Proceedings of the 2006 American Control Conference
Minneapolis, Minnesota, USA, June 14-16, 2006

LMI-Based Neurocontroller for Guaranteed Cost Control of Uncertain Servo System

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Abstract— **This paper considers the robust tracking control problem that is based on the guaranteed cost control approach with additive gain perturbation. Based on the Linear Matrix Inequality (LMI) design approach, a class of a state feedback controller is established and some sufficient conditions for the existence of guaranteed cost controller are derived. A novel contribution is that the guaranteed cost control is applied to uncertain servo system for the first time. As a result, it is shown that the robust stability for uncertain servo system and the reduction of the cost performance are both attained.**

I. INTRODUCTION

To achieve the precise tracking control in a servo system, the accurate modeling of the system is required. In general, however, the modeling error remains as uncertainty and would be caused not only reduction of the tracking performance but also the destruction of the system stability. For that, the problem of the robust control for the uncertain system has been actively investigated [1], [2]. As one of the efficient method, the guaranteed cost control approach [2] is well known. This method has the advantage that an upper bound of the performance index can be obtained. Recently, the Linear Matrix Inequality (LMI) approach has been applied to the guaranteed cost control problem [3]. Although this approach will be asymptotically stable for the closed-loop uncertain system, the upper bound of the performance index becomes quite large due to its uncertainty.

On the other hand, a Neural Network (NN) has been widely exploited to construct an intelligent control system because of its nonlinear mapping approximation ability for the uncertain systems. The adaptive controllers using NNs to identify the system dynamics have been designed within the framework of the adaptive control theory [4]. Both state and output feedback neural regulators for the nonlinear plants have been tackled [5]. As another important studies, the linear quadratic regulator (LQR) problem using the multiple NNs has been investigated [6], [7]. In these approaches, one neural network is dedicated to the forward model for identifying the uncertainties of the controlled plant and the other network may compensate for the influence of the uncertainties that is based on the trained forward model. However, since in these researches the stability of the closed-loop system which includes the neurocontroller has not been considered, it may cause instability of the system. In order to avoid this disadvantage, the stability of the closed-loop system with the neurocontroller has been studied via the LMI-based design approach [8]. However, the application for the servo systems with the additive gain perturbations has not been considered. It should be noted that it is hard to construct the controller for them directly because the controller gain is time-variant.

In this paper, the guaranteed cost control problem of the uncertain discrete-time servo system is discussed. Particularly, the robot manipulator system as an example of the uncertain servo system is investigated. The main contribution is that the tracking problem with the additive gain perturbations can be solved. This control objective is attained by transforming the overall uncertain continuous-time tracking error-dynamics into the discrete-time one. Firstly, a class of a fixed state feedback controller of the uncertain discrete-time servo system with the gain perturbations is established. Secondly, some sufficient conditions to design the guaranteed cost controller is introduced by means of the LMI. The reduction of the large cost caused by the guaranteed cost control is attained by using NN. As a result, although the neurocontroller is included in the uncertain discrete-time servo system, the robust stability of the closed-loop system and the reduction of the cost are both attained. It is worth pointing out that the guaranteed cost control with the additive gain perturbations is applied to the uncertain servo system for the first time. This result seems to be novelty. Finally, in order to demonstrate the efficiency of our design approach, the numerical example that is based on robot manipulator system is given.

II. PROBLEM FORMULATION

The motion equation of the robotic manipulator with n-joints is given by [9]

$$M(\theta)\ddot{\theta} + h(\theta, \dot{\theta}) + g(\theta) = \tau, \tag{1}$$

where $\theta,\ \dot{\theta},\ \ddot{\theta} \in \Re^n$ are the vectors of joint positions, velocities and accelerations, $M(\theta) \in \Re^{n\times n}$ is the matrix of the moment inertia, $h(\theta, \dot{\theta}) \in \Re^n$ is the vector of the centripetal and Coriolis forces, $g(\theta) \in \Re^n$ is the vector of gravitational force and $\tau \in \Re^n$ is the vector of joint torques.

Then, let us define the state vector.

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix}. \tag{2}$$

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The motion equation (1) is transferred into the following form:

$$\dot{x} = \left[\begin{array}{c} x_2 \\ -M^{-1}(x_1)D(x_1, x_2) \end{array} \right] + \left[\begin{array}{c} 0 \\ M^{-1}(x_1) \end{array} \right] \tau, \quad (3)$$

where $D(x_1, x_2) = h(x_1, x_2) + g(x_1)$. This equation represents a typical nonlinear dynamical system with an affine nonlinear input function.

Now, using the desired angle θ_d, let us define

$$\tilde{\theta} = \theta - \theta_d = x_1 - \theta_d \quad (4)$$

as the joint position error.

If this joint position error becomes as small as possible, the real trajectory of the robot manipulator can be more precise. In order to perform the tracking task precisely, we use the following control law:

$$\tau = M(x_1) \left(\ddot{\theta}_d - K_1 \tilde{\theta} - K_2 \dot{\tilde{\theta}} + u_0 \right) + D_0(x_1, x_2), \quad (5)$$

where $D_0(x_1, x_2)$ is the nominal estimate of $D(x_1, x_2)$. The control parameters K_1, K_2 are specified as the diagonal matrices to be designed and u_0 is an auxiliary control signal.

Without loss of generality, the following assumption is made [10].

Assumption 1: There exists a scalar nonnegative function $\mathcal{E}(x_1, x_2)$ such that $\mathcal{E}(0, 0) = 0$ and $\|D_0(x_1, x_2) - D(x_1, x_2)\| \le \mathcal{E}(x_1, x_2)$.

Using (4) and (5), the equation (3) is rewritten

$$\dot{x} = \left[\begin{array}{c} x_2 \\ M^{-1}(x_1) \left[D_0(x_1, x_2) - D(x_1, x_2) \right] + v \end{array} \right], \quad (6)$$

where

$$v = \ddot{\theta}_d - K_1 \tilde{\theta} - K_2 \dot{\tilde{\theta}} + u_0. \quad (7)$$

In order to obtain the linearized form, let us define the system uncertainty as follows.

$$f(t) = M^{-1}(x_1) \left[D_0(x_1, x_2) - D(x_1, x_2) \right]. \quad (8)$$

Using Assumption 1, it is assumed that the system uncertainty $f(t)$ satisfies

$$f^T(t) f(t) \le e^T(t) W^T W e(t), \quad (9)$$

where W is a known constant matrix satisfying $W^T W > 0$. If this constant $W \in \Re^{r \times n}$ becomes large, it can consider higher levels of uncertainty.

Finally, the following simple form can be obtained [9].

$$\dot{x} = \left[\begin{array}{c} x_2 \\ v + f(t) \end{array} \right]. \quad (10)$$

Using (4) and (10), the tracking error dynamics of robot manipulator system can be expressed.

$$\dot{e}(t) = A_c e(t) + B_c u(t) + B_c f(t), \quad (11)$$

where

$$A_c = \left[\begin{array}{cc} 0_{n \times n} & I_n \\ -K_1 & -K_2 \end{array} \right], B_c = \left[\begin{array}{c} 0_{n \times n} \\ I_n \end{array} \right],$$
$$e(t) = \left[\begin{array}{c} \tilde{\theta} \\ \dot{\tilde{\theta}} \end{array} \right], \ u(t) = u_0. \quad (12)$$

Using above formulation, the continuous-time tracking error equation can be obtained. In order to calculate the control signal for the practical plant such as robot manipulator systems, the error equation should be described to the discrete-time. Thus, the continuous-time tracking error dynamics (11) is transformed to the following discrete-time system.

$$e(k+1) = Ae(k) + Bu(k) + Bf(k), \quad (13)$$

where

$$A := I_n + A_c T, \ B := B_c T$$

and $k \ge 0$ is the number of step, T is the sufficient small sampling period in the discrete-time systems.

It should be noted that Euler approximation is used as the discrete-time approximation to simplify the systems description.

III. AN LMI BASED DESIGN APPROACH

The tracking error dynamics of uncertain robot manipulator is given by (13). If the robot manipulator system is not influenced by the uncertainty $f(k)$ (i.e., $f(k) = 0$), the tracking error $e(k)$ will converge to zero as $k \to \infty$. In general, however, most of the error dynamics has the uncertainty. Therefore, the guaranteed cost approach via the additive gain perturbation that is based on the neurocontroller is considered. In this method, we define a control input as

$$u(k) = [K + \Delta K(k)] e(k), \quad (14)$$

where

$$\Delta K(k) = D_k N(k) E_k, \quad (15)$$

and D_k and E_k are known constant matrices, and $N(k) \in \Re^{p \times q}$ is arbitrary function. It is assumed that $N(k)$ satisfies the following condition

$$N^T(k) N(k) \le I_q. \quad (16)$$

Associated with the system (13) is the quadratic cost function

$$J = \sum_{k=0}^{\infty} \left[e^T(k) Q e(k) + u^T(k) R u(k) \right], \quad (17)$$

where Q and R are the positive definite symmetric matrices.

In this situation, the definition of the guaranteed cost control with the additive gain is given below.

Definition 1: For the uncertain discrete-time system (13) and the cost function (17), if there exist a fixed control gain matrix K and a positive scalar J^* such that for the admissible uncertainties and the additive gain perturbation (15), the closed-loop systems is asymptotically stable and the closed-loop value of the cost function (17) satisfies $J < J^*$, then J^* and K are said to be the guaranteed cost and the guaranteed cost control matrix, respectively.

The above definition is very popular for dealing with the time-varying uncertainties and is also used in [2]. The following theorem gives the sufficient condition for the existence of the guaranteed cost control.

Theorem 1: Suppose that the following matrix inequality holds for the uncertain discrete-time system (13)

$$\Phi(P,\ K) = \begin{bmatrix} \tilde{A}^T P \tilde{A} - P + W^T W + Q + \tilde{K}^T R \tilde{K} & \tilde{A}^T P B \\ B^T P \tilde{A} & B^T P B - I_m \end{bmatrix} < 0, \quad (18)$$

where $P > 0$ is the positive symmetric matrix and

$$\tilde{A} = A + B(K + \Delta K),\ \tilde{K} = K + \Delta K.$$

If such condition is satisfied, the matrix K of the controller (14) is the guaranteed control gain matrix associated with the cost function (17). Then, the closed-loop uncertain system

$$e(k+1) = [A + B(K + \Delta K)]\, e(k) + B f(k) \quad (19)$$

is asymptotically stable and achieves

$$J < J^* = e^T(0) P e(0). \quad (20)$$

Proof: Suppose now there exist the symmetric positive definite matrices $P > 0$ such that the matrix inequality (18) holds for all admissible uncertainties (9) and the additive control input (15). Let us define the following Lyapunov function candidate

$$V(e(k)) = e^T(k) P e(k), \quad (21)$$

where $V(e(k))$ satisfies $V(e(k)) > 0$ for all $e(k) \neq 0$.

The corresponding difference along any trajectory of the closed-loop system (19) is given by

$$\begin{aligned} \Delta V(e(k)) &= V(e(k+1)) - V(e(k)) \\ &= e^T(k+1) P e(k+1) - e^T(k) P e(k) \\ &= \Xi^T(k) P \Xi(k) - e^T(k) P e(k), \end{aligned} \quad (22)$$

where

$$\Xi(k) = \left[\{A + B\tilde{K}\} e(k) + B f(k) \right]. \quad (23)$$

Furthermore, the inequality (22) is rewritten as the following form.

$$\begin{aligned} &\Delta V(e(k)) \\ &= \Xi^T(k) P \Xi(k) - e^T(k) P e(k) \\ &\quad + \left\{ e^T(k) W^T W e(k) - f^T(k) f(k) \right\} \\ &\quad - \left\{ e^T(k) W^T W e(k) - f^T(k) f(k) \right\} \\ &= \begin{bmatrix} e(k) \\ f(k) \end{bmatrix}^T \begin{bmatrix} \tilde{A}^T P \tilde{A} - P + W^T W & \tilde{A}^T P B \\ B^T P \tilde{A} & B^T P B - I_m \end{bmatrix} \begin{bmatrix} e(k) \\ f(k) \end{bmatrix} \\ &\quad - \left\{ e^T(k) W^T W e(k) - f^T(k) f(k) \right\}. \end{aligned} \quad (24)$$

Using the assumption (9), we have

$$e^T(k) W^T W e(k) - f^T(k) f(k) > 0. \quad (25)$$

Hence, it follows from (24) immediately that

$$\begin{aligned} &\Delta V(e(k)) \\ &< \begin{bmatrix} e(k) \\ f(k) \end{bmatrix}^T \begin{bmatrix} \tilde{A}^T P \tilde{A} - P + W^T W & \tilde{A}^T P B \\ B^T P \tilde{A} & B^T P B - I_m \end{bmatrix} \begin{bmatrix} e(k) \\ f(k) \end{bmatrix}. \end{aligned} \quad (26)$$

Summing $e^T(k) \left[Q + \tilde{K}^T R \tilde{K} \right] e(k)$ to both sides of (26) results in

$$\begin{aligned} &\Delta V(e(k)) + e^T(k) \left[Q + \tilde{K}^T R \tilde{K} \right] e(k) \\ &< \begin{bmatrix} e(k) \\ f(k) \end{bmatrix}^T \Phi(P,\ K) \begin{bmatrix} e(k) \\ f(k) \end{bmatrix}. \end{aligned} \quad (27)$$

Taking $\Phi(P,\ K) < 0$ into account from the assumption, it follows that

$$\Delta V(e(k)) < -e^T(k) \left[Q + \tilde{K}^T R \tilde{K} \right] e(k) < 0 \quad (28)$$

because $e^T(k) \left[Q + \tilde{K}^T R \tilde{K} \right] e(k) > 0$ for all $e(k)$.

Therefore, the closed-loop system (19) is asymptotically stable. Furthermore, by summing both sides of the inequality (28) from 0 to $N-1$ and using the initial conditions, we have

$$\begin{aligned} &V(e(N)) - V(e(0)) \\ &< -\sum_{k=0}^{N} \left[e^T(k) Q e(k) + u^T(k) R u(k) \right]. \end{aligned} \quad (29)$$

Since the closed-loop system (19) is asymptotically stable, that is, $e(N) \to 0$, when $N \to \infty$, we obtain $V(e(N)) \to 0$. Thus we have

$$\begin{aligned} J &= \sum_{k=0}^{\infty} \left[e^T(k) Q e(k) + u^T(k) R u(k) \right] \\ &< V(e(0)) = e^T(0) P e(0). \end{aligned} \quad (30)$$

The proof of Theorem 1 is completed. ■

The objective of this section is to design the fixed guaranteed cost control gain matrix K for the uncertain system (13) with the cost function (17) via the LMI design approach.

Theorem 2: Consider the uncertain discrete-time system (13) and the cost function (17). Now we assume that there exist the matrices $X \in \Re^{n \times n}$, $Y \in \Re^{m \times n}$ and the positive constant ε_k satisfying LMI (31) for all the arbitrary function $N(k)$ as the neural input. Then, the fixed gain matrix $K = Y X^{-1}$ is the guaranteed cost control gain matrix.

Proof: Since the results of Theorem 2 can be proved by using the similar argument of the proof in [8], it is omitted. ■

Since the LMI (31) consists of a convex solution set of $(\varepsilon_k,\ X,\ Y)$, various efficient convex optimization algorithm can be applied. Moreover, its solutions represent a set of the guaranteed cost control gain matrix K. This parameterized representation can be exploited to design the guaranteed cost control gain which minimizes the value of the guaranteed cost for the closed-loop uncertain system. Consequently, solving the following optimization problem allows us to determine the optimal bound.

$$J < J^* < \min_{(\varepsilon_k,\ X,\ Y)} \alpha, \quad (32)$$

such that (31) and

$$\begin{bmatrix} -\alpha & e^T(0) \\ e(0) & -X \end{bmatrix} < 0. \quad (33)$$

$$\begin{bmatrix} -X+\varepsilon_k BD_kD_k^TB^T & AX+BY & B & \varepsilon_k BD_kD_k^T & 0 & 0 & 0 \\ (AX+BY)^T & -X & 0 & Y^T & XE_k^T & X & X \\ B^T & 0 & -I_m & 0 & 0 & 0 & 0 \\ \varepsilon_k D_kD_k^TB^T & Y & 0 & -R^{-1}+\varepsilon_k D_kD_k^T & 0 & 0 & 0 \\ 0 & E_kX & 0 & 0 & -\varepsilon_k I_q & 0 & 0 \\ 0 & X & 0 & 0 & 0 & -Q^{-1} & 0 \\ 0 & X & 0 & 0 & 0 & 0 & -(W^TW)^{-1} \end{bmatrix} < 0. \tag{31}$$

The problem addressed in this section is defined as follows:

Problem 1: Find the guaranteed cost control gain $K = YX^{-1}$ satisfying the LMIs (31) and (33) to make the cost α become as small as possible.

Since the bound in *Problem 1* depends on the initial condition $e(0)$, it is assumed to remove such condition that $e(0)$ is a zero mean random variable satisfying $E[e(0)e^T(0)] = I_n$.

Then, the LMI (33) yields

$$\begin{bmatrix} -M & I_n \\ I_n & -X \end{bmatrix} < 0, \tag{34}$$

where $E[\cdot]$ denotes the expectation, M is the expectation of α. In this paper, the condition (34) is used instead of (33) in the optimization problem and M would be gotten as small as possible.

In the next section we will discuss about the additive gain perturbation as the neural input.

IV. ADDITIVE GAIN PERTURBATION USING NEURAL NETWORK

The LMI design approach usually yields the conservative controller due to the presence of the additive gain perturbation. In order to reduce the cost bound, the NN as the additive gain perturbation is considered.

A. On-line learning algorithm of neurocontroller

It is expected that the reduction of the cost will be attained if the neurocontroller could be trained that the error dynamics including uncertainty converges to zero rapidly as possible. That is, the neurocontroller is used to improve the transient response.

$$E(k) := \frac{1}{2}e^T(k+1)e(k+1), \tag{35}$$

where the tracking error $e(k)$ is defined in the previous section as (13).

If $E(k)$ can be reduced as small as possible, the tracking error $e(k)$ would become small. That is, robot manipulator will follow the desired trajectory more precisely.

In the learning phase of NN, the weight updating rules can be described as

$$w_g^{ij}(k+1) = w_g^{ij}(k) + \Delta w_g^{ij}(k). \tag{36}$$

On the other hand, the modification of the weight coefficient $w_g^{ij}(k)$ is given by

$$\Delta w_g^{ij}(k) = -\eta \frac{\partial E(k)}{\partial w_g^{ij}(k)}, \tag{37a}$$

$$\frac{\partial E(k)}{\partial w_g^{ij}(k)} = \frac{\partial E(k)}{\partial N(k)} \cdot \frac{\partial N(k)}{\partial w_g^{ij}(k)}, \tag{37b}$$

where η is the learning ratio.

The term $\dfrac{\partial E(k)}{\partial N(k)}$ of the equation (37b) can be calculated from the energy function (35) as follows:

$$\frac{\partial E(k)}{\partial N(k)} = e(k+1)BD_kE_ke(k). \tag{38}$$

Using (38), the NN can be trained so as to decrease the cost J on-line. However, it should be noted that the reduction in the time taken for the system to settle down may result in increasing the computational complexity because of the introduction of Neural Network

B. Multilayered Neural networks

The utilized NN is of a three-layer feed-forward network as shown in Fig. 1. The linear function is utilized in the neurons of the input and the hidden layers, and a sigmoid function in the output layer. The inputs and outputs of each layer can be described as follows.

$$s_g^i(k) := \begin{cases} U_i(k) & \{g = 1(\text{input layer})\} \\ \sum w_1^{(i,j)}(k)o_1^j(k) & \{g = 2(\text{hidden layer})\} \\ \sum w_2^{(i,j)}(k)o_2^j(k) & \{g = 3(\text{output layer})\}, \end{cases}$$

$$o_g^i(k) := \begin{cases} s_1^i(k) & \{g = 1(\text{input layer})\} \\ s_2^i(k) + \theta_1^{(i)}(k) & \{g = 2(\text{hidden layer})\} \\ \dfrac{1 - e^{(-s_3^i(k)+\theta_2^{(i)}(k))}}{1 + e^{(-s_3^i(k)+\theta_2^{(i)}(k))}} & \{g = 3(\text{output layer})\}, \end{cases}$$

where $s_g^i(k)$ and $o_g^i(k)$ are the input and the output of the neuron i in the gth layer at the step k. $w_g^{i,j}(k)$ indicates the weight coefficient from the neuron j in the gth layer to the neuron i in the $(g+1)$th layer. $U_i(k)$ is the input of NN. $\theta_g^{(i)}(k)$ is a positive constant for the threshold of the neuron i in the $(g+1)$th layer. As the additive gain perturbations defined in the formula (14), the outputs of NN are set in the range of $[-1.0,\ 1.0]$.

Finally, it should be noted that the inputs of NN are $e(k+1)$ of (13) and the outputs of NN are $N(k)$ of (15).

V. A NUMERICAL EXAMPLE

In order to verify the effectiveness of proposed method, a numerical example is given. Consider a two-link robot manipulator in Fig. 2 with the system parameters as: link mass $m_1 = 10$[kg], $m_2 = 4$[kg], lengths $l_1 = l_2 = 0.2$[m], joint positions θ_1[rad], θ_2[rad].

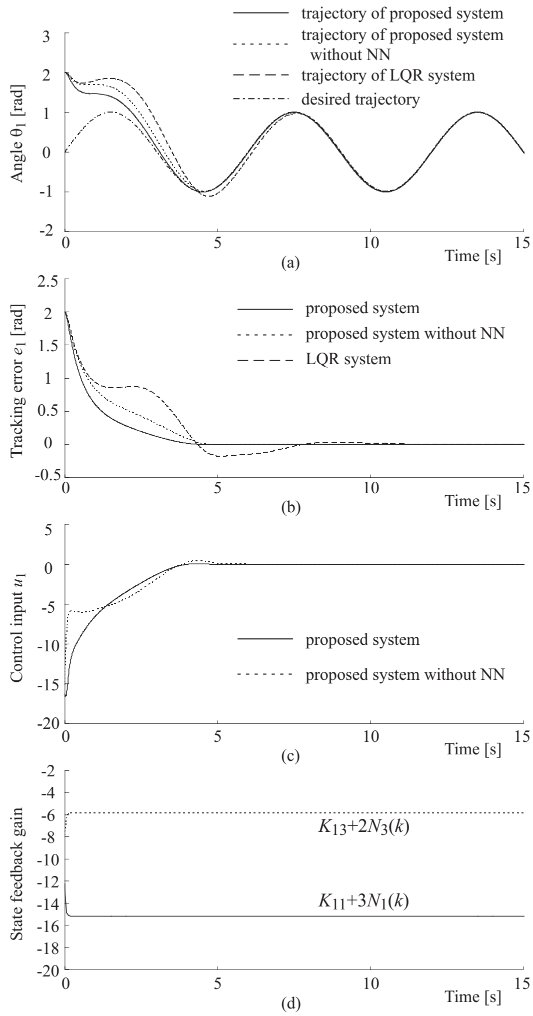

Fig. 3. Simulation results of neurocontroller and without neurocontroller. (a) Angle θ_1. (b) Tracking error. $e_1 = \theta_1 - \theta_{1d}$. (c) Control input $u_1(k)$. (d) State feedback gain with additive gain.

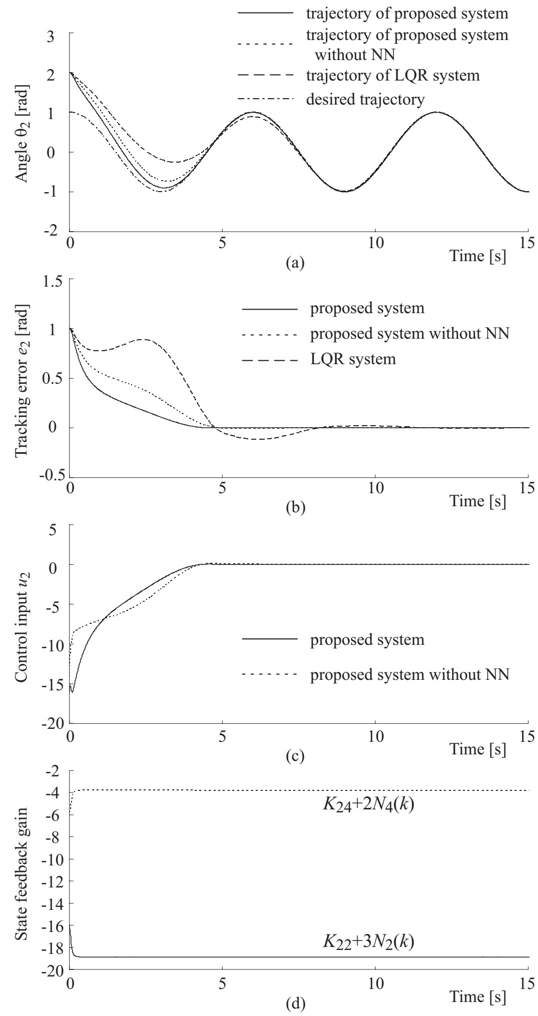

Fig. 4. Simulation results of neurocontroller and without neurocontroller. (a) Angle θ_2. (b) Tracking error. $e_2 = \theta_2 - \theta_{2d}$. (c) Control input $u_2(k)$. (d) State feedback gain with additive gain.

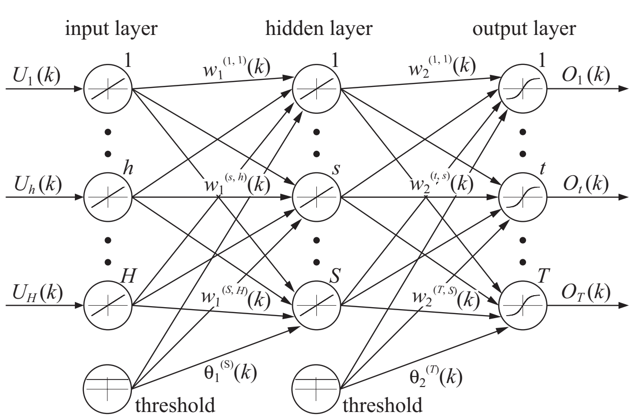

Fig. 1. Structure of the multilayered neural networks

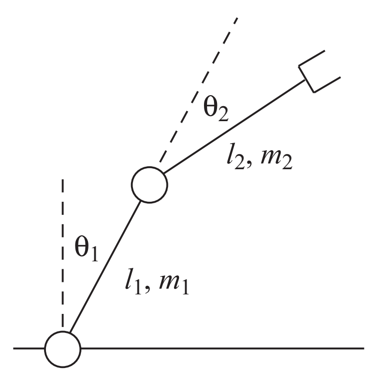

Fig. 2. The two-link robotic manipulator.

The system matrices are given as follows, where $T = 0.01$.

$$A = \begin{bmatrix} 1.00 & 0 & 0.01 & 0 \\ 0 & 1.00 & 0 & 0.01 \\ -0.09 & 0 & 0.90 & 0 \\ 0 & -0.07 & 0 & 0.86 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0.01 & 0 \\ 0 & 0.01 \end{bmatrix}, \; D_k = \begin{bmatrix} 3 & 1 & 2 & 1 \\ 1 & 3 & 1 & 2 \end{bmatrix}, \; E_k = 1,$$

$$N(k) = \begin{bmatrix} N_1(k) & 0 & 0 & 0 \\ 0 & N_2(k) & 0 & 0 \\ 0 & 0 & N_3(k) & 0 \\ 0 & 0 & 0 & N_4(k) \end{bmatrix}.$$

It should be noted that $N_1(k)$, $N_2(k)$, $N_3(k)$ and $N_4(k)$ are the output of NN. The initial conditions are $\theta_1(0) = 2$, $\theta_2(0) = 2$, $\dot{\theta}_1(0) = \dot{\theta}_2(0) = 0$. The weighting matrices are $Q = \mathrm{diag}(100,\ 100,\ 10,\ 10)$ and $R = \mathrm{diag}(1,\ 1)$, respectively. The desired reference trajectories are

$$\theta_{1d} = \sin\left(\frac{2\pi}{600}k\right),\ \theta_{2d} = \cos\left(\frac{2\pi}{600}k\right),$$

where $e(0) = [2\ 1\ {-1}\ 0]^T$.

The system uncertainty is assumed to be a following form.

$$f(k) = 10\|e(k)\| \begin{bmatrix} \sin\left(\dfrac{2\pi}{700}k\right) \\ \dfrac{1}{2}\sin\left(\dfrac{2\pi}{350}k\right) \end{bmatrix}. \tag{39}$$

Therefore, we choose as $W = \mathrm{diag}(W_1,\ W_2,\ W_3,\ W_4) = \mathrm{diag}(10,\ 10,\ 10,\ 10)$.

Using the LMI (31), the fixed control gain matrix K is given by

$$K = \begin{bmatrix} -12.228 & 0.0002 & -7.8245 & 0.0004 \\ 0.0004 & -15.991 & 0.0004 & -5.7138 \end{bmatrix}.$$

In order to verify the obtained results, the proposed method is compared with the LQR approach that does not consider the system uncertainty $f(k)$. The control gain matrix $\hat{K}$ that is based on the LQR approach is given by

$$\hat{u}(k) = \hat{K}e(k), \tag{40}$$

where

$$\hat{K} = \begin{bmatrix} -4.3907 & 0 & -0.8852 & 0 \\ 0 & -5.1614 & 0 & -0.6873 \end{bmatrix}.$$

On the other hand, the control gain matrix $\bar{K}$ without the additive gain perturbation is given by

$$\bar{u}(k) = \bar{K}e(k), \tag{41}$$

where

$$\bar{K} = \begin{bmatrix} -10.797 & 0 & -6.3802 & 0 \\ 0 & -14.101 & 0 & -4.6773 \end{bmatrix}.$$

The neurocontroller consists of four neurons in the input and the output layers and 30 neurons in the hidden layers, respectively. The initial weights are set randomly in the range of $[-0.05, 0.05]$.

The simulation results are shown in Fig. 3 and Fig. 4. The response of the proposed system is stabilized faster than other systems such as the system without the neurocontroller and the LQR system. This result means that the higher tracking performance of the proposed system was obtained. Moreover, the comparison of the cost function is shown in Table I. It is shown that the cost of the proposed system is the smallest than that of the system without the neurocontroller and the LQR system. These results show that our proposed method can attain not only good tracking performance but also the reduction of the cost for the uncertain servo system.

TABLE I

COMPARISON TO VALUE OF COST FUNCTION BY DIFFERENCE OF CONTROL GAIN

	With NN($\eta = 0.8$)	Without NN	LQR
Cost	55604	57267	76487

VI. CONCLUSIONS

The application of the guaranteed cost control problem for the uncertain discrete-time servo system has been investigated. The novel contribution is that the guaranteed cost control has been applied to the robot manipulator system as an example of the uncertain servo system for the first time. Using LMI approach, the class of the state feedback gain has been derived. Substituting the neurocontroller into the additive gain perturbation, the robust stability of the closed-loop system is guaranteed even if the servo system include NN. Furthermore, the reduction of the cost is attained by using neurocontroller. The simulation result has shown that the robust stability and the adequate cost bound of the uncertain servo system has been attained.

REFERENCES

[1] K. Glover and J. C. Doyle, "State-Space Formulae for All Stabilizing Controllers that Satisfy an H_∞-norm Bound and Relations to Risk Sensitivity," *Systems and Control Letters*, vol. 11, 1988, pp. 167-172.

[2] I. R. Petersen, and D. C. McFralane, "Optimal guaranteed cost control and filtering for uncertain linear systems," *IEEE Trans. Automatic Control*, vol. 39, 1994, pp. 1971-1977.

[3] L. Xie, and Y. C. Soh, "Guaranteed cost control of uncertain discrete-time systems," *Control Theory and Advanced Technology.*, vol. 10, 1995, pp. 1235-1250.

[4] K. S. Narendra, and K. Parthasarathy, "Identification and control of dynamical systems using neural networks," *IEEE Trans. Neural Networks*, vol. 1, no. 1, 1990, pp. 4-27.

[5] M. S. Ahmed, M. A. Al-Dajani, "Neural regulator design", *Neural Networks*, vol. 11, 1998, pp. 1695–1709.

[6] Y. Iiguni, H. Sakai, and H. Tokumatsu, "A nonlinear regulator design in the presence of system uncertainties using multilayered neural networks," *IEEE Trans. Neural Networks*, vol. 2, no. 4, 1991, pp. 410-417.

[7] Y. Iiguni, "A robust neurocontroller incorporating a priori knowledge of plant dynamics," *Mathl. Comput. Modeling.*, vol. 23, 1996, pp. 143-157.

[8] H. Mukaidani, Y. Ishii, Y. Tanaka, N. Bu, and Toshio Tsuji, "LMI based neurocontroller for guaranteed cost control of discrete-time uncertain system," *Proc. 43rd IEEE Conf. Decision and Control*, pp. 809-814 Bahamas, Dec. 2004.

[9] B. S.Chen, H. J. Uang, and C. S. Tseng, "Robust Tracking Enhancement of Robot Systems Including Motor Dynamics : A Fuzzy-Based Dynamic Game Approach ," *IEEE Trans. Fuzzy Systems*, vol. 6, no. 4, 1998, pp. 538-552.

[10] F. Lin and R. D. Brandt, "An optimal control approach to robust control of robot manipulators," *IEEE Trans. Robotics and Automation*, vol. 14, no. 1, 1998, pp. 69-77.