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Feedback Control of Nonholonomic Mobile Robots Using Time Base Generator

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Abstract

The present paper proposes a new feedback control strategy of mobile robots with nonholonomic constraints. This method introduces a function generator called a Time Base Generator (TBG) as a time varying feedback gain, and by synchronizing linear and angular velocities of the mobile robot with a scalar variable generated by the TBG, the mobile robot can be positioned to the origin in the state space for any initial condition. Not only the configuration of the robot but also the time behavior of the generated trajectory such as the velocity profile and the movement time from an initial to a target position can be regulated through adjusting the parameters of the TBG.

1 Introduction

Recently, control of robot systems with nonholonomic constraints has been investigated actively in various fields such as mobile robots, underwater vehicles and manipulators with free joints [1]. For controlling a mobile robot with a nonholonomic constraint, Samson [2] and Pomet [3] proposed the feedback law using a time-periodic function and showed that a mobile robot with two driving wheels can be positioned to a given final configuration for any initial condition. Although this approach using the time-varying smooth feedback can assure the stability of the system, the slow convergence may be a defect. Then, Canudas de Wit and Sørdalen [4] proposed the piecewise smooth feedback law using the discontinuous controller and proved that a mobile robot is exponentially stabilized and the convergence to the target point is extremely faster than the time-varying smooth feedback control. Also, Badreddin and Mansour [5] and Casalino et al. [6] showed that a special choice of the polar coordinate system representing the position and orientation of a mobile robot allows to derive a smooth stabilizing control law without contradicting the well known work of Brockett [7].

Although all approaches described above can assure the stability, it is hard to say to be always practical because an unnatural trajectory including many switchings between forward and backward movements may be generated for some initial and target conditions and the time behavior of generated trajectories such as convergence time and velocity profile cannot be regulated.

On the other hand, Morasso, Sanguineti and Tsuji [8], [9] have proposed a planar end-point trajectory generation model for human reaching movements. Generally, when a human subject is instructed to move a hand position from a starting point to a target point, it is well known that a trajectory of the end-point draws almost a straight line and the velocity along the motion direction has a bell-shaped profile with only one peak [10]. Morasso et al. [8], [9] introduced a function generator called a *Time Base Generator* (TBG) having a bell-shaped velocity profile and showed that, by synchronizing translational and angular velocities of the end-point with the TBG, not only straight trajectories but also curved trajectories could be generated within the specified motion duration.

By introducing the piecewise smooth feedback law into the end-point trajectory generation model using the TBG, the goal of this paper is to propose a new feedback control method that can assure the convergence to the target point and regulate the time behavior of the generated trajectory such as convergence time and velocity profile. The effectiveness of the proposed method is shown through some simulation experiments and the behavior of the mobile robot under the proposed control law in the cases of straight and circular trajectories are analyzed.

2 Mobile robot with two driving wheels [1], [4]

Figure 1 shows a mobile robot with two driving wheels, where Σ_0 denotes the world coordinate system fixed on the planar task space. Also Σ_c is defined as the moving coordinate system fixed on the robot, where the origin and the x axis of Σ_c are set at the center of the wheel axis and

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the orientation of the robot, respectively. Under these definitions, the generalized coordinates of the mobile robot consists of three variables: the position (x, y) and the orientation angle θ of the Σ_c represented in the Σ_0.

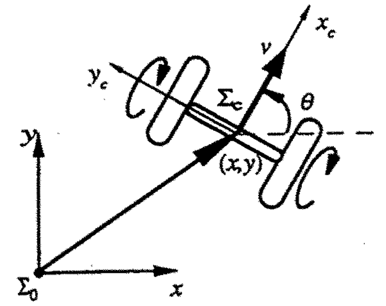

Fig.1 Mobile robot with two wheels

The relationship between the velocity of the generalized coordinates $\boldsymbol{x} = (x, y, \theta)^{\mathrm{T}}$ and the linear and the angular velocities $\boldsymbol{u} = (v, \omega)^{\mathrm{T}}$ is given as

$$\dot{\boldsymbol{x}} = G(\boldsymbol{x})\, \boldsymbol{u}, \tag{1}$$

$$G(\boldsymbol{x}) = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix}, \tag{2}$$

where v and ω denote the linear and the angular velocities of the robot. In (1), it can be seen that one degree of freedom of the system is constrained as

$$\dot{x}\sin\theta - \dot{y}\cos\theta = 0\,. \tag{3}$$

It has been shown that the system (1) is of symmetric Affine and controllable, so that it includes a nonholonomic constraint [1], [4].

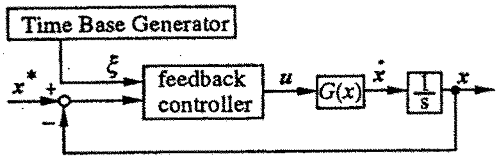

Fig.2 Feedback control System using TBG

3 Feedback control using Time Base Generator

3.1 Time Base Generator

Figure 2 shows a block diagram of a feedback control of the planar mobile robot using the TBG. Generally, for a system with nonholonomic constraints, any smooth time-invariant state feedback stabilizing the system does not exist [7]. In the present paper, this problem is coped with by introducing a scalar variable $\xi(t)$ with a bell-shaped velocity profile generated by the TBG as a time varying feedback gain.

The scalar function $\xi(t)$ is defined as a first differentiable and monotonically non-increasing function satisfying $\xi(0) = 1$ and $\xi(t_f) = 0$, where t_f represents the convergence time of the mobile robot from the initial to the target positions. In this paper, the dynamics of the TBG is defined as follows:

$$\dot{\xi} = -\gamma(\xi(1.0 - \xi))^{\beta}, \tag{4}$$

where γ is defined as a function of the convergence time t_f and β satisfying $0 < \beta < 1$ is a constant that determines the behavior of the TBG.

From (4), $\xi(t)$ has two equilibrium points of $\xi = 0$ and $\xi = 1$. Consequently, $\xi(t)$ always converges stably to $\xi = 0$, when an initial value of ξ is chosen as $\xi(0) = 1 - \varepsilon$ using a very small positive constant ε. Then the convergence time can be calculated as

$$t_f = \int_0^{t_f} dt = \int_1^0 \frac{d\xi}{\dot{\xi}} = \frac{\Gamma^2(1-\beta)}{\gamma\Gamma(2-2\beta)}, \tag{5}$$

where $\Gamma(\cdot)$ is the gamma function (Euler's integral of the second kind). When the parameter γ is chosen as

$$\gamma = \frac{\Gamma^2(1-\beta)}{t_f\,\Gamma(2-2\beta)}, \tag{6}$$

the system converges to the equilibrium point in the finite time t_f and this equilibrium point $\xi = 0$ becomes a terminal attractor [11]. The velocity profile $\dot{\xi}(t)$ satisfies $\dot{\xi}(0) \approx 0$ and $\dot{\xi}(t_f) = 0$, and is bell-shaped with the minimum value $\dot{\xi}(t_f/2) = -\gamma 4^{-\beta}$ at $t = t_f/2$.

Figures 3 and 4 show the changes of the time behavior $\xi(t)$ generated by the TBG depending on the parameters t_f and β. In Fig.3, the time histories of $\xi(t)$ and $\dot{\xi}(t)$ are shown depending on the change of the convergence time

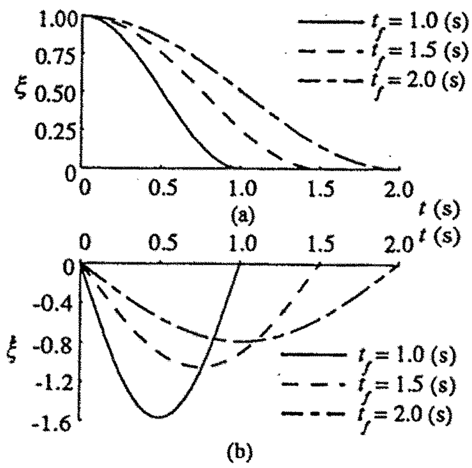

Fig.3 Changes of the ξ behavior depending on the convergence time t_f

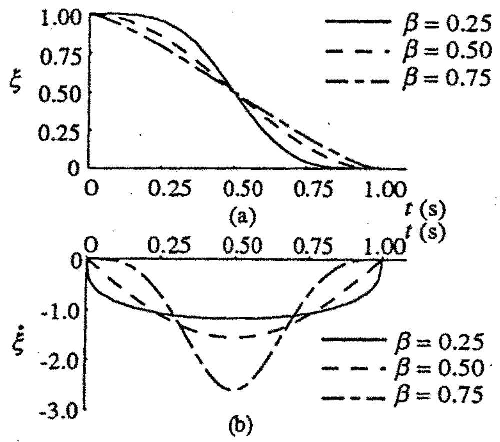

Fig.4 Changes of the ξ behavior depending on the power parameter β

t_f= 1.0, 1.5 and 2.0 (s) under the parameters $\beta = 0.5$ and $\varepsilon = 1.0 \times 10^{-9}$. All trajectories converge to the equilibrium point at the specified time t_f. On the other hand, Fig.4 shows time histories of $\xi(t)$ depending on the change of the power parameter β= 0.25, 0.5 and 0.75 with $t_f = 1.0$ (s) and $\varepsilon = 1.0 \times 10^{-9}$. The time history of $\xi(t)$ can be regulated through the power parameter β while keeping the convergence time constant.

Consequently, using the TBG with two parameters of β and t_f presented here, various kinds of time-varying functions having the bell-shaped velocity profile can be generated. In the following section, the method generating smooth trajectories of the mobile robot with nonholonomic constraints is presented using this TBG.

3.2 System equation and feedback control law

The purpose of the feedback control is to automatically drive the mobile robot from any initial configuration to the given final configuration which we define to be the origin of the generalized coordinates.

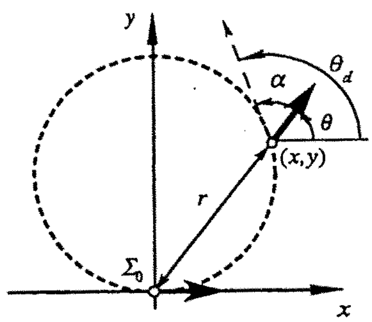

Fig.5 Coordinate transformation

Canudas de Wit and Sørdalen [4] proposed the piecewise smooth feedback control law using the circle family that pass through the origin and the current position of the mobile robot (x, y) and contacts with the x axis of the Σ_0 at the origin as shown in Fig.5. In the figure, θ_d represents the tangential direction in a point (x, y) of this circle and it belongs to $[-\pi, \pi)$. Their control law is based on the idea that the arc length from the origin to the current position should be decreasing and the current angular orientation of the mobile robot should agree with the tangential direction θ_d. In the present paper, instead of the arc length, the distance r from the current position to the origin is used.

Let an error angle between the tangential direction θ_d and the current angular orientation θ be denoted as α. Then the following coordinate transformations from (x, y, θ) are introduced:

$$r(x, y) = \sqrt{x^2 + y^2}\,, \tag{7}$$

$$\alpha\,(x, y, \theta) = e + 2n(e)\pi, \tag{8}$$

$$e = \theta - \theta_d, \tag{9}$$

$$\theta_d = 2\mathrm{atan2}\,(y, x)\,, \tag{10}$$

where $n(e)$ is a function that takes an integer in order to satisfy $\alpha \in [-\pi, \pi)$ and $\mathrm{atan2}(\cdot,\cdot)$ is the scalar function defined as $\mathrm{atan2}\,(a, b) = \arg\,(b + ja)$ where j denotes the imaginary unit and " arg " denotes an argument of a complex number. As a result, a current state of the mobile robot can be represented by

$$z = F(x), \tag{11}$$

$$F(x) = \begin{bmatrix} r\,(x, y) \\ \alpha\,(x, y, \theta) \end{bmatrix}. \tag{12}$$

Also, the target configuration of the mobile robot is transformed to $z_f = (0, 0)^T$. Consequently the stabilization of the system is understood as designing a control law converging to $z_f = (0, 0)^T$ for any initial configuration.

Firstly, from (11), the relationship between $\dot{z}$ and $\dot{x}$ is given as

$$\dot{z} = \frac{\partial F(x)}{\partial x}\dot{x} = J(x)\dot{x}\,, \tag{13}$$

where

$$J(x) = \begin{bmatrix} (x^2+y^2)^{-\frac{1}{2}}x & (x^2+y^2)^{-\frac{1}{2}}y & 0 \\ \frac{2y}{x^2+y^2} & -\frac{2x}{x^2+y^2} & 1 \end{bmatrix} \in R^{2\times 3}. \tag{14}$$

Substituting (1) into the above equation, we have the relationship between $\dot{z}$ and the system input u :

$$\dot{z} = J(x)G(x)\,u = B(x)\,u, \tag{15}$$

where

$$B(x)=\begin{bmatrix} b_1 & 0 \\ b_2 & 1 \end{bmatrix}, \tag{16}$$

$$b_1=(x^2+y^2)^{-\frac{1}{2}}(x\cos\theta+y\sin\theta), \tag{17}$$

$$b_2=\frac{2}{x^2+y^2}(y\cos\theta-x\sin\theta). \tag{18}$$

Thus, the number of the state variables is reduced to the same number as the system input.

For the system of (15), the following feedback control law is proposed in this paper:

$$u=\begin{bmatrix} v \\ \omega \end{bmatrix}=\begin{bmatrix} \dfrac{r\dot{\xi}}{b_1\xi} \\ -b_2v+\dfrac{\alpha\dot{\xi}}{\xi} \end{bmatrix}, \tag{19}$$

where it is assumed that $b_1\neq 0$ for any t except for $t=t_f$.

3.3 Stability analysis

Substituting the control law (19) into the system equation (15), we have

$$\dot{r}=b_1v=r\frac{\dot{\xi}}{\xi}, \tag{20}$$

$$\dot{\alpha}=b_2v+\omega=\alpha\frac{\dot{\xi}}{\xi}. \tag{21}$$

Firstly let us consider the behavior of the distance r with respect to the variable ξ. From (20), we can obtain

$$\frac{dr}{d\xi}=\frac{r}{\xi}. \tag{22}$$

Solving this differential equation, we can get

$$r=r_0\xi, \tag{23}$$

where r_0 is an initial value of r. Thus it can be seen that the distance r is proportional to the $\xi(t)$ under the control law (19). The $\xi(t)$ converges to $\xi\to 0$ at time t_f, so that r also reaches $r\to 0$ at time t_f. On the other hand, the relationship between α and ξ is also given as

$$\alpha=\alpha_0\xi, \tag{24}$$

where α_0 is an initial value of α. In the same way as the distance r, we can see $\alpha\to 0$ at time t_f.

In summary, it has been shown that the mobile robot always converges to the target position at time t_f using the control law proposed in this paper so long as $b_1\neq 0$. Under the proposed method, both the translational and the rotational motion of the mobile robot can be synchronized with the ξ behavior of the TBG and thus the time behavior of the generated trajectory of the robot with nonholonomic constraints can be regulated through the TBG.

4 Simulation experiments

4.1 Generation of straight trajectories

Let us firstly consider a case $x_0=(x_0, 0, 0)^T$, where the initial position is on the x axis and an initial orientation θ is given by $\theta=0$ (rad) as shown in Fig.6.

Then from (8) and (19), we have

$$\alpha_0=2\,\mathrm{atan2}\,(0, x_0)=0. \tag{25}$$

Also under the proposed control law (19), we have $\omega=0$ since $\theta_0=0$. As a result, it can be seen that the mobile robot always exists on the x axis of the Σ_0. Since $y=0$ in this case, from (7) we can derive

$$r=\sqrt{x^2}=|x|, \tag{26}$$

and then from (23), we have

$$|x|=|x_0|\xi. \tag{27}$$

This means that the position of the mobile robot is completely proportional to $\xi(t)$.

Fig.6 Generation of a straight trajectory

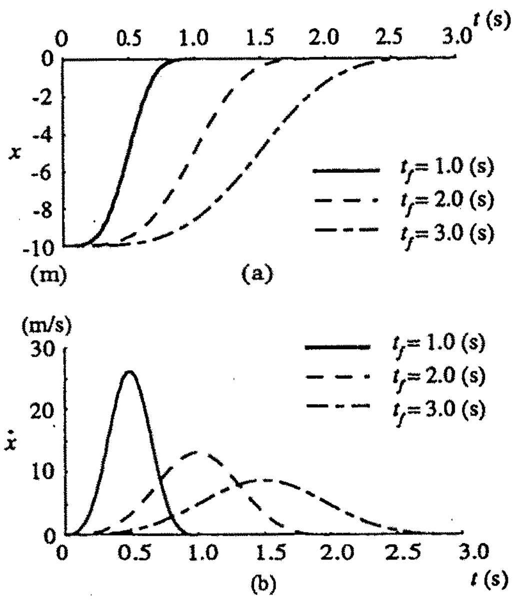

Fig.7 Straight trajectories generated by the proposed method

Figure 7 shows the time histories of x and $\dot{x}$ generated by the proposed method for the initial position of the robot as $x_0 = (-10\ (m),\ 0\ (m),\ 0\ (rad))^T$, where three different convergence time $t_f = 1.0, 2.0, 3.0$ (s) are used with the same power parameter $\beta = 0.75$. It should be noted that the y coordinate and the angular orientation θ of the generated trajectory are always 0, since the trajectory is a straight line along the x axis. From Fig.7, it can be seen that the proposed method can generate a straight trajectory naturally for some specific initial conditions and the dynamic behavior of the robot can be specified by the TBG.

4.2 Generation of curved trajectories

Figures 8 and 9 show the results generating the proposed method for several initial conditions corresponding to different points on the circle with its radius of 10 (m), where the initial orientation angle θ_0 is set as $\theta_0 = \pi/2$ (rad) in Fig.8 and $\theta_0 = 0$ (rad) in Fig.9. Also the set of the parameters of the TBG, $\beta = 0.75$ and $t_f = 1.0$ (s), are used.

To have described in the section 3.2, the control law (19) becomes singular at the point that the parameter b_1 of (17) reduces to 0. From (17), it is clear that the singularity occurs at points where the position vector from the origin of the coordinate system (that is, the target position) to the current position of the mobile robot and the orientation vector of the mobile robot are orthogonal each other (see Fig.5). Then to observe the behavior of the mobile robot in the neighborhood of the singular configuration, the initial conditions starting from the point close to the singular configuration are included in Fig.8 and 9: two trajectories starting from the points close to the x axis in Fig.8 ($x_0 =$ (10 (m), 1.0×10^{-5} (m), $\pi/2$ (rad))^T and $x_0 =$ (-10 (m), -1.0×10^{-5} (m), $\pi/2$ (rad))^T), and two trajectories starting from the points close to the y axis in Fig.9 ($x_0 =$ (1.0×10^{-5} (m), 10 (m), 0 (rad))^T and $x_0 =$ (-1.0×10^{-5} (m), -10 (m), 0 (rad))^T).

In all cases including the trajectories starting from the neighborhood of the singular configuration, the robot can arrive at the target position without encountering with the singularity ($b_1 = 0$) generating the smooth trajectories not containing any switching between forward and backward movements. Especially, it seems that the trajectories corresponding to the initial configurations of $x_0 =$ ($\sqrt{50}$ (m), $\sqrt{50}$ (m), $\pi/2$ (rad))^T and $x_0 =$ ($-\sqrt{50}$ (m), $-\sqrt{50}$ (m), $\pi/2$ (rad))^T in Fig.8 may be circular.

These cases are now examined in detail. Let us consider the case that the initial orientation θ_0 agrees with the tangent direction of the circle passing the initial position and the origin (see Fig.10). From (8), it is clear that $\alpha_0 = 0$, then from (19), we can obtain

$$\omega = -b_2 v. \tag{28}$$

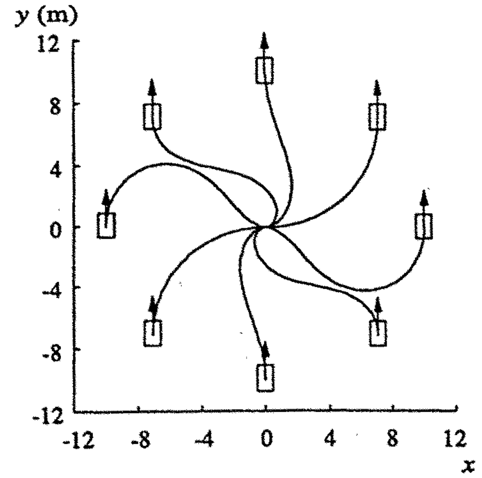

Fig.8 Generated trajectories when the robot is initially located on the circle with $\theta_0 = \pi/2$ (rad)

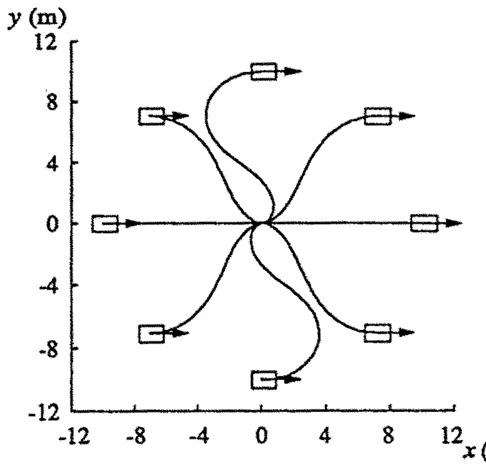

Fig.9 Generated trajectories when the robot is initially located on the circle with $\theta_0 = 0$ (rad)

Also, letting a radius of the circle defined at the initial position be denoted as R_0 (see Fig.10), we have

$$R_0 = \frac{x^2 + y^2}{2y}. \tag{29}$$

Using the relations $\sin\theta = x/R_0$ and $\cos\theta = (R_0 - y)/R_0$, transforming b_2 of (18), the following relationship is derived:

$$b_2 = \frac{2}{x^2 + y^2}\left(\frac{R_0 - y}{R_0} y - \frac{x}{R_0} x\right) = -\frac{1}{R_0}. \tag{30}$$

As a result, the gain b_2 remains a constant. From (28), it can be seen that the mobile robot always exists on the circle defined at the initial position and it converges to the target configuration as generating the complete circular trajectory.

Figure 11 shows the time histories of the x coordinate and the linear velocity v for the initial configuration $x_0 = (\sqrt{50}\ \text{(m)}, \sqrt{50}\ \text{(m)}, \pi/2\ \text{(rad)})^T$ in Fig.8. The solid and dashed lines represent the results using the proposed method and the method by Canudas de Wit and Sørdalen [4], respectively. For the case using the method of Canudas de Wit and Sørdalen, the robot converges slower as approaching the target position. On the other hand, the robot under our method converges just in the appointed time $t_f = 1.0$ (s) and the velocity profile almost becomes bell-shaped.

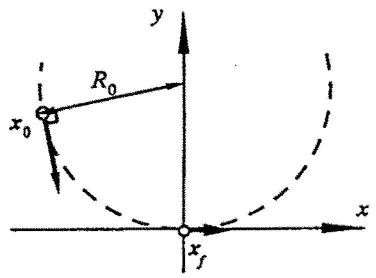

Fig.10 Generation of a circular trajectory

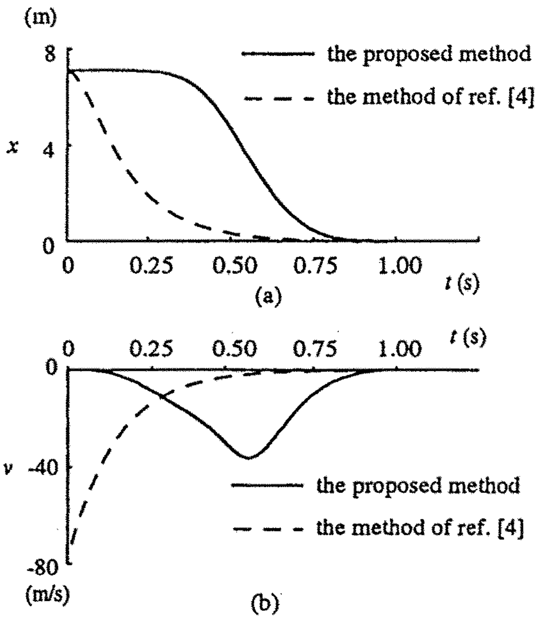

Fig.11 Time histories of x and v for $x_0 = (\sqrt{50}$ (m), $\sqrt{50}$ (m), $\pi/2$ (rad)$)^T$

5 Conclusions

In the present paper, the new feedback control method of the mobile robot with nonholonomic constraints has been proposed. The method has the built-in time function generator called the *Time Base Generator* and the time behavior of the moving vehicle such as the convergence time and the velocity profile can be regulated through adjusting the parameters included in the TBG.

Some discontinuous points in the task space, however, are existing under the proposed method because of the property of the piecewise smooth feedback law. Future research will be directed to the singularity analysis.

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