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Neural-Based Decentralized Robust Control of Large-Scale Uncertain Nonlinear Systems with Guaranteed H_∞ Performance

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***Abstract*— This paper investigates an application of Neural Networks (NNs) to the decentralized guaranteed H_∞ performance for a class of large-scale uncertain nonlinear systems. In order to guarantee the adequate H_∞ performance level for the nonlinear systems, nonlinear linear matrix inequality (NLMI) condition is derived. The linear matrix inequality (LMI) approach instead of the NLMI is used to construct the decentralized local state feedback controllers with additive gain perturbation. The novel contribution is that in order to avoid H_∞ performance degradation caused by the uncertainty, NNs are substituted into the additive gain perturbations. Although the NNs are included in the large-scale uncertain nonlinear systems, it is newly shown that the closed-loop system is internally stable and the adequate H_∞ performance bound is attained. Finally, a numerical example is given to verify the efficiency.**

I. Introduction

In recent years, the problem of the decentralized robust control of large-scale uncertain systems has been widely studied (see, for example, [1] and the references therein). When controlling such plant, it is desirable that the control systems guarantee not only a robust stability, but also an adequate level of performance. One approach to this problem is the so-called quadratic guaranteed cost control [2]. This approach has the advantage of providing an upper bound on a given performance index. Recent advance in theory of linear matrix inequality (LMI) has allowed a revisiting of the guaranteed cost control approach [3], [4] for the large-scale uncertain systems. Particularly, the robust non-fragile decentralized controller design for uncertain large-scale interconnected systems with time-delay has been established [3], [4]. However, the disturbance inputs have not been considered in these researches. In fact, in order to attain the desired performance against the external disturbance, the consideration of the disturbance inputs is important for implementing the actual control systems.

The nonlinear H_∞ control problem has been considered extensively [5], [6]. Particularly, L_2-gain analysis of nonlinear system has been tackled [5]. On the other hand, the solutions to the nonlinear H_∞ control problems were characterized in terms of the nonlinear matrix inequalities (NLMIs) [6]. However, in general, it is very hard to solve Hamilton-Jacobi inequality or NLMIs for obtaining the controller. Moreover, although these approaches have the advantage of providing an controller, the H_∞ performance degradation incurred by the uncertainty has not been considered.

Neural networks (NNs) have been utilized for an intelligent control system because NNs have nonlinear mapping approximation property. The numerous control methodologies utilizing NNs have been proposed by combining the modern control theory. For example, the linear quadratic regulator (LQR) problem using multiple NNs has been investigated [7]. However, there is a possibility that NNs may result in the unstable system because the stability of the closed-loop system which includes the neurocontroller has not been considered. It should be noted that the system stability may destroy when the degree of system nonlinearity is strong [7]. In order to avoid this problem, the stability of the closed-loop system with the neurocontroller has been considered [8], [9]. However, since much effort has been made towards finding the guaranteed cost controller with additive gain perturbations, the guaranteed H_∞ performance bound has not been discussed.

In this paper, the guaranteed H_∞ performance analysis and control synthesis of the decentralized robust control for the large-scale uncertain nonlinear systems with the neurocontroller is discussed. The crucial difference between the existing results [8], [9] and the proposed method is that the disturbance input is newly considered. As a result, the L_2-gain condition of the large-scale uncertain nonlinear systems is satisfied. The novel contributions are as follows. First, in order to guarantee the adequate H_∞ performance level, NLMI condition is established. Second, a class of the fixed state feedback controller of the large-scale uncertain nonlinear systems with the gain perturbations is derived by means of the LMI. As a result, since the LMI is used instead of the NLMI, it is easy to obtain the fixed gains. Finally, in order to compensate for the degradation of the given disturbance attenuation level caused by the parameter uncertainties, NNs are used. Although the neurocontrollers are included in the large-scale uncertain nonlinear systems, it is newly shown that the robust internal stability of the closed-loop system and the adequate H_∞ performance bound are both attained. In order to verify the effectiveness of our design approach, the numerical example is given.

The notations used in this paper are fairly standard. The superscript T denotes the matrix transpose. $I_n \in \Re^{n\times n}$ denotes the identity matrices. $\|z\|^2$ denotes the square norm of a vector $z \in \Re^k$. The notation $L_2(0,\ T)$ will be also used

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for vector-valued function [5].

II. Preliminary

Consider the continuous-time large-scale uncertain nonlinear systems that is a special kind of the multimachine power systems [1], which consist of N subsystems of the following form.

$$\begin{aligned}\dot{x}_i(t) &= [A_i + \Delta A_i(t)]\, x_i(t) + [B_i + \Delta B_i(t)]\, u_i(t) \\ &\quad + [G_i + \Delta G_i(t)]\, g_i(\boldsymbol{x}(t)) + H_i d_i(t), \qquad (1a)\\ u_i(t) &= [K_i + \Delta K_i(t)]\, x_i(t), \qquad (1b)\\ z_i(t) &= C_i x_i(t) + D_i u_i(t),\ i = 1,\ \ldots,\ N, \qquad (1c)\end{aligned}$$

where for the ith subsystem $x_i \in \Re^{n_i}$, $\boldsymbol{x}(t) := \begin{bmatrix} x_1^{\mathrm{T}}(t) & \cdots & x_N^{\mathrm{T}}(t) \end{bmatrix}^{\mathrm{T}} \in \Re^{\bar{n}}$ is the state, $u_i \in \Re^{m_i}$ is the input, $z_i \in \Re^{p_i}$ is the controlled output, $d_i \in \Re^{q_i}$ is the external disturbance, respectively. The matrices A_i, B_i, G_i, H_i, C_i and D_i are known real constant matrices of appropriate dimensions that describe the nominal model. K_i is the fixed gain matrix. $\Delta A_i(t)$, $\Delta B_i(t)$ and $\Delta G_i(t)$ are real time varying parameter uncertainties. $\Delta K_i(t)$ is the additive gain such as neural inputs. $g_i(x) \in \Re^{l_i}$ is unknown nonlinear vector functions that represent nonlinearity between the ith subsystem and the interactions of other subsystems.

The uncertain matrices $\Delta A_i(t)$, $\Delta B_i(t)$ and $\Delta G_i(t)$ and the additive gain $\Delta K_i(t)$ are assumed to be of the following structure:

$$\begin{aligned}\begin{bmatrix} \Delta A_i(t) & \Delta B_i(t) \end{bmatrix} &= L_i F_i(t) \begin{bmatrix} E_{A_i} & E_{B_i} \end{bmatrix}, \qquad (2a)\\ \Delta G_i(t) &= L_{G_i} F_{G_i}(t) E_{G_i}, \qquad (2b)\\ \Delta K_i(t) &= L_{K_i} N_i(t) E_{K_i} \qquad (2c)\end{aligned}$$

with $F_i(t) \in \Re^{r_i \times s_i}$, $F_{G_i}(t) \in \Re^{t_i \times l_i}$ and $N_i(t) \in \Re^{v_i \times w_i}$ are unknown matrix functions with Lebesgue measurable elements and satisfying

$$F_i^{\mathrm{T}}(t) F_i(t) \le I_{s_i},\ F_{G_i}^{\mathrm{T}}(t) F_{G_i}(t) \le I_{l_i},\ N_i^{\mathrm{T}}(t) N_i(t) \le I_{w_i},$$

where L_i, E_{A_i}, E_{B_i}, L_{G_i}, E_{G_i}, L_{K_i} and E_{K_i} are known real constant matrices with appropriate dimensions.

It may be noted that the systems (1) include the disturbance input compared with the existing results [1]. That is, the considered systems (1) are an extension of [1]. Without loss of generality, the following assumptions concerning the unknown nonlinear vector functions are made [1].

Assumption 1: There exist known constant matrices W_{ij} such that for all $x_j \in \Re^{n_j}$

$$\|g_i(\boldsymbol{x})\| \le \sum_{j=1}^{N} W_{ij} \|x_j\|, \qquad (3)$$

for all i, j and for all $t \ge 0$.

Let us introduce the following partitioned matrices.

$$\boldsymbol{u}(t) := \begin{bmatrix} u_1(t) \\ \vdots \\ u_N(t) \end{bmatrix} \in \Re^{\bar{m}},\ \boldsymbol{d}(t) := \begin{bmatrix} d_1(t) \\ \vdots \\ d_N(t) \end{bmatrix} \in \Re^{\bar{p}},$$

$$\boldsymbol{z}(t) := \begin{bmatrix} z_1(t) \\ \vdots \\ z_N(t) \end{bmatrix} \in \Re^{\bar{q}},\ \boldsymbol{g}(\boldsymbol{x}(t)) := \begin{bmatrix} g_1(\boldsymbol{x}) \\ \vdots \\ g_N(\boldsymbol{x}) \end{bmatrix} \in \Re^{\bar{l}},$$

$$\boldsymbol{A}_\Delta := \boldsymbol{A} + \boldsymbol{L}\boldsymbol{F}(t)\boldsymbol{E}_A,\ \boldsymbol{B}_\Delta := \boldsymbol{B} + \boldsymbol{L}\boldsymbol{F}(t)\boldsymbol{E}_B,$$

$$\boldsymbol{G}_\Delta := \boldsymbol{G} + \boldsymbol{L}_G\boldsymbol{F}_G(t)\boldsymbol{E}_G,\ \boldsymbol{K}_\Delta := \boldsymbol{K} + \boldsymbol{L}_K\boldsymbol{N}(t)\boldsymbol{E}_K,$$

$$\boldsymbol{A} := \begin{bmatrix} A_1 & & \boldsymbol{O} \\ & \ddots & \\ \boldsymbol{O} & & A_N \end{bmatrix},\ \boldsymbol{L} := \begin{bmatrix} L_1 & & \boldsymbol{O} \\ & \ddots & \\ \boldsymbol{O} & & L_N \end{bmatrix},$$

$$\boldsymbol{E}_A := \begin{bmatrix} E_{A_1} & & \boldsymbol{O} \\ & \ddots & \\ \boldsymbol{O} & & E_{A_N} \end{bmatrix},\ \boldsymbol{B} := \begin{bmatrix} B_1 & & \boldsymbol{O} \\ & \ddots & \\ \boldsymbol{O} & & B_N \end{bmatrix},$$

$$\boldsymbol{E}_B := \begin{bmatrix} E_{B_1} & & \boldsymbol{O} \\ & \ddots & \\ \boldsymbol{O} & & E_{B_N} \end{bmatrix},\ \boldsymbol{G} := \begin{bmatrix} G_1 & & \boldsymbol{O} \\ & \ddots & \\ \boldsymbol{O} & & G_N \end{bmatrix},$$

$$\boldsymbol{L}_G := \begin{bmatrix} L_{G_1} & & \boldsymbol{O} \\ & \ddots & \\ \boldsymbol{O} & & L_{G_N} \end{bmatrix},\ \boldsymbol{E}_G := \begin{bmatrix} E_{G_1} & & \boldsymbol{O} \\ & \ddots & \\ \boldsymbol{O} & & E_{G_N} \end{bmatrix},$$

$$\boldsymbol{H} := \begin{bmatrix} H_1 & & \boldsymbol{O} \\ & \ddots & \\ \boldsymbol{O} & & H_N \end{bmatrix},\ \boldsymbol{C} := \begin{bmatrix} C_1 & & \boldsymbol{O} \\ & \ddots & \\ \boldsymbol{O} & & C_N \end{bmatrix},$$

$$\boldsymbol{D} := \begin{bmatrix} D_1 & & \boldsymbol{O} \\ & \ddots & \\ \boldsymbol{O} & & D_N \end{bmatrix},\ \boldsymbol{K} := \begin{bmatrix} K_1 & & \boldsymbol{O} \\ & \ddots & \\ \boldsymbol{O} & & K_N \end{bmatrix},$$

$$\boldsymbol{L}_K := \begin{bmatrix} L_{K_1} & & \boldsymbol{O} \\ & \ddots & \\ \boldsymbol{O} & & L_{K_N} \end{bmatrix},\ \boldsymbol{E}_K := \begin{bmatrix} E_{K_1} & & \boldsymbol{O} \\ & \ddots & \\ \boldsymbol{O} & & E_{K_N} \end{bmatrix},$$

$$\boldsymbol{F}(t) := \begin{bmatrix} F_1(t) & & \boldsymbol{O} \\ & \ddots & \\ \boldsymbol{O} & & F_N(t) \end{bmatrix},$$

$$\boldsymbol{F}_G(t) := \begin{bmatrix} F_{G_1}(t) & & \boldsymbol{O} \\ & \ddots & \\ \boldsymbol{O} & & F_{G_N}(t) \end{bmatrix},$$

$$\boldsymbol{N}(t) := \begin{bmatrix} N_1(t) & & \boldsymbol{O} \\ & \ddots & \\ \boldsymbol{O} & & N_N(t) \end{bmatrix},\ \bar{n} := \sum_{i=1}^{N} n_i,\ \bar{m} := \sum_{i=1}^{N} m_i,$$

$$\bar{p} := \sum_{i=1}^{N} p_i,\ \bar{q} := \sum_{i=1}^{N} q_i,\ \bar{l}_i := \sum_{j=1,\ j\ne i}^{N} l_j,\ \bar{l} := \sum_{i=1}^{N} \bar{l}_i.$$

Using the above notations, the large-scale uncertain nonlinear systems (1) can be rewritten as

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}_\triangle \boldsymbol{x}(t) + \boldsymbol{B}_\triangle \boldsymbol{u}(t) + \boldsymbol{G}_\triangle \boldsymbol{g}(\boldsymbol{x}(t)) + \boldsymbol{H}\boldsymbol{d}(t), \quad (4a)$$

$$\boldsymbol{u}(t) = \boldsymbol{K}_\triangle \boldsymbol{x}(t), \quad (4b)$$

$$\boldsymbol{z}(t) = \boldsymbol{C}\boldsymbol{x}(t) + \boldsymbol{D}\boldsymbol{u}(t). \quad (4c)$$

It is assumed that the performance of large-scale uncertain nonlinear systems (4) is measured in term of L_2-gain in this paper.

It will be given the following definition of finite L_2-gain [5].

Definition 1: Let $\gamma \geq 0$. System (4) with initial state $\boldsymbol{x}(0) = 0$ is said to have L_2-gain less than or equal to γ if

$$\int_0^T \|\boldsymbol{z}(t)\|^2 dt \leq \gamma^2 \int_0^T \|\boldsymbol{d}(t)\|^2 dt, \quad (5)$$

for all $T \geq 0$ and $\boldsymbol{d}(t) \in L_2(0,\ T)$.

The following lemma will play an important role in solving the nonlinear state feedback H_∞ control problem [5].

Lemma 1: Consider the class of nonlinear system given by (6).

$$\dot{x}(t) = f(x) + g(x)d(t) \quad (6a)$$

$$z(t) = h(x), \quad f(x_0) = 0, \quad h(x_0) = 0, \quad (6b)$$

where $x \in \Re^n$, $d \in \Re^q$, $z \in \Re^p$ and $g(x) \in \Re^{n\times q}$.

Let $\gamma > 0$. If there exists a smooth solution $V \geq 0$ of Hamilton-Jacobi inequality

$$\frac{\partial V}{\partial x}(x)f(x) + \frac{1}{2}\frac{1}{\gamma^2}\frac{\partial V}{\partial x}(x)g(x)g^{\mathrm{T}}(x)\frac{\partial^{\mathrm{T}} V}{\partial x}(x) + \frac{1}{2}h^{\mathrm{T}}(x)h(x) \leq 0, \quad V(x_0) = 0, \quad (7)$$

then the nonlinear system (6) has L_2-gain less than or equal to γ.

The following theorem indicates the sufficient condition for existence of the H_∞ control synthesis for uncertain nonlinear systems.

Theorem 1: For a given continuous matrix-valued function $\boldsymbol{P}_i(\boldsymbol{x})$, suppose that the following nonlinear matrix inequality holds for the large-scale uncertain nonlinear systems (1)

$$\begin{bmatrix} \Phi_i & \boldsymbol{P}_i(\boldsymbol{x})\boldsymbol{G}_{\triangle i} \\ \boldsymbol{G}_{\triangle i}^{\mathrm{T}}\boldsymbol{P}_i(\boldsymbol{x}) & -I_{\bar{l}_i} \end{bmatrix} < 0, \ i = 1,\ \ldots,N, \quad (8)$$

where

$$\Phi_i = \boldsymbol{P}_i(\boldsymbol{x})\left(\boldsymbol{A}_{\triangle i} + \boldsymbol{B}_{\triangle i}\boldsymbol{K}_{\triangle i}\right) + \left(\boldsymbol{A}_{\triangle i} + \boldsymbol{B}_{\triangle i}\boldsymbol{K}_{\triangle i}\right)^{\mathrm{T}}\boldsymbol{P}_i(\boldsymbol{x}) + \frac{1}{\gamma^2}\boldsymbol{P}_i(\boldsymbol{x})\boldsymbol{H}_i\boldsymbol{H}_i^{\mathrm{T}}\boldsymbol{P}_i(\boldsymbol{x}) + \left(\boldsymbol{C}_i + \boldsymbol{D}_i\boldsymbol{K}_{\triangle i}\right)^{\mathrm{T}}\left(\boldsymbol{C}_i + \boldsymbol{D}_i\boldsymbol{K}_{\triangle i}\right) + W_i,$$

$$W_i = N\sum_{j=1}^{N} W_{ij}^{\mathrm{T}}W_{ij} > 0.$$

If such condition is met, the large-scale uncertain nonlinear systems (4) have L_2-gain less than or equal to γ.

In order to prove Theorem 1, the following inequality is needed.

$$N\sum_{j=1}^{N} x_j^{\mathrm{T}}W_{ij}^{\mathrm{T}}W_{ij}x_j \geq g_i^{\mathrm{T}}g_i. \quad (9)$$

It should be noted that it is easy to verify that the above inequality holds under Assumption 1. Now, let us prove Theorem 1.

Proof: Suppose now there exists the positive definite continuous matrix-valued function $\boldsymbol{P}(\boldsymbol{x})$ such that

$$\frac{\partial \boldsymbol{V}(\boldsymbol{x}(t))}{\partial \boldsymbol{x}(t)} = \boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}(\boldsymbol{x}), \quad (10)$$

where

$$\boldsymbol{P}(\boldsymbol{x}) = \begin{bmatrix} P_1(\boldsymbol{x}) & & \boldsymbol{O} \\ & \ddots & \\ \boldsymbol{O} & & P_N(\boldsymbol{x}) \end{bmatrix},$$

$$P_i(\boldsymbol{x}) > 0, \ i = 0,\ \ldots,N. \quad (11)$$

Comparing (4) and vector form of (6), it follows that

$$f(x) = \left(\boldsymbol{A}_\triangle + \boldsymbol{B}_\triangle\boldsymbol{K}_\triangle\right)\boldsymbol{x}(t) + \boldsymbol{G}_\triangle\boldsymbol{g}(\boldsymbol{x}(t)), \quad (12a)$$

$$g(x) = \boldsymbol{H}, \quad (12b)$$

$$h(x) = \left(\boldsymbol{C} + \boldsymbol{D}\boldsymbol{K}_\triangle\right)\boldsymbol{x}(t), \quad (12c)$$

$$\frac{\partial V}{\partial x}(x) = \boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}(\boldsymbol{x}). \quad (12d)$$

Applying (12) to (7), the following equation holds.

$$\begin{aligned}\mathcal{N} = &\frac{1}{2}\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}(\boldsymbol{x})\left[\left(\boldsymbol{A}_\triangle + \boldsymbol{B}_\triangle\boldsymbol{K}_\triangle\right)\boldsymbol{x}(t) + \boldsymbol{G}_\triangle\boldsymbol{g}(\boldsymbol{x}(t))\right] \\ &+\frac{1}{2}\left[\left(\boldsymbol{A}_\triangle + \boldsymbol{B}_\triangle\boldsymbol{K}_\triangle\right)\boldsymbol{x}(t) + \boldsymbol{G}_\triangle\boldsymbol{g}(\boldsymbol{x}(t))\right]^{\mathrm{T}}\boldsymbol{P}(\boldsymbol{x})\boldsymbol{x}(t) \\ &+\frac{1}{2}\frac{1}{\gamma^2}\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}(\boldsymbol{x})\boldsymbol{H}\boldsymbol{H}^{\mathrm{T}}\boldsymbol{P}(\boldsymbol{x})\boldsymbol{x}(t) \\ &+\frac{1}{2}\boldsymbol{x}^{\mathrm{T}}(t)\left(\boldsymbol{C} + \boldsymbol{D}\boldsymbol{K}_\triangle\right)^{\mathrm{T}}\left(\boldsymbol{C} + \boldsymbol{D}\boldsymbol{K}_\triangle\right)\boldsymbol{x}(t).\end{aligned}$$

Moreover, it is easy to verify the following equality.

$$\begin{aligned}\mathcal{M} &= N\sum_{i=1}^{N}\sum_{j=1}^{N}\left(x_j^{\mathrm{T}}W_{ij}^{\mathrm{T}}W_{ij}x_j - g_i^{\mathrm{T}}g_i\right) \\ &= \boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{W}\boldsymbol{x}(t) - \boldsymbol{g}^{\mathrm{T}}\boldsymbol{g} \geq 0.\end{aligned}$$

Adding the above equality to $\mathcal{N}$ and using (9), it follows that

$$\begin{aligned}\mathcal{N} &= \mathcal{N} + \frac{1}{2}(\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{W}\boldsymbol{x}(t) - \boldsymbol{g}^{\mathrm{T}}\boldsymbol{g} - \mathcal{M}) \\ &< \frac{1}{2}\begin{bmatrix}\boldsymbol{x}(t) \\ \boldsymbol{g}(\boldsymbol{x}(t))\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}\Phi & \boldsymbol{P}(\boldsymbol{x})\boldsymbol{G}_\triangle \\ \boldsymbol{G}_\triangle^{\mathrm{T}}\boldsymbol{P}(\boldsymbol{x}) & -I_{\bar{l}}\end{bmatrix}\begin{bmatrix}\boldsymbol{x}(t) \\ \boldsymbol{g}(\boldsymbol{x}(t))\end{bmatrix},\end{aligned} \quad (13)$$

where

$$\begin{aligned}\Phi = &\boldsymbol{P}(\boldsymbol{x})\left(\boldsymbol{A}_\triangle + \boldsymbol{B}_\triangle\boldsymbol{K}_\triangle\right) + \left(\boldsymbol{A}_\triangle + \boldsymbol{B}_\triangle\boldsymbol{K}_\triangle\right)^{\mathrm{T}}\boldsymbol{P}(\boldsymbol{x}) \\ &+\frac{1}{\gamma^2}\boldsymbol{P}(\boldsymbol{x})\boldsymbol{H}\boldsymbol{H}^{\mathrm{T}}\boldsymbol{P}(\boldsymbol{x}) \\ &+\left(\boldsymbol{C} + \boldsymbol{D}\boldsymbol{K}_\triangle\right)^{\mathrm{T}}\left(\boldsymbol{C} + \boldsymbol{D}\boldsymbol{K}_\triangle\right) + \boldsymbol{W}.\end{aligned}$$

Hence, the following nonlinear matrix inequality (14) results in $\mathcal{N} < 0$.

$$\begin{bmatrix} \Phi & \boldsymbol{P}(\boldsymbol{x})\boldsymbol{G}_\Delta \\ \boldsymbol{G}_\Delta^{\mathrm{T}}\boldsymbol{P}(\boldsymbol{x}) & -I_{\bar{l}} \end{bmatrix} < 0.$$

Next, the matrix inequality (14) is written by the following form

$$\begin{bmatrix} \Phi_1 & & \boldsymbol{O} & P_1(\boldsymbol{x})\boldsymbol{G}_{\Delta 1} & & \boldsymbol{O} \\ & \ddots & & & \ddots & \\ \boldsymbol{O} & & \Phi_N & \boldsymbol{O} & & P_N(\boldsymbol{x})\boldsymbol{G}_{\Delta N} \\ \boldsymbol{G}_{\Delta 1}^{\mathrm{T}}P_1(\boldsymbol{x}) & & \boldsymbol{O} & -I_{\bar{l}_1} & & \boldsymbol{O} \\ & \ddots & & & \ddots & \\ \boldsymbol{O} & & \boldsymbol{G}_{\Delta N}^{\mathrm{T}}P_N(\boldsymbol{x}) & \boldsymbol{O} & & -I_{\bar{l}_N} \end{bmatrix} < 0. \qquad (14)$$

Using Schur complement [10] for the equation (14), it is easy to verify that the nonlinear matrix inequality (14) holds iff the following reduced-order nonlinear matrix inequality (15) holds.

$$\begin{bmatrix} \Phi_i & P_i(\boldsymbol{x})\boldsymbol{G}_{\Delta i} \\ \boldsymbol{G}_{\Delta i}^{\mathrm{T}}P_i(\boldsymbol{x}) & -I_{\bar{l}_i} \end{bmatrix} < 0, \; i = 1, \; \dots, N. \qquad (15)$$

Therefore, since the condition (7) holds when the nonlinear matrix inequalities (8) are satisfied, the considered systems (4) have L_2-gain less than or equal to γ. The proof of Theorem 1 is completed. ∎

III. LMI Condition for the Existence of the Controller

In view of the previous section, the guaranteed H_∞ performance analysis involve solving the NLMIs (8). This property also implies that the complicated computational effort is needed. However, it is well-known that it is very hard to solve the NLMIs. In this section, the LMI condition will be established for the guaranteed H_∞ performance analysis and control synthesis instead of the NLMIs.

Theorem 2: Consider the large-scale uncertain nonlinear systems (1a) and additive gain perturbation (1b). Now suppose that there exist the matrices $X_i \in \Re^{n_i \times n_i}$, $Y_i \in \Re^{m_i \times n_i}$ and positive constants ε_{i1}, ε_{i2} and ε_{iK} satisfying the LMI (16) for all uncertain functions $F_i(t)$ and $F_{ij}(t)$, and the arbitrary function $N_i(t)$ as the neural input. Then, the fixed gain matrix $K_i = Y_i X_i^{-1}$ attains L_2-gain less than or equal to γ.

Proof: Let us introduce matrices $X_i = P_i^{-1}$ and $Y_i = K_i P_i^{-1}$. Since Theorem 2 is proved by using the Schur Complement [10], it is omitted (see, e.g., [9]). ∎

IV. Neural Networks for Additive Gain Perturbation

In general, the existence of the parameter uncertainties and the gain perturbations yield the performance degradation. The main purpose of this paper is to improve the degradation of H_∞ performance using NNs. It should be noted that the proposed neurocontroller regulates its outputs in real-time under the internal stability guaranteed by the LMI approach.

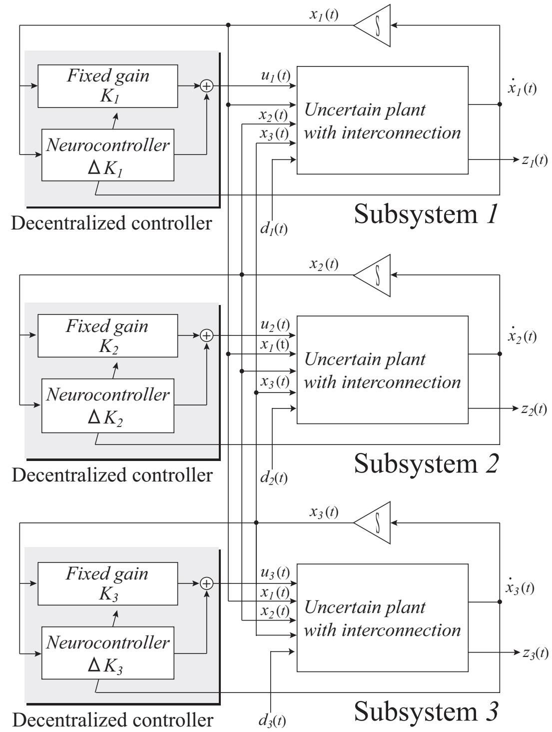

Fig. 1. Block diagram of proposed system composed of three-dimensional subsystems

The decentralized neurocontroller for the large-scale uncertain nonlinear systems is considered. As a specific example, the block diagram of the proposed control systems that have three-dimensional subsystems is given by Fig. 1. Fig. 1 shows that each neurocontroller uses only the state values of each subsystem as its input. It should be noted that this example is also used in the next section.

In order to calculate the control signal for the large-scale uncertain nonlinear systems, the equation (1) should be described to the discrete-time system. Thus, the continuous-time dynamics (1) is transformed to the following discrete-time system.

$$\begin{aligned} x_i(k+1) &= \left[\bar{A}_i + \Delta\bar{A}_i(k)\right]x_i(k) + \left[\bar{B}_i + \Delta\bar{B}_i(k)\right]u_i(k) \\ &\quad + \left[\bar{G}_i + \Delta\bar{G}_i(k)\right]g_i(x) + \bar{H}_i d_i(k), \qquad (16a) \\ u_i(k) &= [K_i + \Delta K_i(k)]x_i(k), \qquad (16b) \\ z_i(k) &= C_i x_i(k) + D_i u_i(k), \qquad (16c) \end{aligned}$$

where $\bar{A}_i := T_c A_i + I_{n_i}$, $\Delta\bar{A}_i(k) := T_c \Delta A_i(k)$, $\bar{B}_i := T_c B_i$, $\Delta\bar{B}_i(k) := T_c \Delta B_i(k)$, $\bar{G}_i := T_c G_i$, $\Delta\bar{G}_i(k) := T_c \Delta G_i(k)$, $\bar{H}_i := T_c H_i$ and $k \geq 0$ is number of step, $T_c \in \Re$ is a sufficient small sampling period in discrete-time systems.

It should be noted that Euler approximation is used as a discrete-time approximation to simplify the systems description. For each subsystem, the NNs should be trained in real-time so that the norm of the state discrepancy that is given by $\left\|x_i(k+1)\right\|$ between the operating point $x_i = 0$ and the present state value becomes as small as possible at each step k.

$$\begin{bmatrix} \Psi_i & G_i & H_i & \Theta_i^{\mathrm{T}} & \Xi_i^{\mathrm{T}} & 0 & X_i E_{K_i}^{\mathrm{T}} & X_i \\ G_i^{\mathrm{T}} & -I_{\bar{l}_i} & 0 & 0 & 0 & E_{G_i}^{\mathrm{T}} & 0 & 0 \\ H_i^{\mathrm{T}} & 0 & -\gamma^2 I_{q_i} & 0 & 0 & 0 & 0 & 0 \\ \Theta_i & 0 & 0 & -I_{p_i} + \varepsilon_{iK} D_i L_{K_i} L_{K_i}^{\mathrm{T}} D_i^{\mathrm{T}} & \varepsilon_{iK} D_i L_{K_i} L_{K_i}^{\mathrm{T}} E_{B_i}^{\mathrm{T}} & 0 & 0 & 0 \\ \Xi_i & 0 & 0 & \varepsilon_{iK} E_{B_i} L_{K_i} L_{K_i}^{\mathrm{T}} D_i^{\mathrm{T}} & -\varepsilon_{i1} I_{s_i} + \varepsilon_{iK} E_{B_i} L_{K_i} L_{K_i}^{\mathrm{T}} E_{B_i}^{\mathrm{T}} & 0 & 0 & 0 \\ 0 & E_{G_i} & 0 & 0 & 0 & -\varepsilon_{i2} I_{\bar{s}_i} & 0 & 0 \\ E_{K_i} X_i & 0 & 0 & 0 & 0 & 0 & -\varepsilon_{iK} I_{v_i} & 0 \\ X_i & 0 & 0 & 0 & 0 & 0 & 0 & -W_i^{-1} \end{bmatrix} < 0,$$

where $\Psi_i = A_i X_i + B_i Y_i + (A_i X_i + B_i Y_i)^{\mathrm{T}} + \varepsilon_{i1} L_i L_i^{\mathrm{T}} + \varepsilon_{i2} L_{G_i} L_{G_i}^{\mathrm{T}} + \varepsilon_{iK} B_i L_{K_i} L_{K_i}^{\mathrm{T}} B_i^{\mathrm{T}}$,

$\Theta_i = (C_i X_i + D_i Y_i) + \varepsilon_{iK} D_i L_{K_i} L_{K_i}^{\mathrm{T}} B_i^{\mathrm{T}}$, $\Xi_i = (E_{A_i} X_i + E_{B_i} Y_i) + \varepsilon_{iK} E_{B_i} L_{K_i} L_{K_i}^{\mathrm{T}} B_i^{\mathrm{T}}$, $\bar{s}_i = \sum_{j=1, j\neq i}^{N} s_{ij}$.

$N_i(k)$ of the equation (17) can be expressed as a nonlinear function of the state $x_i(k)$, the weight coefficient $w_i(k)$ of NN and the threshold $\theta_i(k)$ as follows.

$$N_i(k) = f\left(x_i(k), w_i(k), \theta_i(k)\right). \tag{17}$$

For each subsystem, an energy function $\mathcal{E}_i(k)$ is defined as the square norm of the state discrepancy. At each step, the weight coefficients are modified so as to minimize $\mathcal{E}_i(k)$ given by

$$\mathcal{E}_i(k) := \frac{1}{2} x_i^{\mathrm{T}}(k+1) x_i(k+1) = \frac{1}{2} \left\| x_i(k+1) \right\|^2. \tag{18}$$

$\mathcal{E}_i(k)$ can be calculated by using the observed state value $x_i(k+1)$. Therefore, it is not necessary to consider the behavior of the uncertain matrices $F_i(k)$ and $F_{ij}(k)$. If $\mathcal{E}_i(k)$ can be minimized as small as possible for each subsystem, the discrepancy $\left\| x_i(k+1) \right\|^2$ would also be minimized.

In the learning of NN, the modification of weight coefficient $\Delta w_i(k)$ is given by

$$w_i(k+1) = w_i(k) - \eta_i \frac{\partial \mathcal{E}_i(k)}{\partial w_i(k)}, \tag{19a}$$

$$\frac{\partial \mathcal{E}_i(k)}{\partial w_i(k)} = \frac{\partial \mathcal{E}_i(k)}{\partial N_i(k)} \frac{\partial N_i(k)}{\partial w_i(k)}, \tag{19b}$$

where $\eta_i,\ i = 1,\ 2,\ 3$ is the learning ratio. The term $\frac{\partial \mathcal{E}_i(k)}{\partial N_i(k)}$ in the equation (19b) can be calculated from the energy function (18) as follows.

$$\frac{\partial \mathcal{E}_i(k)}{\partial N_i(k)} = x_i(k+1)[\bar{B}_i + T_c L_i F_i(k) E_{B_i}] L_{K_i} E_{K_i} x_i(k). \tag{20}$$

It should be noted that since $F_i(k)$ is the unknown matrix function, (20) that is used to learn for the NN can not be calculated. To remove this problem, suppose there exists the parameter $\alpha(k)$ such that $\bar{B}_i + T_c L_i F_i(k) E_{B_i} \approx \alpha_i(k) \bar{B}_i$, where $\alpha_i(k)$ is the matrix value function and its elements are positive scalar. Then, the above equation (20) can be rewritten as follows.

$$\frac{\partial \mathcal{E}_i(k)}{\partial N_i(k)} \approx \alpha(k) x_i(k+1) \bar{B}_i L_{K_i} E_{K_i} x_i(k). \tag{21}$$

However, since there exists the functions $\alpha_i(k)$, it is difficult to decide the learning rule of the NN. Then, the modification

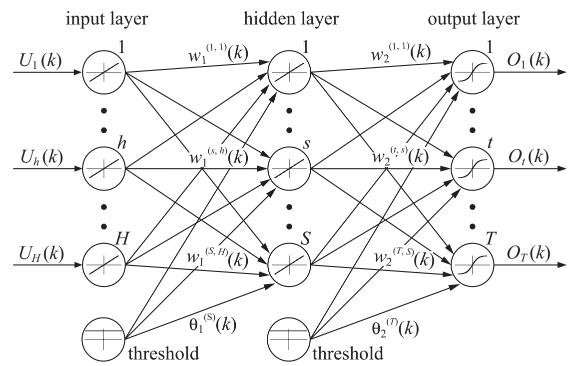

Fig. 2. Structure of the multilayered neural networks.

of the weight coefficient of the equation (19b) is newly defined as follows.

$$\Delta w_i(k) \approx -\mu_i x_i(k+1) \bar{B}_i L_{K_i} E_{K_i} x_i(k) \frac{\partial N_i(k)}{\partial w_i(k)}, \tag{22}$$

where $\mu_i := \eta_i \alpha_i(k)$ is defined as a new learning ratio. ε_i is used instead of deciding η_i according to $\alpha_i(k)$. On the other hand, $\frac{\partial N_i(k)}{\partial w_i(k)}$ can be calculated using the chain rule on the NN. As a result, using (20), NN can be trained so as to decrease the guaranteed H_∞ performance on-line.

The utilized NN are of a three-layer feed-forward network as shown in Fig. 2. A linear function is utilized in the neurons of the input and the hidden layers, and a sigmoid function in the output layer. For each subsystem i, inputs and outputs of each layer can be described as follows.

$$s_{iq}^y(k) = \begin{cases} U_i^y(k) & \{q = 1(\text{input layer})\} \\ w_{i1}^{(y,z)}(k) o_{i1}^z(k) & \{q = 2(\text{hidden layer})\} \\ w_{i2}^{(y,z)}(k) o_{i2}^z(k) & \{q = 3(\text{output layer})\} \end{cases}$$

$$o_{iq}^y(k) = \begin{cases} s_{i1}^y(k) & \{q = 1(\text{input layer})\} \\ s_{i2}^y(k) + \theta_{i1}^y(k) & \{q = 2(\text{hidden layer})\} \\ \frac{1 - e^{(-s_{i3}^y(k) + \theta_{i2}^y(k))}}{1 + e^{(-s_{i3}^y(k) + \theta_{i2}^y(k))}} & \{q = 3(\text{output layer})\} \end{cases}$$

where $s_{iq}^y(k)$ and $o_{iq}^y(k)$ are the input and output of neuron y in the qth layer at step k, $w_{iq}^{(y,z)}(k)$ indicates the weight coefficient from neuron z in the qth layer to neuron y in the $(q+1)$th layer, $U_i^y(k)$ is the input of NN, $\theta_{iq}^y(k)$ is a positive constant for the threshold of neuron y in the $(q+1)$th layer. As the additive gain perturbations defined in the formula (1b), the outputs of NN are chosen adaptively in the range of $[-1.0,\ 1.0]$.

V. Numerical Example

In order to demonstrate the efficiency of the proposed algorithm, a numerical example is given. Consider the interconnected uncertain nonlinear systems (1) composed of three two-dimensional subsystems. The system matrices and the nonlinear functions with the uncertainties are given as follows.

$$A_1 = \begin{bmatrix} -0.2 & 1 \\ -1 & -0.5 \end{bmatrix},\ B_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix},\ L_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix},$$
$$E_{A_1} = \begin{bmatrix} 1 & 1 \end{bmatrix},\ E_{B_1} = 1,\ G_1 = \begin{bmatrix} -0.1 & 0.05 \\ -0.1 & 0.09 \end{bmatrix},$$
$$L_{G_1} = \begin{bmatrix} 0.2 \\ 0.1 \end{bmatrix},\ E_{G_1} = \begin{bmatrix} 0.2 & 0.2 \end{bmatrix},\ L_{K_1} = \begin{bmatrix} 0.8 & 0.8 \end{bmatrix},$$
$$E_{K_1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ H_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix},\ C_1 = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix},\ D_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} -0.5 & -1 \\ 2 & -1 \end{bmatrix},\ B_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix},\ L_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix},$$
$$E_{A_2} = \begin{bmatrix} 1 & 1 \end{bmatrix},\ E_{B_2} = 0.7,\ G_2 = \begin{bmatrix} 0.2 & 0 \\ 0.01 & 0.03 \end{bmatrix},$$
$$L_{G_2} = \begin{bmatrix} 1 \\ 1 \end{bmatrix},\ E_{G_2} = \begin{bmatrix} 0.04 & 0.05 \end{bmatrix},\ L_{K_2} = \begin{bmatrix} 0.7 & 0.7 \end{bmatrix},$$
$$E_{K_2} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ H_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix},\ C_2 = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix},\ D_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$
$$A_3 = \begin{bmatrix} -1.8 & -1.1 \\ 0.63 & -1.91 \end{bmatrix},\ B_3 = \begin{bmatrix} 1 \\ 1 \end{bmatrix},\ L_3 = \begin{bmatrix} 0.8 \\ 0.7 \end{bmatrix},$$
$$E_{A_3} = \begin{bmatrix} 0.4 & 0.7 \end{bmatrix},\ E_{B_3} = 1,\ G_3 = \begin{bmatrix} 0.3 & -0.1 \\ -0.2 & 0.1 \end{bmatrix},$$
$$L_{G_3} = \begin{bmatrix} 0.3 \\ 0.5 \end{bmatrix},\ E_{G_3} = \begin{bmatrix} 0.6 & 0.01 \end{bmatrix},\ L_{K_3} = \begin{bmatrix} 0.6 & 0.6 \end{bmatrix},$$
$$E_{K_3} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ H_3 = \begin{bmatrix} 1 \\ 1 \end{bmatrix},\ C_3 = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix},\ D_3 = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$
$$x_i(t) = \begin{bmatrix} x_{i1}(t) \\ x_{i2}(t) \end{bmatrix},\ N_i(t) = \begin{bmatrix} N_{i1}(t) & 0 \\ 0 & N_{i2}(t) \end{bmatrix},$$
$$F_1(t) = F_{G_2}(t) = \sin\left(\frac{2\pi}{5}t\right),\ F_{G_1}(t) = F_3(t) = 1,$$
$$F_2(t) = F_{G_3}(t) = \cos\left(\frac{2\pi}{5}t\right),\ W_{11} = W_{22} = W_{33} = 1,$$
$$W_{12} = W_{13} = W_{21} = W_{23} = W_{31} = W_{32} = 0.5,$$
$$g_i(\boldsymbol{x}) = \begin{bmatrix} W_{i1}\sin\left(\begin{bmatrix} 1 & 0 \end{bmatrix} x_1\right) + \cdots + W_{iN}\sin\left(\begin{bmatrix} 1 & 0 \end{bmatrix} x_N\right) \\ W_{i1}\sin\left(\begin{bmatrix} 0 & 1 \end{bmatrix} x_1\right) + \cdots + W_{iN}\sin\left(\begin{bmatrix} 0 & 1 \end{bmatrix} x_N\right) \end{bmatrix},$$
$$x_1(0) = x_2(0) = x_3(0) = \begin{bmatrix} 0 & 0 \end{bmatrix}^{\mathrm{T}}.$$

The unknown functions $g_i(x)$ satisfy

$$\|g_i(\boldsymbol{x})\| = W_{i1}\|x_1\| + \cdots + W_{iN}\|x_N\|.$$

The three basic quantities for the subsystem are $\gamma_1 = 7.221892$, $\gamma_2 = 1.733101$ and $\gamma_3 = 1.665494$ respectively. Thus, for every boundary value $\gamma > \bar{\gamma} = \max\{\gamma_1,\ \gamma_2,\ \gamma_3\} = 7.221892$, the reduced-order LMIs (16) have the solutions.

The disturbance attenuation level is chosen as $\gamma = 10.0$. In this case, the fixed state feedback gain $\boldsymbol{K}$ that is based on the proposed LMI (16) is given by

$$K_1 = \begin{bmatrix} -3.5278 & -1.6955 \end{bmatrix},$$
$$K_2 = \begin{bmatrix} -2.0991 & -1.9808 \end{bmatrix},$$
$$K_3 = \begin{bmatrix} -1.3974 & -1.5805 \end{bmatrix}.$$

In order to compare the obtained result, a different gain is obtained. The interconnected nonlinear systems that ignore the uncertainty are given as follows.

$$\dot{\tilde{\boldsymbol{x}}}(t) = \boldsymbol{A}\tilde{\boldsymbol{x}}(t) + \boldsymbol{B}\tilde{\boldsymbol{u}}(t) + \boldsymbol{G}\boldsymbol{g}(\tilde{\boldsymbol{x}}(t)) + \boldsymbol{H}\boldsymbol{d}(t), \tag{23a}$$
$$\tilde{\boldsymbol{u}}(t) = \tilde{\boldsymbol{K}}\tilde{\boldsymbol{x}}(t), \tag{23b}$$
$$\tilde{\boldsymbol{z}}(t) = \boldsymbol{C}\tilde{\boldsymbol{x}}(t) + \boldsymbol{D}\tilde{\boldsymbol{u}}(t), \tag{23c}$$

where

$$\tilde{\boldsymbol{x}}(t) := \begin{bmatrix} \tilde{x}_1(t) \\ \tilde{x}_2(t) \\ \tilde{x}_3(t) \end{bmatrix} \in \Re^{\bar{n}},\ \tilde{\boldsymbol{u}}(t) := \begin{bmatrix} \tilde{u}_1(t) \\ \tilde{u}_2(t) \\ \tilde{u}_3(t) \end{bmatrix} \in \Re^{\bar{m}},$$
$$\tilde{\boldsymbol{z}}(t) := \begin{bmatrix} \tilde{z}_1(t) \\ \tilde{z}_2(t) \\ \tilde{z}_3(t) \end{bmatrix} \in \Re^{\bar{q}},\ \tilde{\boldsymbol{K}} = \begin{bmatrix} \tilde{K}_1 & & \boldsymbol{O} \\ & \tilde{K}_2 & \\ \boldsymbol{O} & & \tilde{K}_3 \end{bmatrix}.$$

Using the similar technique used in Theorem 2, the state feedback gain $\tilde{\boldsymbol{K}}$ by means of LMI that is based on (24) is given below.

$$\tilde{K}_1 = \begin{bmatrix} -2.8715 & -1.2738 \end{bmatrix},$$
$$\tilde{K}_2 = \begin{bmatrix} -1.9310 & -1.8354 \end{bmatrix},$$
$$\tilde{K}_3 = \begin{bmatrix} -1.1571 & -1.2989 \end{bmatrix},$$

where $\tilde{K}_i := \tilde{Y}_i\tilde{X}_i^{-1}$, $\tilde{\Phi}_i = A_i\tilde{X}_i + B_i\tilde{Y}_i + (A_i\tilde{X}_i + B_i\tilde{Y}_i)^{\mathrm{T}}$,

$$\begin{bmatrix} \tilde{\Phi}_i & G_i & H_i & (C_i\tilde{X}_i + D_i\tilde{Y}_i)^{\mathrm{T}} & \tilde{X}_i \\ G_i^{\mathrm{T}} & -I_{\bar{l}_i} & 0 & 0 & 0 \\ H_i^{\mathrm{T}} & 0 & -\gamma^2 I_{q_i} & 0 & 0 \\ C_i\tilde{X}_i + D_i\tilde{Y}_i & 0 & 0 & -I_{p_i} & 0 \\ \tilde{X}_i & 0 & 0 & 0 & -W_i^{-1} \end{bmatrix} < 0. \tag{24}$$

For each subsystem, the neurocontroller is composed of 30 neurons in the hidden layer. On the other hand, there exist two neurons in the input and the output layers, respectively. The state variables of each subsystem are used as the NN's inputs and the learning ratio $\mu_1 = 8.0$, $\mu_2 = 6.0$ and $\mu_3 = 6.0$. The initial weights are randomly chosen in the range of $[\ -0.05,\ 0.05\ ]$. Moreover, the external disturbances are chosen as follows.

$$d_1(t) = \sin\left(\frac{2\pi}{3}t\right),\ d_2(t) = \sin\left(\frac{2\pi}{6}t\right),\ d_3(t) = \sin\left(\frac{2\pi}{9}t\right).$$

The simulation results are shown in Fig. 3-5. It is easy to verify that the influence of the external disturbance to the

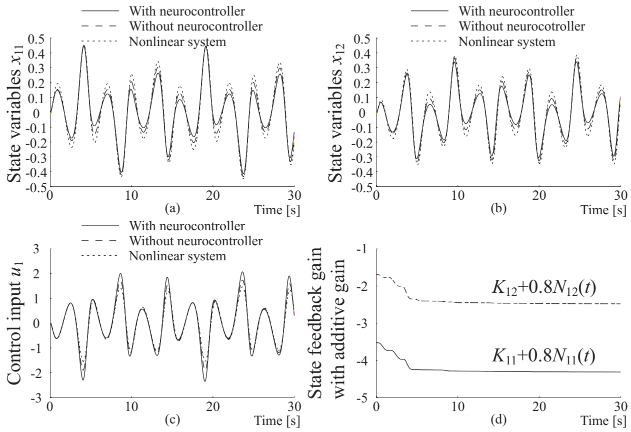

Fig. 3. Simulation results of subsystem 1.

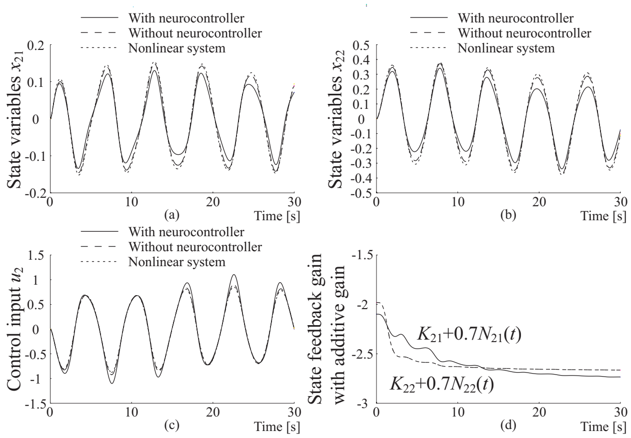

Fig. 4. Simulation results of subsystem 2.

proposed system that is based on the NLMI is smaller than that of the large-scale systems using the gain $\tilde{\boldsymbol{K}}$. Moreover, the disturbance attenuation of the proposed neurocontroller is observed. Therefore, it is shown that the proposed NLMI method with the neurocontroller result in the effectiveness on the suppression of the disturbance.

Finally, it is easy to verify that L_2-gain of the proposed closed-loop system is 1.176006 that is less than $\gamma = 10.0$. Thus, it is shown that the proposed method satisfy the boundary of L_2-gain. That is, it is emphasized that the desired disturbance attenuation level can be attained.

VI. Conclusions

In this paper, the guaranteed H_∞ performance analysis and control synthesis of the large-scale uncertain nonlinear systems with the neurocontroller has been studied. Using the NLMIs approach, the sufficient condition that is related to the existence of the guaranteed H_∞ controller for the large-scale uncertain nonlinear systems with the additive gain perturbations has been derived. Moreover, the class of the fixed state feedback controller has been newly established by means of the LMIs. As another important feature, the robust stability of the closed-loop system is guaranteed even

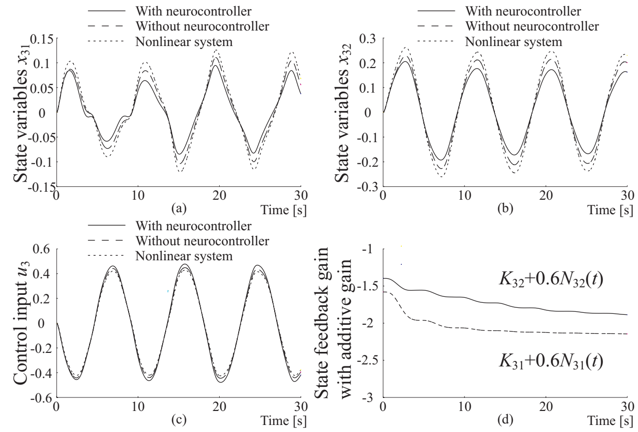

Fig. 5. Simulation results of subsystem 3.

if the systems include NNs. Moreover, the guaranteed H_∞ control synthesis with NNs has succeeded in reducing the degradation of the given disturbance attenuation level that is caused by the parameter uncertainties. Finally, the numerical example have shown the excellent result.

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