DECENTRALIZED GUARANTEED COST CONTROL FOR DISCRETE–TIME UNCERTAIN LARGE–SCALE SYSTEMS USING NEURAL NETWORKS

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Abstract: This paper investigates an application of neural networks to the guaranteed cost control problem of decentralized robust control for a class of discrete–time uncertain large–scale systems. Based on the Linear Matrix Inequality (LMI) design approach, a class of decentralized local state feedback controllers with additive gain perturbations is established. The novel contribution of this paper is that to reduce the large cost caused by the LMI conditions Neural Networks (NNs) are substituted for the additive gain perturbations. Although the NNs are included in the uncertain large–scale systems, the closed–loop system is asymptotically stable. Furthermore, it is shown that the closed–loop cost function is not more than the specified upper bound for all admissible uncertainties.

Keywords: Large–scale systems, Discrete–time systems, Uncertain linear systems, Decentralized control, Robust control, Neural networks

1. INTRODUCTION

In recent years, the decentralized robust control of large–scale systems has been intensively studied (Shu *et al.*, 1982; LEE *et al.*, 1988; Wang *et al.*, 1997). Compared with the centralized control approaches, the decentralized control approaches have been known to be efficient in cases where such approaches are applied to the large–scale dynamic systems.

When controlling a real plant, it is desirable that the control systems guarantee not only a robust stability, but also an adequate level of performance. One approach to this problem is the so–called quadratic guaranteed cost control (Petersen and McFarlane, 1994). Recently, the theory of the Linear Matrix Inequality (LMI) has allowed advancement in the guaranteed cost control. The LMI–based guaranteed cost stabilization for the uncertainty–free large–scale systems with time delay has been discussed in Park (2004a). However, due to the presence of the parameter uncertainties, it is well known that the cost performance becomes quite large.

Neural networks (NNs) have been utilized for an intelligent control system because NNs have nonlinear mapping approximation property. Then some control methodologies utilizing NNs have been proposed by combining with modern control approaches. For example, a decentralized controller using NNs which identify the unknown parameters for a class of large–scale nonlinear systems was studied (Alessandri *et al.*, 1997; Huang *et al.*, 2003). The linear quadratic regulator (LQR) problem using multiple NNs has been investigated (Iiguni *et al.*, 1991). However, in these researches, there is a possibility that NNs may cause the system unstable, because the stability of the closed–loop system which includes the neurocontroller has not been considered. For example, it has been shown that the system stability is destroyed when the degree of system nonlinearity is strong (Iiguni *et al.*, 1991). In order to avoid this problem, the stability of the closed–loop system with the neurocontroller was studied (Ishii *et al.*, 2004; Mukaidani *et al.*, 2004b). However, these researches have not been investigated for the decentralized control of large–scale systems.

In this paper, the decentralized guaranteed cost control problem of the discrete–time uncertain large–scale systems with the neurocontroller is discussed. The crucial difference between the method in Park (2004b) and the proposed method is that the decomposition of the optimization based on the LMI is newly considered and the neurocontroller is substituted for the additive gain perturbations. Our contributions are as follows. Firstly, a class of the fixed state feedback controller of the discrete–time uncertain large–scale systems with the gain perturbations is derived. Secondly, some sufficient conditions to design the decentralized guaranteed cost controller are newly established by means of the LMI. Finally, in order to reduce the large cost caused by the parameter uncertainties, NNs are used. As a result, although the neurocontrollers are included in the discrete–time uncertain large–scale systems, it is newly shown that the robust stability of the closed–loop system and the reduction of the cost are attained.

2. PRELIMINARY

Consider discrete–time uncertain large–scale interconnected systems, which consist of N subsystems of the form.

$$x_i(k+1) = (A_i + \Delta A_i(k))x_i(k) + B_i u_i(k) + \sum_{j=1,\ j\neq i}^{N} (A_{ij} + \Delta A_{ij}(k))\, x_j(k), \quad (1a)$$

$$u_i(k) = (K_i + \Delta K_i(k))x_i(k), \quad (1b)$$

where $x_i(k) \in \Re^{n_i}$ and $u_i(k) \in \Re^{m_i}$ are the state and control input of the ith subsystem $i = 1, \ldots, N$, respectively. A_i, B_i and A_{ij} are constant matrices of appropriate dimensions and A_{ij} are interconnected matrices between the ith subsystems and other subsystems, and $K_i \in \Re^{m_i \times n_i}$ is the fixed gain matrix for the controller (1b). $\Delta A_i(k)$ and $\Delta A_{ij}(k)$ are the parameter uncertainties, and $\Delta K_i(k)$ is the neurocontroller. The parameter uncertainties and the neurocontroller considered here are assumed to be of the following form

$$\begin{aligned}&[\Delta A_i(k)\ \ \Delta A_{ij}(k)\ \ \Delta K_i(k)]\\ =&[D_{ai}F_{ai}(k)E_{ai}\ D_{aij}F_{aij}(k)E_{aij}\ D_{ki}N_i(k)E_{ki}]\,,\end{aligned} \quad (2)$$

where $D_{ai}, E_{ai}, D_{aij}, E_{aij}, D_{ki}$ and E_{ki} are known constant real matrices of appropriate dimensions, and $F_{ai}(k) \in \Re^{g_i \times h_i}$ and $F_{aij}(k) \in \Re^{p_i \times r_i}$ are unknown matrix functions, $N_i(k) \in \Re^{s_i \times t_i}$ is the output of NN. It is assumed that $F_{ai}(k), F_{aij}(k)$ and $N_i(k)$ satisfy

$$\begin{aligned}&F_{ai}^T(k)F_{ai}(k) \le I_{h_i},\ \ F_{aij}^T(k)F_{aij}(k) \le I_{r_i},\\ &N_i^T(k)N_i(k) \le I_{t_i}.\end{aligned} \quad (3)$$

Associated with the system (1) is the cost function

$$J = \sum_{i=1}^{N}\left(\sum_{k=0}^{\infty}(x_i^T(k)Q_i x_i(k) + u_i^T(k)R_i u_i(k))\right), \quad (4)$$

where $Q_i \in \Re^{n_i \times n_i}$ and $R_i \in \Re^{m_i \times m_i}$ are given by the positive definite symmetric matrices.

In this situation, the definition of the guaranteed cost control for the uncertain large–scale systems (1) and the cost function (4) is given below.

Definition 1: For the discrete–time uncertain large–scale systems (1) and the cost function (4), if there exist a control gain matrix K_i and positive scalar J^* such that for the admissible uncertainties and neurocontroller (2), the closed–loop system is asymptotically stable and the closed–loop value of the cost function (4) satisfies $J < J^*$, then J^* and K_i are said to be a guaranteed cost and a guaranteed cost control gain matrix, respectively.

The following theorem gives the sufficient condition for existence of the guaranteed cost control.

Theorem 1: Consider the uncertain large–scale interconnected systems (1) with uncertainties and neurocontroller (2). Suppose that the matrix inequality (5) has solutions such as symmetric positive definite matrices $P_i \in \Re^{n_i \times n_i}$ for all matrices $F_{ai}(k)$, $F_{aij}(k)$ and $N_i(k)$.

$$\mathcal{M}_i = \begin{bmatrix} \tilde{A}_i^T P_i \tilde{A}_i + \Theta_i & \tilde{A}_i^T P_i \tilde{A}_{i1} & \cdots & \tilde{A}_i^T P_i \tilde{A}_{iN} \\ \tilde{A}_{i1}^T P_i \tilde{A}_i & \tilde{A}_{i1}^T P_i \tilde{A}_{i1} - I_{n1} & \cdots & \tilde{A}_{i1}^T P_i \tilde{A}_{iN} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{A}_{iN}^T P_i \tilde{A}_i & \tilde{A}_{iN}^T P_i \tilde{A}_{i1} & \cdots & \tilde{A}_{iN}^T P_i \tilde{A}_{iN} - I_{nN} \end{bmatrix} < 0, \quad (5)$$

where there exists no matrix $\tilde{A}_{ii}^T P_i \tilde{A}_{ii}$ in $\mathcal{M}_i$ and

$$\mathcal{M}_i \in \Re^{\tilde{N}\times\tilde{N}}, \quad \tilde{N} := \sum_{j=1}^{N} n_j,$$
$$\Theta_i := -P_i + (N-1)I_{n_i} + \tilde{Q}_i,$$
$$\tilde{Q}_i := Q_i + (K_i + \Delta K_i(k))^T R_i (K_i + \Delta K_i(k)),$$
$$\tilde{A}_i := A_i + B_i K_i + \Delta A_i(k) + B_i \Delta K_i(k),$$
$$\tilde{A}_{ij} := A_{ij} + \Delta A_{ij}(k).$$

If such conditions are met, the control laws $u_i(k) = (K_i + \Delta K_i(k))x_i(k)$, $i = 1, \dots, N$ are said to be the guaranteed cost controller. In this case, the corresponding value of the cost function (4) satisfies the following inequality (6) for admissible uncertainties.

$$J < J^* = \sum_{i=1}^{N} x_i^T(0) P_i x_i(0). \tag{6}$$

Proof: With the control law (1b), the resulting closed–loop subsystem becomes

$$x_i(k+1) = \tilde{A}_i x_i(k) + \sum_{j=1,\ j\neq i}^{N} \tilde{A}_{ij} x_j(k). \tag{7}$$

In order to prove the asymptotic stability of the closed–loop system (7), let us define the following Lyapunov function candidate.

$$V(x(k)) = \sum_{i=1}^{N} x_i^T(k) P_i x_i(k), \tag{8}$$

where $x(k) := [x_1^T(k) \ \cdots \ x_N^T(k)]^T$, and P_i, $i = 1, \dots, N$ is the positive definite matrix. Note that $V(x(k)) > 0$ whenever $x(k) \neq 0$. Since the proof can be done by using the similar approach in Mukaidani *et al.,* (2004a), it is omitted. ■

The objective of this section is to design a fixed guaranteed cost control gain matrix K_i for the uncertain large–scale system (1) with the LMI design approach.

Theorem 2: Consider the uncertain large–scale systems (1) and cost function (4). Suppose that for all $i = 1, \dots, N$, the LMI (9) has a solution set such as symmetric positive definite matrices $X_i \in \Re^{n_i \times n_i}$, the matrices $Y_i \in \Re^{m_i \times n_i}$, and the positive scalars $\epsilon_{ai}, \ \epsilon_{ki}, \ \epsilon_{ai1}, \ \dots, \ \epsilon_{aiN} > 0$.

If such conditions are met, $K_i = Y_i X_i^{-1}$ is the guaranteed cost control gain matrix for the closed–loop uncertain large–scale interconnected systems. Furthermore, the value of the cost function (4) satisfies the following inequality (10).

$$J < J^* = \sum_{i=1}^{N} x_i^T(0) X_i^{-1} x_i(0). \tag{10}$$

Proof : Applying the Schur complement (Zhou, 1998) to the matrix inequality (5) yields (11). Using a standard matrix inequality (Wang *et al.*, 1998) to the LMI (12) and applying Schur complement, the inequality (11) holds. Moreover, pre– and post–multiplying both sides of the inequality by the positive definite matrix **block diag** $[\ P_i^{-1} \ I_{h_i} \ I_{t_i} \ I_{n_i} \ \cdots \ I_{n_i} \ I_{n_i} \ I_{m_i} \ I_{n_i} \ I_{r_i} \ \cdots \ I_{n_i} \ I_{r_i} \ I_{n_i}\ ]$ and introducing the matrices $X_i = P_i^{-1}$, $Y_i = K_i P_i^{-1}$, the matrix inequality (12) results in the LMI (9).

On the other hand, since the result of the cost bound (10) can be proved by using the similar argument for the proof of Theorem 1, it is omitted. Thus, K_i is the guaranteed cost control gain matrix. ■

Since the LMI (9) consists of a convex solution set $(X_i, \ Y_i, \ \epsilon_{ai}, \ \epsilon_{ki}, \ \epsilon_{ai1}, \ \dots, \ \epsilon_{aiN})$, various efficient convex optimization algorithms can be applied. Moreover, its solutions represent a set of the guaranteed cost control gain matrix K_i. Consequently, let us consider the optimization problem that allows us to determine the optimal bound.

Problem A: Consider the LMI (9) and the following constrained conditions.

$$\begin{bmatrix} -\alpha_i & x_i^T(0) \\ x_i(0) & -X_i \end{bmatrix} < 0. \tag{13}$$

Moreover, also consider the convex set $\mathcal{X}_i \in (X_i, \ Y_i, \ \epsilon_{ai}, \ \epsilon_{ki}, \ \epsilon_{ai1}, \ \dots, \ \epsilon_{aiN})$ such that $\epsilon_{ai}, \epsilon_{ki}, \epsilon_{ai1}, \ \dots, \ \epsilon_{aiN} > 0$ holds. Find $K_i = Y_i X_i^{-1}$, $i = 1, \ \dots, N$ such that the LMIs (9) and (13) are satisfied, and the cost $\sum_{i=1}^{N} \alpha_i \in \mathcal{X}_i$ becomes as small as possible. That is, the problem addressed in this paper is

$$J < J^* < \min_{\sum_{i=1}^{N} \mathcal{X}_i} \sum_{i=1}^{N} \alpha_i, \tag{14}$$
$$\mathcal{X}_i \in (X_i, \ Y_i, \ \epsilon_{ai}, \ \epsilon_{ki}, \ \epsilon_{ai1}, \ \dots, \ \epsilon_{ai1})$$

It should be noted that the existing result (Park, 2004b) has not considered the decomposition of the LMI optimization problem. In this paper, it is shown that the LMI optimization problem related with the guaranteed cost control can be decomposed. That is, it is possible to replace the Problem **A** with each optimization problem for all i by using the following result because the Problem **A** can be decomposed.

Theorem 3: If the above optimization problem **A** has the solution $\mathcal{X}_i$ and α_i, then the fixed gain matrices K_i are the decentralized state feedback control gain matrices which ensure the minimization of the guaranteed cost (10) for the uncertain large–scale interconnected systems. Moreover, the optimization problem (14) can be changed to the following problem.

$$\min_{\sum_{i=1}^{N} \mathcal{X}_i} \sum_{i=1}^{N} \alpha_i = \sum_{i=1}^{N} \min_{\mathcal{X}_i} \alpha_i.$$

$$
\left[\begin{array}{cccccccccccccc}
-X_i & (E_{ai}X_i)^T & (E_{ki}X_i)^T & X_i^T & \cdots & X_i^T & X_i^T & Y_i^T & 0 & 0 & \cdots & 0 & 0 & (A_iX_i+B_iY_i)^T \\
E_{ai}X_i & -\epsilon_{ai}I_{h_i} & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\
E_{ki}X_i & 0 & -\epsilon_{ki}I_{t_i} & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\
X_i & 0 & 0 & -I_{n_i} & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
X_i & 0 & 0 & 0 & \cdots & -I_{n_i} & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\
X_i & 0 & 0 & 0 & \cdots & 0 & -Q_i^{-1} & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\
Y_i & 0 & 0 & 0 & \cdots & 0 & 0 & -R_i^{-1}+\epsilon_{ki}D_{ki}D_{ki}^T & 0 & 0 & \cdots & 0 & 0 & \epsilon_{ki}D_{ki}D_{ki}^TB_i^T \\
0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & -I_{n_i} & E_{ai1}^T & \cdots & 0 & 0 & A_{i1}^T \\
0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & E_{ai1} & -\epsilon_{ai1}I_{r_i} & \cdots & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots & -I_{n_i} & E_{aiN}^T & A_{i1}^T \\
0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots & E_{aiN} & -\epsilon_{aiN}I_{r_i} & 0 \\
A_iX_i+B_iY_i & 0 & 0 & 0 & \cdots & 0 & 0 & \epsilon_{ki}B_iD_{ki}D_{ki}^T & A_{i1} & 0 & \cdots & A_{iN} & 0 & -X_i+\Phi
\end{array}\right]
< 0, \qquad (9)
$$

where $\quad \Phi := \epsilon_{ai}D_{ai}D_{ai}^T + \epsilon_{ki}B_iD_{ki}D_{ki}^TB_i^T + \sum_{j=1}^{N}\epsilon_{aij}D_{aij}D_{aij}^T.$

$$
(5) \Leftrightarrow \left[\begin{array}{ccccccc}
-P_i+(N-1)I_{n_i} & I_{n_i} & (K_i+\Delta K_i(k))^T & 0 & \cdots & 0 & \tilde{A}_i^T \\
I_{n_i} & -Q_i^{-1} & 0 & 0 & \cdots & 0 & 0 \\
(K_i+\Delta K_i(k)) & 0 & -R_i^{-1} & 0 & \cdots & 0 & 0 \\
0 & 0 & 0 & -I_{n1} & \cdots & 0 & \tilde{A}_{i1}^T \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & 0 & 0 & \cdots & -I_{nN} & \tilde{A}_{iN}^T \\
\tilde{A}_i & 0 & 0 & \tilde{A}_{i1} & \cdots & \tilde{A}_{iN} & -P_i^{-1}
\end{array}\right] < 0. \qquad (11)
$$

$$
\Leftarrow \left[\begin{array}{cccccccccccccc}
-P_i & E_{ai}^T & E_{ki}^T & I_{n_i} & \cdots & I_{n_i} & I_{n_i} & K_i^T & 0 & 0 & \cdots & 0 & 0 & (A_i+B_iK_i)^T \\
E_{ai} & -\epsilon_{ai}I_{h_i} & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\
E_{ki} & 0 & -\epsilon_{ki}I_{t_i} & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\
I_{n_i} & 0 & 0 & -I_{n_i} & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
I_{n_i} & 0 & 0 & 0 & \cdots & -I_{n_i} & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\
I_{n_i} & 0 & 0 & 0 & \cdots & 0 & -Q_i^{-1} & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\
K_i & 0 & 0 & 0 & \cdots & 0 & 0 & -R_i^{-1}+\epsilon_{ki}D_{ki}D_{ki}^T & 0 & 0 & \cdots & 0 & 0 & \epsilon_{ki}D_{ki}D_{ki}^TB_i^T \\
0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & -I_{n_i} & E_{ai1}^T & \cdots & 0 & 0 & A_{i1}^T \\
0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & E_{ai1} & -\epsilon_{ai1}I_{r_i} & \cdots & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots & -I_{n_i} & E_{aiN}^T & A_{i1}^T \\
0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots & E_{aiN} & -\epsilon_{aiN}I_{r_i} & 0 \\
A_i+B_iK_i & 0 & 0 & 0 & \cdots & 0 & 0 & \epsilon_{ki}B_iD_{ki}D_{ki}^T & A_{i1} & 0 & \cdots & A_{iN} & 0 & -P_i^{-1}+\Phi
\end{array}\right]
< 0. \qquad (12)
$$

Proof: Since the proof can be done by using the similar approach in Mukaidani *et al.*, (2004a), it is omitted. ■

Remark: It can be noted that the bound obtained in Theorem 3 depends on the initial condition $x_i(0)$. It is assumed to remove such condition that $x_i(0)$ is a zero mean random variable satisfying $E[x_i(0)x_i^T(0)] = I_{n_i}$, where $E[\cdot]$ denotes the expectation. Then, the LMI (13) becomes

$$
\begin{bmatrix} -M_i & I_{n_i} \\ I_{n_i} & -X_i \end{bmatrix} < 0, \qquad (15)
$$

where M_i is the expectation of α_i.

In this paper the condition (15) will be used instead of (13) in the optimization problem.

3. NEURAL NETWORKS FOR ADDITIVE GAIN PERTURBATIONS

The LMI approach for the uncertain large–scale systems usually results in the conservative controller design due to the existence of the parameter uncertainties and the gain perturbations, which cause the large cost. The main purpose of this paper is to improve the cost with a learning method using NN (Mukaidani *et al.*,, 2004b; Ishii *et al.*,, 2004). Note that the proposed neurocontroller regulates its outputs in real–time under the robust stability guaranteed by the LMI approach.

3.1 On–line learning Algorithm of neurocontroller

It is expected that the reduction of the cost will be attained when the dynamics of the uncertain large–scale systems are close to a nominal one. That is, the neurocontroller is required to compensate the conservative controller to perform as the nominal system in the uncertain large–scale systems.

Let us consider the following nominal discrete-time large–scale systems.

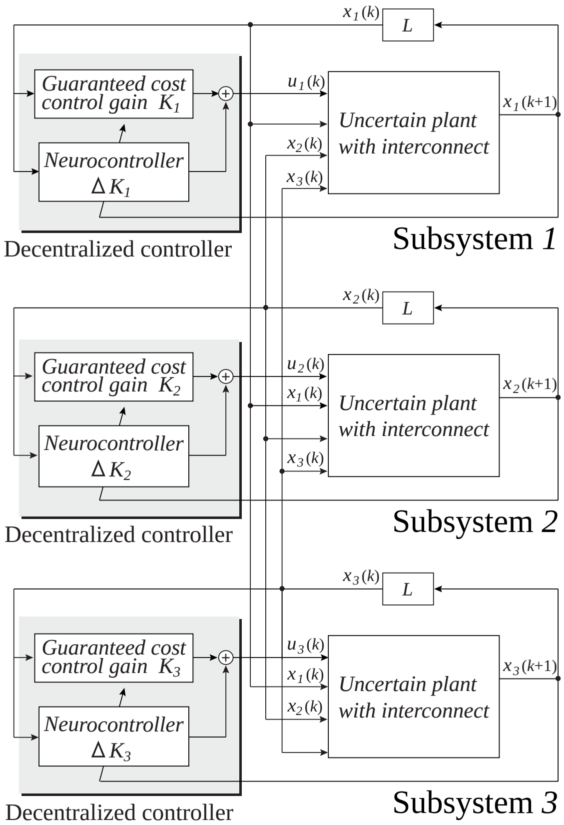

Fig. 1. Block diagram of proposed system composed of three–dimensional subsystems

$$\hat{x}_i(k+1) = A_i\hat{x}_i(k) + B_i\hat{u}_i(k) + \sum_{j=1,\ j\neq i}^{N} A_{ij}\hat{x}_j(k), \qquad (16a)$$

$$\hat{u}_i(k) = \hat{K}_i\hat{x}_i(k), \qquad (16b)$$

where $\hat{x}_i(k) \in \Re^{n_i}$ is the state and $\hat{u}_i(k) \in \Re^{m_i}$ is the control input of the ith subsystem $i = 1, \ \ldots\ , N$. $\hat{K}_i \in \Re^{m_i \times n_i}$ is the state feedback gain derived by any LMI approach for the nominal system (16). For the nominal system (16) and the cost function (4), it is known that the guaranteed cost $\hat{J}^*$ of the nominal system is smaller than that of the cost J^* for the uncertain large–scale systems.

In this paper, the decentralized neurocontroller for the discrete–time uncertain large–scale systems is considered. As a specific example, the block diagram of the proposed control systems that have three–dimensional subsystems is given by Fig. 1. Note that L is a time lag diagram. Fig. 1 shows that each neurocontroller uses only the observed state values of each subsystem as its input. It should be noted that this example is also used in the next section.

For each subsystem, the NN should be trained in real–time so that the norm of the state discrepancy, which is given by $\|\hat{x}_i(k+1) - x_i(k+1)\|$ between the behavior of the nominal system and the uncertain large–scale system becomes as small as possible at each step k. $N_i(k)$, in equation (2), can be expressed as a nonlinear function of the state $x_i(k)$, the weight coefficient of NN $w_i(k)$, and the threshold $\theta_i(k)$ as follows

$$N_i(k) = f\left(x_i(k), w_i(k), \theta_i(k)\right). \qquad (17)$$

For each subsystem, an energy function $E_i(k)$ is defined as the square norm of the state discrepancy. At each step, the weight coefficients are modified so as to minimize $E_i(k)$ given by

$$E_i(k) = \frac{1}{2}(\hat{x}_i(k+1) - x_i(k+1))^T \times(\hat{x}_i(k+1) - x_i(k+1)) \qquad (18)$$

$E_i(k)$ can be calculated by using the observed state value $x_i(k+1)$. Therefore, it is not necessary to consider the behavior of the uncertain matrices $F_{ai}(k)$ and $F_{aij}(k)$. If $E_i(k)$ can be minimized as small as possible for each subsystem, the discrepancy $\left\|\hat{x}_i(k+1) - x_i(k+1)\right\|^2$ would also be minimized so that the cost of the uncertain large–scale system is close to the cost of the nominal large–scale systems.

In the learning of NN, the modification of weight coefficient $\Delta w_i(k)$ is given by

$$w_i(k+1) = w_i(k) + \Delta w_i(k), \qquad (19a)$$

$$\Delta w_i(k) = -\eta_i \frac{\partial E_i(k)}{\partial w_i(k)}, \qquad (19b)$$

$$\frac{\partial E_i(k)}{\partial w_i(k)} = \frac{\partial E_i(k)}{\partial N_i(k)}\frac{\partial N_i(k)}{\partial w_i(k)}, \qquad (19c)$$

where $\eta_i,\ i = 1,\ 2,\ 3$ is the learning ratio. The term $\frac{\partial E_i(k)}{\partial N_i(k)}$ can be calculated from the energy function (18) as follows.

$$\frac{\partial E_i(k)}{\partial N_i(k)} = -\big(\hat{x}_i(k+1) - x_i(k+1)\big) \times\ B_i D_{ki} E_{ki} x_i(k) \qquad (20)$$

On the other hand, $\frac{\partial N_i(k)}{\partial w_i(k)}$ can be calculated using the chain rule on the NN. As a result, using (16)–(20), NN can be trained so as to decrease the cost J on–line.

3.2 Multilayered Neural networks

The utilized NN are of a three–layer feed–forward network as shown in Fig. 2. A linear function is utilized in the neurons of the input and the hidden layers, and a sigmoid function in the output layer. For each subsystem i, inputs and outputs of each layer can be described as follows

$$s_{iq}^{y}(k) = \begin{cases} U_i^y(k) & \{q = 1(\text{input layer})\} \\ \sum w_{i1}^{(y,z)}(k) o_{i1}^{z}(k) & \{q = 2(\text{hidden layer})\} \\ \sum w_{i2}^{(y,z)}(k) o_{i2}^{z}(k) & \{q = 3(\text{output layer})\}, \end{cases}$$

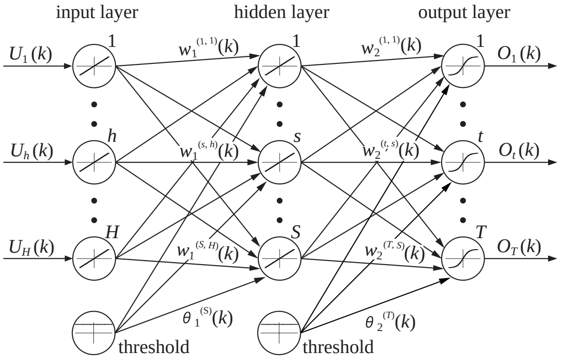

Fig. 2. Structure of the multilayered neural networks.

$$o_{iq}^{y}(k) = \begin{cases} s_{i1}^{y}(k) & \{q = 1(\text{input layer})\} \\ s_{i2}^{y}(k) + \theta_{i1}^{y}(k) & \{q = 2(\text{hidden layer})\} \\ \dfrac{1 - e^{(-s_{i3}^{y}(k) + \theta_{i2}^{y}(k))}}{1 + e^{(-s_{i3}^{y}(k) + \theta_{i2}^{y}(k))}} & \{q = 3(\text{output layer})\}, \end{cases}$$

where $s_{iq}^{y}(k)$ and $o_{iq}^{y}(k)$ are the input and output of neuron y in the qth layer at step k, $w_{iq}^{(y,z)}(k)$ indicates the weight coefficient from neuron z in the qth layer to neuron y in the $(q+1)$th layer, $U_i^{y}(k)$ is the input of NN, $\theta_{iq}^{y}(k)$ is a positive constant for the threshold of neuron y in the $(q+1)$th layer. As the additive gain perturbations defined in the formula (3), the outputs of NN are chosen adaptively in the range of $[-1.0,\ 1.0]$.

In general, it should be noted that the number of neurons and the learning ratio that is a constant as neural networks architecture should be chosen appropriately. However, since our proposed method is independent of such choice, the proposed method is reliable and useful.

4. CONCLUSIONS

The application of neural networks to the guaranteed cost control problem of the discrete–time uncertain large–scale interconnected systems has been investigated. Using the LMI approach, the class of the decentralized state feedback gain has been derived. Substituting the neurocontroller into the gain perturbations, the reduction of the cost is attained by using them. Moreover, the robust stability of the closed–loop system is guaranteed even if the systems include NN. It is worth pointing out that the decentralized controller is constructed by using the new decomposed LMI optimization technique compared to existing results (Park, 2004b). The numerical example have shown the excellent result that the NN have succeeded in reducing the large cost caused by the LMI.

The implementation of the proposed feedback control law may result in increasing the computational complexity. As a result, the computing time required to carry out the control algorithm should be clarified. It will be demonstrated in the near future by showing the practical example.

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