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Dynamic Modeling of Closed-Chain Mechanisms for Multi-Arm Robots

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A method for the dynamic simulation of multiarm robots by taking the mechanism of various contact types into consideration is proposed. The system of multi-arm robots grasping a common object forms a multiple closed-chain mechanisms. Each arm and the object can be modeled to be an open-chain with kinematic constraints on its end-effector motion. The dynamics of each arm is expressed using the Appel's method, where the end-effector's constraints of each arm are derived from the object motion. As a result, a parallel computation of joint acceleration for each arm can be performed, and various contact types between the end-effectors and the object can be expressed by constraint equations. Numerical examples are carried out to evaluate the validity of the proposed method.

Key Words: multi-arm robot, dynamic simulation, closed-chain mechanism, Appel's method

1. Introduction

There are growing needs for the development of multi-arm robots which are expected to be applied to many tasks in manufacturing, such as assembly operations, tool manipulations and material handling tasks where the objects involved are very large and voluminous. The application of a single robot in automating a task is typically achieved with intricate jigs or fixtures which represent a significant fraction of the overall start-up costs (in some cases, more than the robot itself). An alternative, more flexible approach is the use of multi-arm robots. This is especially true, for unstructured environments such as the space and the ocean where the auxiliary equipments are not available.

Widespread use of multi-arm robots will require the development of sophisticated control algorithms in order to achieve a true coordinated control scheme, and a number of interesting works have been appeared. For example, Hayati[1] proposed an extension of hybrid control for a multi-arm robot by partitioning the object and regarding each segment as the last link of each arm. Yoshikawa and Zheng[2] proposed a cooperative dynamic hybrid position/force control method for a set of multi-arm robots or multifingered robots grasping a common object, which took the manipulator dynamics and object dynamics into consideration. On the other hand, the effective implementation of complex robot control systems requires extensive computer simulations and experiments prior to implementation in the field. It is expected that, through computer simulations, one can freely evaluate different control schemes without extensive setup time and costs.

The dynamic computer simulation of the multi-arm robots have been studied by several researchers. For example, Anderson[3] presented a method for the forward dynamics of two cooperating manipulators by regarding the interactive forces as the external forces exerted on the end-effectors. Oh and Orin[4], Rodriguez[5] and Murphy et al.[6] developed the methods of dynamic computer simulation for an arbitrary number of robot arms grasping a common object. It is noted that an unified approach was formulated by Oh and Orin[4], such that a single set of equations may describe dynamics of both multi-arm robots and legged vehicles. However, all of the previ-

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ous works have concentrated on the multi-arm robots with only the rigid contact between the end-effectors and the object.

The present paper analyzes the dynamic equations of motion for multi-arm robot grasping a common object. When the multi-arm robot grasping a common object, multiple closed-chain mechanism will be formed. Each arm and the object can be considered as an open-chain with kinematic constraints on its end-effector motion. Therefore, the joint acceleration can be calculated by using the forward dynamics of each open-chain, the motion equations of the object, the kinematic relationships between the object and the end-effectors and the kinematic relationships between the end-effectors and the joints. The forward dynamics of each open-chain is derived using the Appel's method. The method proposed here has the following features:

(a) Using the Appel's method, no explicit differentiation in the dynamic equation of the linked body system is required, such that allows a better adaptation of model forming procedure to the system considered[7].

(b) Using the Appel's method, we do not need the extensive formulation and computation of the dynamic interaction forces/moments between end-effectors and the object specifically. We just add the dynamic interaction forces/moments in the dynamic equation of each arm as an external forces/moments exerted on the constrained end-effector, and this dynamic interaction forces/moments are computed at the same time with the computation of the joint accelerations.

(c) The dynamic of each arm can be derived separately, such that each arm can be simulated simultaneously, and the parallel computation can be implemented. As a result, the simulation time for the multi-arm robots is reduced.

(d) The consideration of the contact mechanism between the end-effectors and the object in the model can be taken easily, where the various contact-types can be expressed in the kinematic relationships between the object and the end-effectors.

First of all, the application of the Appel's method for dynamic analysis of a single-arm robot with kinematic constraints is reviewed, followed by the derivation of the kinematic relationships of the multi-arm robots. Then, the forward dynamics of multi-arm robots grasping a common object is derived. Finally, numerical analysis are carried out and the validity of this method is shown.

2. Dynamic Model of Single-Arm Robot: Unconstrained Case

In this section, the single-arm dynamic model for the unconstrained end-effector based on Appel's equations will be reviewed. This model was developed in its final form by Potkonjak and Vukobratovic[8].

Consider a mechanism of the open-chain type, formed by n rigid bodies of arbitrary form, without branching, and mechanism segments are interconnected by the rotational or translational joint, such as shown in **Fig. 1**. The kinematic variables, such as angular velocity $\tilde{\omega}_i$, angular acceleration $\tilde{\varepsilon}_i$, and linear acceleration $\tilde{a}_i$ are determined with respect to the local coordinate system (the body-fixed coordinate system) of the i-th link. The relations determining these kinematic quantities are derived in the same way as in the Newton-Euler's method

$$\tilde{\omega}_i = A_{i,i-1}\tilde{\omega}_{i-1} + \dot{q}_i(1-s_i)\tilde{e}_i \tag{1}$$

$$\tilde{\varepsilon}_i = A_{i,i-1}\tilde{\varepsilon}_{i-1} + [\ddot{q}_i\tilde{e}_i + \dot{q}_i(\tilde{\omega}_i \times \tilde{e}_i)](1-s_i) \tag{2}$$

$$\tilde{a}_i = A_{i,i-1}[\tilde{a}_{i-1} - \tilde{\varepsilon}_{i-1} \times \tilde{r}_{i-1,i} - \tilde{\omega}_{i-1} \times (\tilde{\omega}_{i-1} \times \tilde{r}_{i-1,i})] + \tilde{\varepsilon}_i \times \tilde{r}^l_{ii} + \tilde{\omega}_i \times (\tilde{\omega}_i \times \tilde{r}^l_{ii}) + [\ddot{q}_i\tilde{e}_i + 2\dot{q}_i(\tilde{\omega}_i \times \tilde{e}_i)]s_i \tag{3}$$

$$\tilde{r}^l_{ii} = \tilde{r}_{ii} + q_i\tilde{e}_i s_i \tag{4}$$

where A_{ij} is the transition matrix from the j-th body-fixed coordinate system to the i-th body-fixed

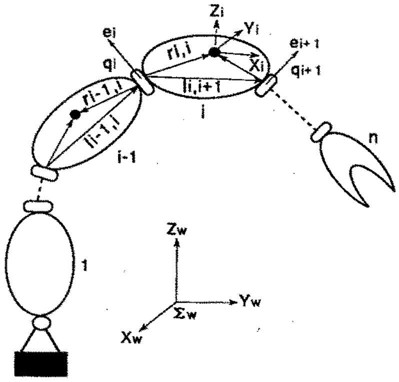

Fig. 1 Single-arm robot: unconstrained case

coordinate system and s_i is the joint type ($s_i=0$ if the joint is rotational, and $s_i=1$ if the joint is translational). $\tilde{e}_i$ denotes the unit vector of the joint axis respect to the body-fixed coordinate system, $\tilde{r}_{i-1,i}$ denotes the vector from the i-th joint to the center of mass of the $(i-1)$-th link respect to the body-fixed coordinate system and q_i denotes the generalized coordinate of the i-th joint. "×" denotes the vector cross product.

By introducing the matrices $\Omega, \Phi, \beta, \Theta$ and the generalized coordinate vector $q=[q_1\, q_2 \cdots q_n]^T$, we can write the linear acceleration and the angular acceleration in the form:

$$\tilde{a}_i=\beta\ddot{q}+\Theta \tag{5}$$

$$\tilde{\varepsilon}_i=\Omega\ddot{q}+\Phi \tag{6}$$

where $\beta=[\beta_1^i\, \beta_2^i \cdots \beta_i^i\, 0\, 0 \cdots 0]$, $\Theta=[\delta^i]$, $\Omega=[\Omega_1^i\, \Omega_2^i \cdots \Omega_i^i\, 0\, 0 \cdots 0]$ and $\Phi=[\gamma^i]$. Ω_j^i, β_j^i, δ^i, and γ^i are derived from the recursive expressions (1)~(4). Thus, for the i-th iteration:

$$\Omega_i^i=\tilde{e}_i(1-s_i) \tag{7}$$

$$\beta_i^i=\Omega_i^i\times\tilde{r}_{ii}+\tilde{e}_i s_i \tag{8}$$

$$\Omega_j^i=A_{i,i-1}\Omega_j^{i-1};\ j=(1, 2, \cdots(i-1)) \tag{9}$$

$$\gamma^i=A_{i,i-1}\gamma^{i-1}+\dot{q}_i(\tilde{\omega}_i\times\tilde{e}_i)(1-s_i) \tag{10}$$

$$\beta_j^i=A_{i,i-1}\beta_j^{i-1}+A_{i,i-1}\Omega_j^{i-1}\times\tilde{r}_{i-1,i}+\Omega_j^i\times\tilde{r}_{ii} \tag{11}$$

$$\delta^i=A_{i,i-1}\delta^{i-1}+A_{i,i-1}(\gamma^{i-1}\times\tilde{r}_{i-1,i})+\gamma^i\times\tilde{r}_{ii}+h_i \tag{12}$$

$$h_i=-A_{i,i-1}\tilde{\omega}_{i-1}\times(\tilde{\omega}_{i-1}\times\tilde{r}_{i-1,i})+\tilde{\omega}_i\times(\tilde{\omega}_i\times\tilde{r}_{ii}^l)+2\dot{q}_i(\tilde{\omega}_i\times\tilde{e}_i)s_i \tag{13}$$

Now, using (5) and (6), the Gibbs-Appel's "acceleration energy" function, S, for the single-arm robot will have the form:

$$S=1/2\ddot{q}^TW\ddot{q}+V\ddot{q}+D \tag{14}$$

where $W=\sum_{i=1}^{n}W_i$, $V=\sum_{i=1}^{n}V_i$ and $D=\sum_{i=1}^{n}D_i$. W_i, V_i, and D_i are given by

$$W_i=m_i\Omega^T\Omega+\beta^T\tilde{I}_i\beta \tag{15}$$

$$V_i=m_i\Theta^T\Omega+\Phi^T\tilde{I}_i\beta-\tilde{u}^T\beta \tag{16}$$

$$D_i=1/2m_i\Theta^T\Theta+1/2\Phi^T\Phi-\tilde{u}^T\Phi \tag{17}$$

$$\tilde{u}=(\tilde{I}_i\cdot\tilde{\omega}_i)\times\tilde{\omega}_i \tag{18}$$

where $\tilde{I}_i$ is the inertia tensor of the i-th link respect to the body-fixed coordinate system, m_i is the mass of the i-th link and "•" denotes the vector dot product.

On the other hand, the Appel's equation can be written in the matrix form

$$\frac{\partial S}{\partial\ddot{q}}=Q \tag{19}$$

where Q is the vector of generalized forces. Substituting (14) into (19), we can obtain the dynamic equation of the unconstrained single-arm robot

$$W\ddot{q}=Q-V^T \tag{20}$$

The vector Q has the form

$$Q=P+Y \tag{21}$$

where P is the joint torque vector. $Y=[y_1\, y_2 \cdots y_n]^T$ is the gravitational torque vector which calculated independent of P, and given by

$$y_i=(e_i\cdot g\sum_{k=0}^{n-1}m_{i+k})s_i+(1-s_i)\sum_{k=0}^{n-1}|m_{i+k}g,\ e_i,\ r_k^i| \tag{22}$$

$$r_k^i=\sum_{p=0}^{k}r_{i+p,i+p}^l-\sum_{p=0}^{k-1}r_{i+p,i+p+1} \tag{23}$$

where "| |" denotes the vector box product and $g\in R^3$ is the gravity acceleration vector. It should be noted that the generalized forces are calculated recursively in the algorithm, and this calculation can be performed in the world coordinate system or in the body-fixed coordinated system.

3. Representation for the Constrained Case

Now, we show how the results obtained for the unconstrained case can be used for solving the dynamics of the single-arm robot, where some or all of the end-effector directions are constrained by the environment. This method was proposed by Masuda et al.[9].

Consider a single-arm robot constrained by the environment shown in **Fig. 2**. In this case, there are interaction forces/moments between the end-effector and the environment according to the constrained directions.

Let us define $\lambda\in R^{l_c}$ is the vector of the forces/moments exerted on the constrained end-effector, and

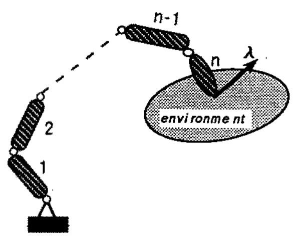

Fig. 2 Single-arm robot with kinematic constraints on its end-effector

l_c is the number of the end-effector directions constrained by the environment. We treat these forces/moments as the external forces/moments acting to the arm, such that the dynamic equation of the robot arm becomes

$$W\ddot{q}=P+Y-V^T+J_c^T\lambda \tag{24}$$

where $J_c \in R^{l_c \times n}$ is the Jacobian matrix corresponding to the constrained directions of the end-effector.

On the other hand, we have the kinematic relationships for the contrained directions such as given by

$$\dot{X}_c=J_c\dot{q} \tag{25}$$

where $\dot{X}_c$ is the constrained end-effector velocity vector. By differentiation we can find

$$\ddot{X}_c=J_c\ddot{q}+\dot{J}_c\dot{q} \tag{26}$$

So, we can rewrite (24) and (26) in the compact form

$$\begin{bmatrix} W & -J_c^T \\ -J_c & 0 \end{bmatrix}\begin{bmatrix} \ddot{q} \\ \lambda \end{bmatrix}=\begin{bmatrix} P+Y-V^T \\ \dot{J}_c\dot{q}-\ddot{X}_c \end{bmatrix} \tag{27}$$

where $\ddot{X}_c$ can be computed from the constraint condition. For example, if the end-effector cannot move in any direction, then

$$\ddot{X}_c=0 \tag{28}$$

Equation (27) expresses the dynamic equation of the single-arm robot constrained by the environment. The Jacobian matrix J_c, and $\dot{J}_c\dot{q}$ can be computed using the recursive expressions which are similar as in (7)~(13).

The manipulator mass matrix W is a positive definite matrix and invertible, such that the coefficient matrix of (27) is not singular. Therefore, the joint acceleration vector $\ddot{q}$ and the force/moment vector λ can be solved by

$$\begin{bmatrix} \ddot{q} \\ \lambda \end{bmatrix}=\begin{bmatrix} W & -J_c^T \\ -J_c & 0 \end{bmatrix}^{-1}\begin{bmatrix} P+Y-V^T \\ \dot{J}_c\dot{q}-\ddot{X}_c \end{bmatrix} \tag{29}$$

4. Dynamic Simulation for Multi-Arm Robots

The multi-arm robot grasping a common object to be simulated is shown in **Fig. 3**. The number of arm is m, and n_k is the number of joint of the k-th arm. In the present paper, it is assumed that the contact points between the end-effectors and the object are constant, i.e., there is no slip motion between the end-effectors and the object. Also, it is assumed that the contact between the end-effectors and the object is always occur, i.e., as a fixed joint for the rigid grasping or as a free joint for the point contact type.

We define a set of Cartesian coordinate systems as follows: (i) the world coordinate system, Σ_w, is an immobile external coordinate system as a reference frame, (ii) the transmission coordinate system[10], Σ_{trk}, is a coordinate system on the object at the k-th contact point where the z axis is normal to the object and the others are tangential to the object, and (iii) the object coordinate system, Σ_o, is a mobile coordinate system according to the motion of the object.

4.1 Kinematic Relationships of Multi-Arm Robots

The kinematic relationships of the multi-arm robot grasping a common object can be summarized in **Fig. 4**.

The matrix $G_k \in R^{l \times l_{ck}}$ specifies the relationships between the object force/motion and the transmission force/motion of the k-th arm, depending, on the location of the contact points, the contact type and

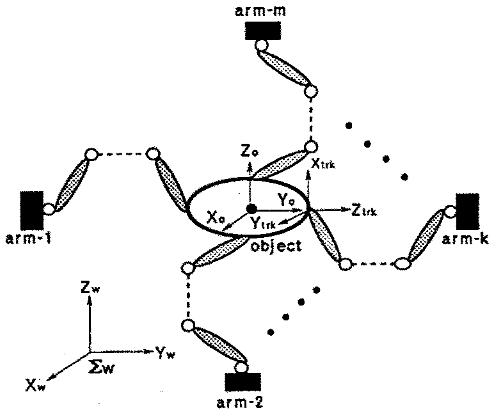

Fig. 3 Multi-arm robot grasping a common object

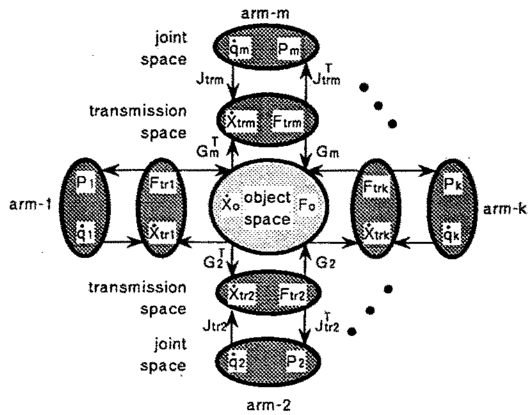

Fig. 4 Kinematics of the multi-arm robot

the reference point on the object such as the center of mass. l_{ck} is the number of the end-effector directions of the k-th arm constrained by the object, and l is the dimension of the task space. The matrix G_k is given by

$$G_k = B_k R^o_{trk} H^T_k \quad (30)$$

where R^o_{trk} is the rotation matrix from the transmission coordinate system, Σ_{trk}, to the object coordinate system, Σ_o. Furthermore, the matrix B_k is given by

$$B_k = \begin{bmatrix} E & 0 \\ (r_{ck})\chi & E \end{bmatrix} \quad (31)$$

where $E \in R^{3\times3}$ denotes a unit matrix, $0 \in R^{3\times3}$ is the null matrix, $r_{ck} = [r_{ckx}\ r_{cky}\ r_{ckz}]^T$ is the position vector of the k-th contact point from the origin of the object coordinate system, Σ_o, and $(r_{ck})\chi \in R^{3\times3}$ is an anti-symmetric matrix obtained by applying the cross operator, "χ", to the position vector, r_{ck}, as the following

$$(r_{ck})\chi = \begin{bmatrix} 0 & -r_{ckz} & r_{cky} \\ r_{ckz} & 0 & -r_{ckx} \\ -r_{cky} & r_{ckx} & 0 \end{bmatrix} \quad (32)$$

The matrix $H_k \in R^{l_{ck}\times l}$ expresses the filter characteristics which filter out some forces/moments of the k-th end-effector and transmit other forces/moments to the object depending on the contact type.

The matrix J_{trk} is the Jacobian matrix relating the joint displacements to the transmission displacements of the k-th arm, represented in the transmission coordinate system, Σ_{trk}, and given by

$$J_{trk} = H_k R^{trk}_w J_{wk} \in R^{l_{ck}\times n_k} \quad (33)$$

where $J_{wk} \in R^{l\times n_k}$ is the Jacobian matrix relating the joint displacements to the end-effector displacements of the k-th arm, represented in the world coordinate system, Σ_w, and R^{trk}_w is the rotation matrix from the world coordinate system, Σ_w, to the transmission coordinate system, Σ_{trk}.

4.2 Motion Equations

The dynamic equations of the multi-arm robot grasping a common object are formed by the dynamic equation of each arm, the motion equation of the object and the constraint equation relating the object motion and the motion of each arm.

When the multi-arm robot grasping a common object, multiple closed-chain mechanisms will be formed. Each arm and the object can be considered as an open-chain with kinematic constraints on its end-effector. Therefore, from (27), the dynamic equation of the k-th arm is given by

$$\begin{bmatrix} W_k & -J^T_{trk} \\ -J_{trk} & 0 \end{bmatrix} \begin{bmatrix} \ddot{q}_k \\ \lambda_{trk} \end{bmatrix} = \begin{bmatrix} P_k + Y_k - V^T_k \\ \dot{J}_{trk}\dot{q}_k - \ddot{X}_{trk} \end{bmatrix} \quad (34)$$

where $q_k = [q_{1k}\ q_{2k} \cdots q_{n_k k}]^T$ is the generalized coordinate vector of the k-th arm, $\ddot{X}_{trk}$ and λ_{trk} are the vector of the transmission acceleration of the k-th arm and the vector of the forces/moments exerted on the constrained end-effector of the k-th arm, respectively, which are represented in the transmission coordinate system, Σ_{trk}, and this forces/moments can be interpreted as a dynamic interaction forces/moments between the k-th end-effector and the object. The term $\dot{J}_{trk}\dot{q}_k$ is computed by

$$\dot{J}_{trk}\dot{q}_k = H_k(\dot{R}^{trk}_w J_{wk} + R^{trk}_w \dot{J}_{wk})\dot{q}_k \quad (35)$$

The motion equation of the object is given by

$$\begin{bmatrix} m_o E & 0 \\ 0 & I_o \end{bmatrix} \ddot{X}_o = \begin{bmatrix} m_o g \\ -\omega_o \times (I_o \cdot \omega_o) \end{bmatrix} + F_o \quad (36)$$

where m_o, I_o and ω_o are the mass, the moment inertia tensor and the angular velocity vector of the object, respectively. $\ddot{X}_o$ and F_o are the vector of the object accelerations and the vector of the forces/moments acting on the object, respectively, which are represented in the object coordinate system, Σ_o.

The free-body diagram of the object and the k-th arm is shown in **Fig. 5**. The dynamic interaction forces/moments, λ_{trk}, are the action-reaction forces/moments. Therefore, the transmitted forces/moments from the end effector to the object at the contact point through the contact mechanism is equal to $-\lambda_{trk}$.

Since the object and the end-effectors form the parallel link structure as shown in Fig. 4, the net forces/moments acting on the object is given by

$$F_o = -\sum_{k=1}^{m} G_k \lambda_{trk} \quad (37)$$

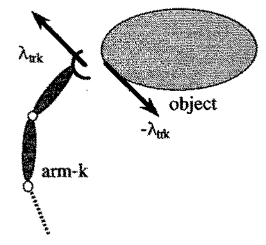

Fig. 5 Free-body diagram of the object and the k-th arm

On the other hand, the relationships between the transmission velocity of the k-th arm and the object velocity is given by

$$\dot{X}_{trk}=G_k^T\dot{X}_o \quad (38)$$

Because of the matrix G_k^T is a constant matrix, then we can obtain

$$\ddot{X}_{trk}=G_k^T\ddot{X}_o \quad (39)$$

Equations (34), (36), (37) and (39) express the dynamic equations of the multi-arms robot grasping a common object, where (39) can be interpreted as a constraint equation of each arm.

Consequently, the algorithm for the forward dynamics of the multi-arm robot proposed in this paper is summarized as the following:

step 1 At the time $t=0$, the initial states of the generalized coordinate vector of positions, $q_k(t=0)$, velocities, $\dot{q}_k(t=0)$, of each arm, and the object acceleration, $\ddot{X}_o(t=0)$, are given.

step 2 Compute the transmission acceleration of each arm, $\ddot{X}_{trk}(t)$, using (39).

step 3 Using (34), compute $\ddot{q}_k(t)$ and $\lambda_{trk}(t)$ for each arm, where the joint torques, $P_k(t)$, of each arm are given according to the control law, and then integrate the $\ddot{q}_k(t)$ to obtain $\dot{q}_k(t+\Delta t)$ and $q_k(t+\Delta t)$.

step 4 Using (36), compute the object acceleration, $\ddot{X}_o(t+\Delta t)$, where $F_o(t+\Delta t)$ is computed using (37), and $\lambda_{trk}(t+\Delta t)$ in (37) is obtained from step 3. Increase t by Δt, then return to step (2).

On the other hand, the inverse dynamics of the multi-arm robot proposed in this paper is summarized as the following:

step 1 At the time $t=0$, the initial states of the generalized coordinate vector of positions, $q_k(t=0)$, velocities, $\dot{q}_k(t=0)$, are given.

step 2 The object acceleration, $\ddot{X}_o(t)$, is derived from the given desired object trajectory, $X_o^d(t)$.

step 3 The transmission acceleration of the k-th arm, $\ddot{X}_{trk}(t)$, is computed using (39). Then, the joint acceleration of each arm $\ddot{q}_k(t)$ is computed by

$$\ddot{q}_k(t)=(J_{trk}(t))^+[\ddot{X}_{trk}(t)-\dot{J}_{trk}(t)\dot{q}_k(t)] \quad (40)$$

step 4 Using (36), compute the object force, $F_o(t)$, required for the desired object motion, and then the desired dynamic interaction forces/moments, $\lambda_{trk}(t)$, is computed using the following equation

$$\lambda_{tr}(t)=G^+F_o \quad (41)$$

where "+" denotes the pseudo-inverse matrix.

$\lambda_{tr}(t)=[\lambda_{tr1}^T(t)\ \lambda_{tr2}^T(t)\cdots\lambda_{trm}^T(t)]^T$, $G=[G_1\ G_2\cdots G_m]$ and G_k; $(k=1,2,\cdots,m)$ is given in (30).

step 5 Compute the joint torque of each arm, $P_k(t)$, using the following equation

$$P_k(t)=W_k(t)\ddot{q}_k(t)-Y_k(t)+V_k^T(t)-J_{trk}^T(t)\lambda_{trk}(t) \quad (42)$$

Increase t by Δt, then return to step (2).

As we can see, the computation of joint acceleration, $\ddot{q}_k$, of each arm can be performed independently. It means that the dynamics of each arm can be simulated in a parallel way. On the other hand, by introducing the transmission space in the kinematic relationships of the multi-arm robot enable us to express the contact type between the end-effectors of each arm and the object using the matrix H_k given in (30) and (33), such that the simulation method presented in this paper can be applied for various contact types between the end-effectors and the object.

5. Numerical Analysis

To evaluate the validity of the proposed method, the numerical examples was performed for the case of a planar dual-arm robot ($l=3$). The first example is intended to evaluate the computation error of the proposed method.

Consider a planar dual arm robot ($n_1=n_2=3$) grasping a common object rigidly, i.e., all of the end-effector forces/moments can be transmitted to the object ($l_{c1}=l_{c2}=3$). The link parameters of the dual-arm robot and the object parameters are shown in **Table 1**.

The goal is to move the object from the initial position $X_o(0)=[0.223205\,(\mathrm{m})\ 0.446410\,(\mathrm{m})\ 0.0\,(\mathrm{rad})]^T$ as shown in **Fig. 6**, to the desired final position $X_o^d(t_f)=[0.273205\,(\mathrm{m})\ 0.346410\,(\mathrm{m})\ 0.0\,(\mathrm{rad})]^T$, through the given trajectories

$$X_{o1}^d(t)=0.223205+0.5t^3-0.75t^4+0.3t^5\,(\mathrm{m}) \quad (43)$$

Table 1 Link parameters of a planar dual-arm robot and the object parameters

	arm-k ($k=1,2$) link i ($i=1,2,3$)	object
length (m)	0.2000	0.1000
mass (kg)	0.5000	5.0000
center of mass (m)	0.1000	0.0500
moment of inertia (kg·m²)	0.0015	0.5000

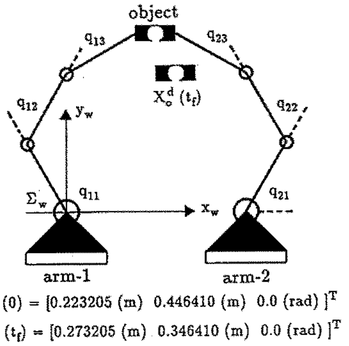

Fig. 6 Planar dual-arm robot grasping a common object: initial posture for the first example

$$X^d_{o2}(t)=0.446410-1.0t^3+1.5t^4-0.6t^5\,(\mathrm{m}) \tag{44}$$

$$X^d_{o3}(t)=0.0\,(\mathrm{rad}) \tag{45}$$

where the time duration t_f is set to 5.0 sec.

The simulation is composed by two stages. For each sampling time, firstly, the joint torque, $P_k(t)$, of each arm is calculated using the inverse dynamics algorithm described in the previous section. Then, using the forward dynamics algorithm, the joint position, $q_k(t)$, of each arm is computed. It should be noted that by differentiation, the object acceleration is computed directly from the given object trajectory. In the simulation, the sampling time Δt is set to 0.0001 sec.

The comparison between the desired values and the simulation results for the object position is shown in **Table 2**. Based on those results, the computation errors of the proposed method are plotted and shown in **Fig. 7**. The position error is calculated by Ref. 9).

$$E_p=\sqrt{(X_{o1}-X^d_{o1})^2+(X_{o2}-X^d_{o2})^2}\Big/\sum_{k=1}^{2}(L_{tk}+\|r_{ck}\|)$$
$$+\sqrt{(X_{o3}-X^d_{o3})^2}/2\pi \tag{46}$$

where L_{tk} is the total length of the k-th arm and "$\|\ \|$" denotes the metric norm.

As shown in Fig. 7, we can see that the position error of the proposed method are about in 10^{-6} order.

The effectiveness of the proposed method for various contact types is shown in the next example. Again, we simulated the cooperative motion of a planar dual-arm robot ($n_1=n_2=6$) grasping a common object. The contact type between the arm-1 and the object is the rigid grasping ($l_{c1}=3$) and between the arm-2 and the object is the point contact with friction type, i. e., the end-effector forces can be transmitted in any direction to the object, but the end-effector moment cannot be transmitted ($l_{c1}=2$). The link parameters of the dual-arm robot and the object parameters are the same as in Table 1, except that the length and the center of mass of the object are 0.18 (m) and 0.09 (m) respectively.

The initial posture is shown in **Fig. 8**, where $X_o(0)=[0.203225\,(m)\ 0.598174\,(m)\ 0.0\,(rad)]^T$. The task

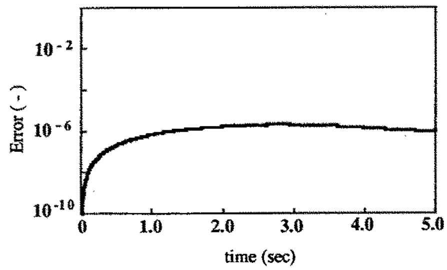

Fig. 7 Computational error of the proposed method

Table 2 Comparison between the desired object positions and the simulation results

time (s)	Desired object positions			Simulation results		
	$X^d_{o1}(t)$ (m)	$X^d_{o2}(t)$ (m)	$X^d_{o3}(t)$ (rad)	$X_{o1}(t)$ (m)	$X_{o2}(t)$ (m)	$X_{o3}(t)$ (rad)
0.0	0.2232050	0.4464100	0.0000000	0.2232050	0.4464100	0.0000000
1.0	0.2261010	0.4406180	0.0000000	0.2261014	0.4406173	0.0000000
2.0	0.2390770	0.4146660	0.0000000	0.2390779	0.4146645	0.0000000
3.0	0.2573330	0.3781540	0.0000000	0.2573339	0.3781540	0.0000000
4.0	0.2703090	0.3522020	0.0000000	0.2703094	0.3522022	0.0000000
5.0	0.2732050	0.3464100	0.0000000	0.2732050	0.3464113	0.0000000

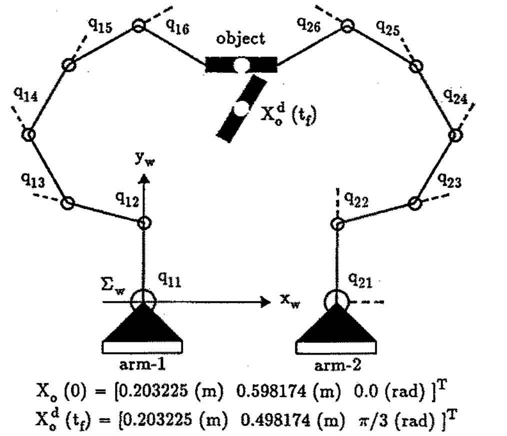

Fig. 8 Planar dual-arm robot grasping a common object: initial posture for the second example

objective is to move the object from its initial position to the desired final position $X_o^d(t_f)=[0.203225\,(m)\ 0.498174\,(m)\ \pi/3\,(rad)]^T$ and $t_f=5.0$ sec.

We choose a PD controller in the task space for positioning the object, where the joint torques of each arm are calculated using the following equations

$$P_k(t)=J_{trk}^T F_{trk} \tag{47}$$

$$F_{tr}(t)=G^+[K(X_o^d(t)-X_o(t))+B(\dot{X}_o^d(t)-\dot{X}_o)] \tag{48}$$

where $F_{tr}(t)=[F_{tr1}^T(t)\ F_{tr2}^T(t)]^T, G=[G_1\ G_2]$ and G_k; $(k=1, 2)$ is given in (30). $K\in R^{3\times3}$ and $B\in R^{3\times3}$ are the position and the velocity feedback gain matrices, given as $K=$diag. [100.0 (N/m) 100.0 (N/m) 100.0 (N. m/rad)] and $B=$diag. [10.0 (N. sec/m) 10.0 (N. sec/m) 10.0 (N. m. sec/rad)], where "diag. [ ]" denotes a diagonal matrix.

The desired trajectory of the object is given by

$$X_{o1}^d(t)=0.203225\,(\mathrm{m}) \tag{49}$$

$$X_{o2}^d(t)=0.598174-1.0t^3+1.5t^4-0.6t^5\,(\mathrm{m}) \tag{50}$$

$$X_{o3}^d(t)=10.0(\pi/3)t^3-5.0\pi t^4+2.0\pi t^5\,(\mathrm{rad}) \tag{51}$$

The sampling time Δt is set to 0.0001 sec.

Fig. 9 shows the stick pictures of the dual-arm robot under the PD controller, and **Table 3** shows the angle between the axes of the last link of each arm and the axes of the object (the contact-angle).

It can be seen that for the rigid grasping, the contact-angle between the end-effector of arm-1 and the object is constant. On the other hand, for the point contact with friction, since the end-effector's moment of arm-2 cannot be transmitted to the object, the end-effector can rotate freely. As a result, the contact-angle between the end-effector of arm-2 and the object changes during control.

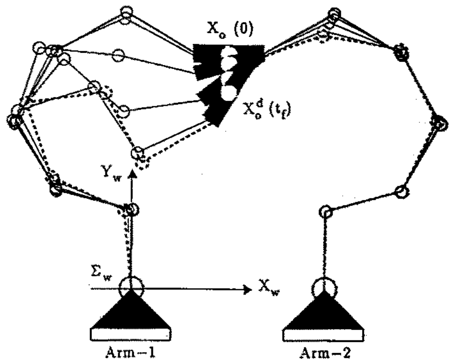

Fig. 9 Stick pictures of a planar dual-arm robot under the PD controller

Table 3 Angles between the axes of the last link of each arm and the axes of the object

time (s)	arm-1	arm-2
0.0	0.523599 (rad)	−0.523599 (rad)
1.0	0.523599 (rad)	−0.475667 (rad)
2.0	0.523599 (rad)	−0.210124 (rad)
3.0	0.523599 (rad)	0.223329 (rad)
4.0	0.523599 (rad)	0.578254 (rad)
5.0	0.523599 (rad)	0.706632 (rad)

6. Conclusion

We have proposed the dynamic simulation method of a multi-arm robot using Appel's method. It was shown that the dynamic equation for a multi-arm robot grasping a common object is formed by the dynamic equation of each arm, the motion equation of the object and the constraint equation.

The dynamic equation of each arm has a form identical to the dynamic equation of an open-loop single-arm with kinematic constraints on its end-effector, where the end-effector's constraint eqation is obtained from the object motion. By using the Appel's method, each arm of the multi-arm robot can be simulated in a parallel way, such that the parallel computation can be implemented. Also, The proposed method can be applied for various contact types between the end-effectors and the object, since the mechanism of the forces/moments transmission between the end-effectors and the object is taken into consideration.

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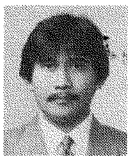

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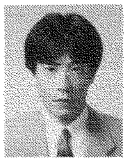

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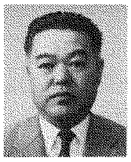

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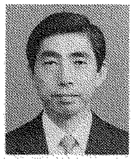

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