the end-point of the connected arm. Since similar joint arms can be defined for the arm 1 and an arm 3,..., an arm N, so in the end, it is possible to express a closed link structure of multiple manipulators as N-1 connected arms composed by the arm 1 and arms j (j=2, 3, ..., N).

If l_c denotes the degree of freedom with which the end-point of the connected arm j (the base of the arm j) is constrained then the relationship between the end-point displacement $dX_j \in R^{l_c}$ and the vector $d\theta \in R^M$ representing the joint angle displacements of all the arms can be obtained in the following manner. First, the end-point displacement vector of the arm 1 is denoted as $dX_{e1} \in R^{lc}$*,thentherelationship*

$$dX_{e1} = J_1 d\theta \quad \dots\dots\dots\dots\dots\dots \quad (1)$$

holds. Here, $J_1 \in R^{l_c \times M}$ is the Jacobian matrix of the arm 1.

On the other hand, if the displacement vector at a representative point of an object (its center of gravity, for example) is written as $dX_0 \in R^{l_c}$, then its relationship with dX_{e1} is given by

$$dX_0 = (S_1^T)^{-1} dX_{e1} \quad \dots\dots\dots\dots\dots\dots \quad (2)$$

Here, $S_1 \in R^{l_c \times l_c}$ is a non-singular transformation matrix expressing the geometric relationship between the position of the end-point of the arm 1 and the representative point of the object[6,7]. Similarly, the relationship between dX_0 and the end-point displacement vectors $dX_{ej} \in R^{l_c}$ of the arms j (j=2, 3, ..., N) becomes

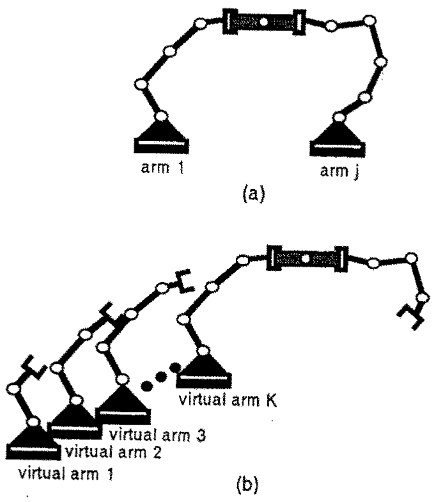


Fig. 2. Virtual arms of a dual arm robot grasping a common object.

$$dX_{ej} = S_j^T dX_0 \quad \dots\dots\dots\dots\dots\dots \quad (3)$$

In contrast, the end-point displacement dX_j of the connected arm j can be resolved into

$$dX_j = dX_{ej} + J_j' d\theta \quad \dots\dots\dots\dots\dots \quad (4)$$

where j=2, 3, ..., N, and $J_j' \in R^{l_c \times M}$ is the Jacobian matrix of the arm whose end-point is considered as the base of the arm j and whose base is considered as the end-point of the arm j. By substituting Eqs.(1) to (3) into Eq.(4), the following relationship is obtained:

$$dX_j = (S_j^T (S_1^T)^{-1} J_1 + J_j') d\theta \quad \dots\dots\dots\dots \quad (5)$$

If Eq.(5) is concatenated together for all the connected arms (j= 2, 3..., N), then we have

$$dX = J d\theta \quad \dots\dots\dots\dots\dots\dots\dots \quad (6)$$

where $dX \in R^{l_c(N-1)}$ and $J \in R^{l_c(N-1) \times M}$ are given by

$$dX = \begin{bmatrix} dX_2 \\ dX_3 \\ \cdot \\ \cdot \\ \cdot \\ dX_N \end{bmatrix}, \ J = \begin{bmatrix} S_2^T(S_1^T)^{-1}J_1 + J_2' \\ S_3^T(S_1^T)^{-1}J_1 + J_3' \\ \cdot \\ \cdot \\ \cdot \\ S_N^T(S_1^T)^{-1}J_1 + J_N' \end{bmatrix} \quad \dots\dots\dots \quad (7)$$

2.2. Definitions of Virtual Arms

An example of virtual arm for a closed link structure is shown in **Fig.2**. In the figure, (a) is the connected arm j and (b) is a virtual arm defined on the connected arm. The parameters for each virtual arm such as the connected angles and link lengths correspond to those of its joint arm. Here, K different virtual arms are defined as a general case. By defining multiple virtual arms this way, it now becomes possible to express the posture of the manipulators as a set of the virtual end-points in the task space[3].

Now, if the end-points displacement vector of a virtual arm k is given by $dX_{vk} \in R^{l}$, then the relationship between dX_{vk} and $d\theta$ can be written as

$$dX_{vk} = J_{vk} d\theta \quad \ldots\ldots\ldots\ldots (8)$$

where $J_{vk} \in R^{l \times M}$ is the Jacobian matrix of the virtual arm k[3], and l represents the degree of freedom of the task space.

When Eq.(8) is concatenated for all the virtual arms, then the following relationship is obtained:

$$dX_{v} = J_{v} d\theta \quad \ldots\ldots\ldots\ldots (9)$$

where

$$dX_v = \begin{bmatrix} dX_{v1} \\ dX_{v2} \\ \cdot \\ \cdot \\ \cdot \\ dX_{vK} \end{bmatrix}, \; J_v = \begin{bmatrix} J_{v1} \\ J_{v2} \\ \cdot \\ \cdot \\ \cdot \\ J_{vK} \end{bmatrix} \quad \ldots\ldots\ldots\ldots (10)$$

Here, $dX_v \in R^{lK}$ is a vector combining the end-point displacements of all the virtual arms, while $J_v \in R^{lK \times M}$ is a matrix combining the Jacobian matrices of all the virtual arms.

3. Trajectory Generation Method for Closed Link Structures based on Virtual Arms

First, the condition for maintaining a closed link structure will be derived. If the kinematic Eq.(6) for connected arms derived in Sec.2.1 is solved with respect to $d\theta$, then

$$d\theta = J^{+} dX + (I_M - J^{+}J) z \quad \ldots\ldots\ldots\ldots (11)$$

Here, $J^{+} \in R^{M \times l_c(N-1)}$ is a pseudoinverse matrix, $I_M \in R^{M \times M}$ is a unit matrix, and $z \in R^{M}$ is an arbitrary vector. Since the end-point of the connected arms are really the bases of the arms 2, 3, ..., N, the end-point displacement dX of Eq.(11) must be 0. Hence,

$$d\theta = (I_M - J_c^{+} J_c) z \quad \ldots\ldots\ldots\ldots (12)$$

This equation is the condition for maintaining the closed link structure of multiple manipulators. In other words, the trajectory of multiple manipulators can be determined by means of an arbitrary vector z. Here, this vector z is determined by using virtual arms.

By substituting Eq.(12) in Eq.(9), the result is as follows:

$$dX_v = Gz \quad \ldots\ldots\ldots\ldots (13)$$

where $G \equiv J_v(I_M - J^{+}J) \in R^{lK \times M}$. If it is assumed here that a target displacement dX^{*}_{v} is given to the virtual arms, then the simultaneous linear equations in Eq.(3) can be categorized into three cases the virtual arms: redundant, over constrained, and singular cases[3]. If the maximum rank decomposition of the matrix G is used, then the general solution for the arbitrary vector z is given by

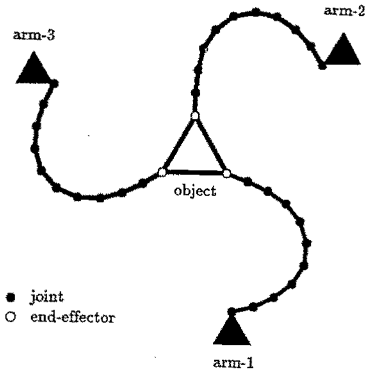


Fig. 3. Initial postures of three arms grasping a triangular object.

$$z = G_b^{+}(G_a^{T} W G_a) G_a^{T} W dX_v^{*} \quad \ldots\ldots\ldots\ldots (14)$$

where $G = G_a G_b$ and $rank(G) = rank(G_a) = rank(G_b) = p$. $G_a \in R^{lK \times p}$ is the over constrained part of G, and $G_b \in R^{p \times M}$ represents the redundant part of G. $W \in R^{lk \times lk}$ is a non-singular weight matrix which gives a degree of priority to each virtual end-point. Then, the joint angle displacement $d\theta$ is obtained from Eqs.(12) and (14) to be

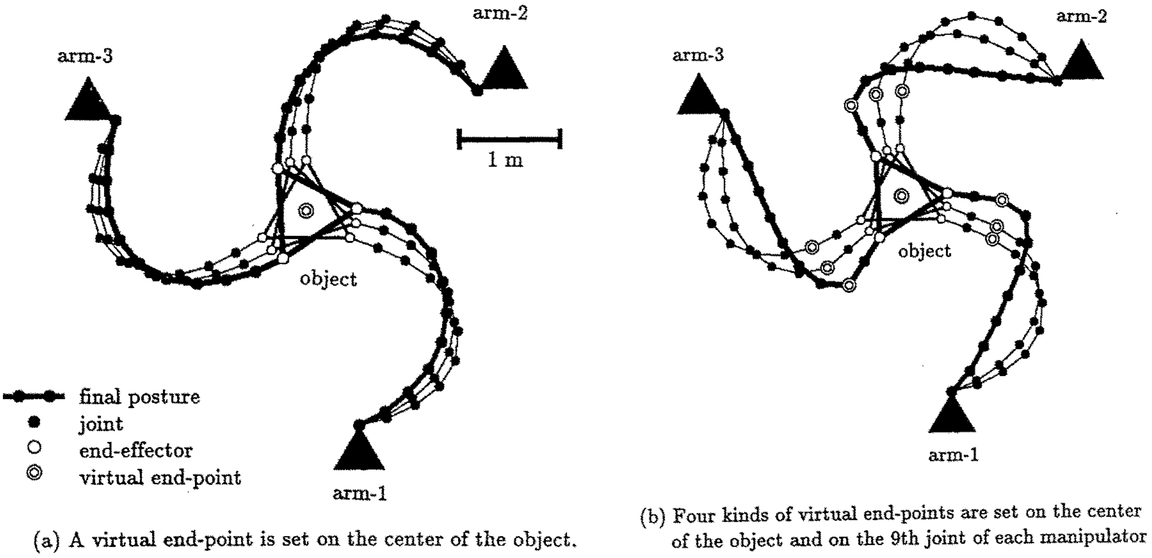
$$d\theta = (I_M - J^{+}J) G_b^{+}(G_a^{T} W G_a) G_a^{T} W dX_v^{*} \quad \ldots\ldots (15)$$

The above equation makes it possible, under the condition of maintaining the closed link structure, not only to give joint angle displacements to multiple manipulators but also to make positive use of the redundant degrees of freedom of the manipulators by means of the virtual arms. Thus, if the target virtual end-point displacement dX^{*}_{v} is given, then the joint angle displacement $d\theta$ of the multiple manipulators can be obtained by using Eq.(15).

4. Simulation Experiments

In order to show the effectiveness of this method, a computer simulation was carried out. Three planar manipulators with 10 joints each ($m_1 = m_2 = m_3 = 10$, $IM = 30$, $N = 3$, $l_c = 2$, $l = 3$) were used, and the length of each link was set at 0.3m. In addition, as an object to be grasped, a right-triangular object in which the length from each vortex to the center of gravity is 0.5m was used. The initial posture is shown in **Fig.3**.

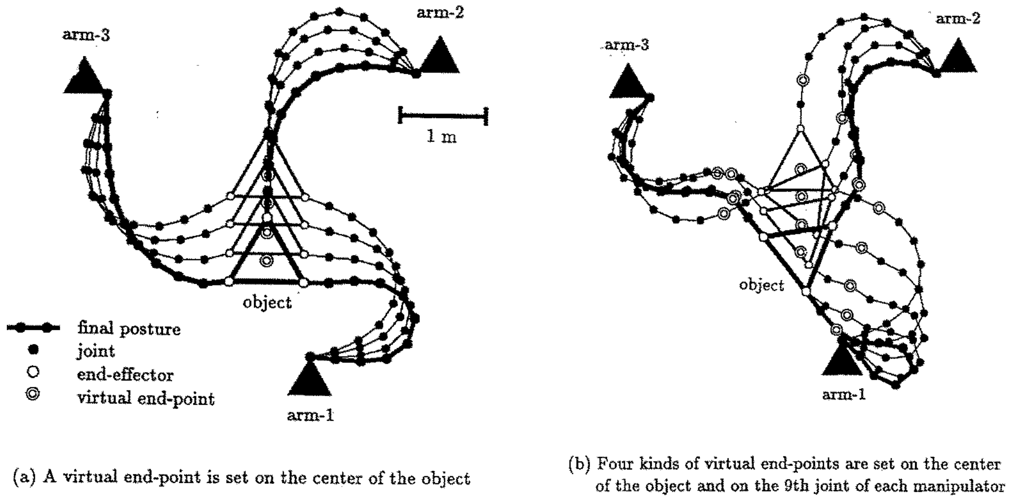
Generated trajectories are shown in **Fig.4** where the object is given a target displacement of a 0.53*rad* rotation in a counter-clockwise direction without changing the position of the object. In the figure, (a) indicates a case in which a virtual arm is placed at the center of gravity of the object (K=1), while (b) represents a case where virtual arms are placed at the center of gravity of the object and at the ninth joint of each of the manipulators (K=4), with the target displacement of the virtual end-points used on the manipulator being given a vector in a direction away from the object's center of gravity. In the case of (a), all joints move smoothly. In (b), on the other hand, the virtual end-points tend to move away from the center of gravity of the object, so the ninth joint of each manipulator draws a circular trajectory with its center at the center of gravity.



(a) A virtual end-point is set on the center of the object.

(b) Four kinds of virtual end-points are set on the center of the object and on the 9th joint of each manipulator

Fig. 4. Simulation results of object rotation.



(a) A virtual end-point is set on the center of the object

(b) Four kinds of virtual end-points are set on the center of the object and on the 9th joint of each manipulator

Fig. 5. Simulation results of positioning tasks.

Figure 5 gives results coming a situation where the object's center of gravity was moved 1m downward, with all the virtual arms used exactly the same as in Fig.4., except that in Fig.5(b), the target displacement for the virtual end-point on each manipulator was given a vector directing towards the base of the corresponding manipulator. It can be seen that whereas in (a) the system reaches the goal while moving each joint smoothly, in (b) the ninth joints are approaching the bases of the manipulators. Thus, it is clear that by using virtual arms, it is possible to generate trajectories for multiple manipulators while making use of redundant degrees of freedom.

5. Conclusions

This paper has proposed a method of generating joint trajectories of multiple arms which form a closed link structure. This method attempts to derive joint trajectories of multiple arms by using multiple virtual arms on each arm and planning trajectories of the end-points of these virtual arms. As such, the method can be applied easily to robotic systems having any number of arms. In addition, it has been made clear that it is possible to realize the following features:

(1) By regarding two arms composing a closed link structure as a virtually supposed single manipulator and defining it as a connected arm, it is possible to generate trajectories of multiple arms while maintaining the closed link structure.

(2) By using virtual arms, it is possible to make positive use of redundant degree of freedom of the system as a whole.

(3) It is possible to freely choose various postures of multiple manipulators within the limitation of maintaining the closed link structure while the control of the object being grasped is being carried out.

For the future, this method will be applied not only for developing a method of having multiple arms avoid an obstacle, a method of distributed control, and others but also for developing a method for trajectory generation that enables distributed execution by each of the arms composing a system.

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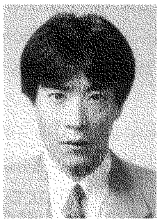

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