A MODEL FOR THE GENERATION OF TARGET SIGNALS IN TRAJECTORY FORMATION

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INTRODUCTION

It is known from experiments on reaching that intermediate positions of the end-effector must be generated by the motor planner in addition to the final one and this can be performed by some kind of integration mechanism that is driven by the difference between the target and the current positions of the end-effector. From this point of view, we think that the central generator of kinematic patterns should be viewed as a *dynamical system* and not as some kind of *static* mechanism, like in the minimum-jerk model (Flash & Hogan, 1985) or the power-law model (Viviani & Terzuolo, 1982). An example is the non-linear leaky integrator described by (Bullock & Grossberg, 1989) in their *Vector Integration To Endpoint* (VITE) model: $\dot{x}(t) = G(t)(x_f - x)$, where x is the present position whereas x_f is the desired final position. The non-linearity is provided here by the gating action of a *GO* signal, $G(t)$, which is a scalar, increasing function of time that controls the initiation of the movement and the speed profile of the planned trajectory. In the case of complex motor tasks that may involve several concurrent targets, a single *GO* signal is also responsible for their synchronization. The *GO* signal, in particular, should generate a symmetric bell-shaped speed profile in the case of straight line trajectories; moreover, the integration process weighted by $G(t)$ should be able to reach the final target position in a finite time.

Non-linear leaky integrators of target error signals are not conceptually restricted to simple reaching movements, but can be extended to curved movements as well, including a target term depending on a directional error in addition to the basic one depending on the positional error. The model that we propose is a non-linear dynamical system that is capable of generating, as motor primitives, a family of curved trajectories that include straight lines as special cases. The dynamics of the model quite constraints the range of possible dynamic behaviours: differently from the VITE model, that can generate any kind of speed profile by an appropriate choice of the $G(t)$ function, the ξ-model is intrinsically based on a symmetric mechanism and it only has 3 free parameters of immediate cognitive significance.

TRAJECTORY GENERATION MODEL

The shape of a trajectory depends on the way in which curvature varies with the curvilinear coordinate. The model links shape and speed by means of a suitable time base generator that drives two equations: a linear-speed equation and a turning-speed equation.

The *time base generator* is modelled as a scalar dynamical system in the normalized variable ξ, obtained simply by adding a 2/3-power non-linearity to an integrator of logistic type:

$$\dot{\xi} = \gamma\,[\xi(1-\xi)]^{2/3} \qquad (1)$$

The system is supposed to generate smooth sigmoidal signals $\xi = \xi(t)$ from $\xi(0) = 0$ to $\xi(T_f) = 1$ with a symmetric, bell-shaped velocity profile and a desired (finite) movement duration T_f, due to its Non-Lipschitzian dynamics [1]. The γ parameter has a double function: it controls the

[1] Non-Lipschitzian systems are dynamical systems that have point attractors of infinite stability, in the sense

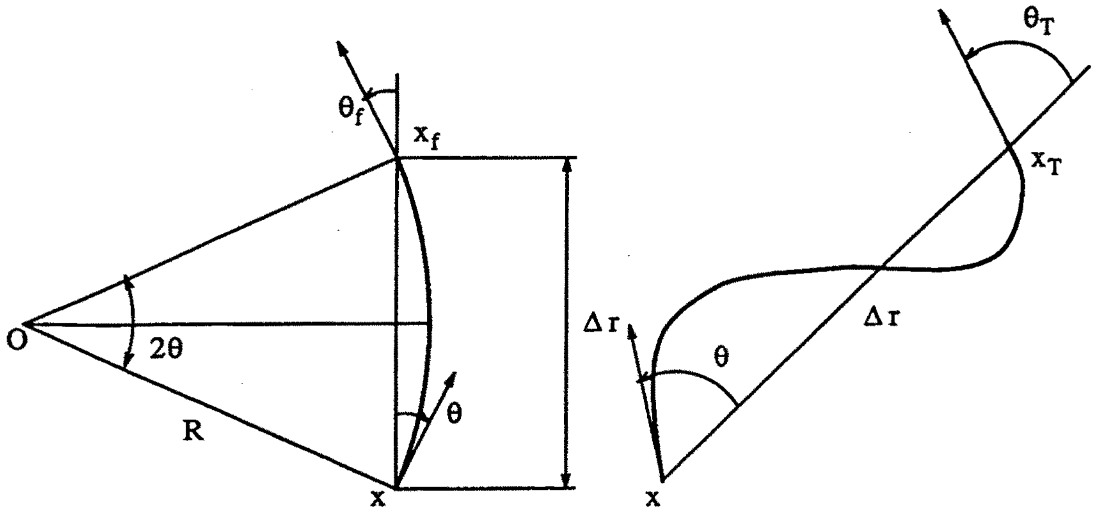

Figure 1: Circular (left) and Noncircular (right) trajectories

movement duration and can also be used to reset the time base generator and make it excitable for subsequent activation cycles.

The linear speed v and the turning speed ω can be linked to a common goal (reaching a target with a desired orientation) by using the same time base generator and a pair of error measures: the linear error Δr and the angular error $\Delta\theta$. The former one is simply the norm of the difference between the final and the current position: $\Delta r = ||x_f - x(t)||$. With this notation, it is possible to demonstrate that the following linear speed equation

$$v = \Delta r\,\dot{\xi}/(1-\xi) \tag{2}$$

allows $x(t)$ to reach x_f at the same time in which $\xi(t)$ reaches 1, provided that the turning speed equation allows Δr to decrease in a monotonic way. In the limit case of a straight trajectory, this is equivalent to a simple interpolation [2]. with no rotation: $x(t) = x_0 + (x_f - x_0)\xi(t)$

For general movements, in which the initial and/or the final directions are not aligned with $(x_f - x_0)$ vector, a sufficient condition for the trajectory to hit the target with the right direction is that at some point in the trajectory (i.e. for $\xi < 1$), the running point $x = x(t)$ reaches the configuration which characterizes a circular motion: $\theta(t) = -\theta_f$ (see Fig. 1, left). In this case, the following relation between ω and v holds: $\omega = v/R$ (R is the radius of the circle) and since $\Delta r = 2R\sin\theta$, we get $\omega = 2\sin\theta\,\dot{\xi}/(1-\xi)$. In general, we can write a turning speed equation

$$\omega = \Delta\theta\dot{\xi}/(1-\xi) \tag{3}$$

where the angular error term is given by $\Delta\theta(t) = 2\sin\theta(t)$ in the limit case of a circular motion. If the current conditions do not support a circular motion (see Fig. 1, right), then a possible strategy is to smoothly drive $\theta(t)$ towards such symmetric condition. The two criteria (approaching the condition of symmetry and following a circular path) can be combined in the following hierarchical way ($\Delta\theta_{sym} = -(\theta + \theta_T)$):

$$\Delta\theta = \Delta\theta_{sym} + 2\sin\theta e^{-\Delta\theta_{sym}{}^2/2\sigma^2} \tag{4}$$

that the gradient of their corresponding Lyapunov function diverges at equilibrium points; a consequence is that they reach equilibrium in finite time (i.e. their equilibrium configuration is a *terminal attractor*).

[2]It is interesting to note that, in this form, if we put $G(t) = \dot{\xi}/(1-\xi)$, the ξ-model is a special case of VITE.

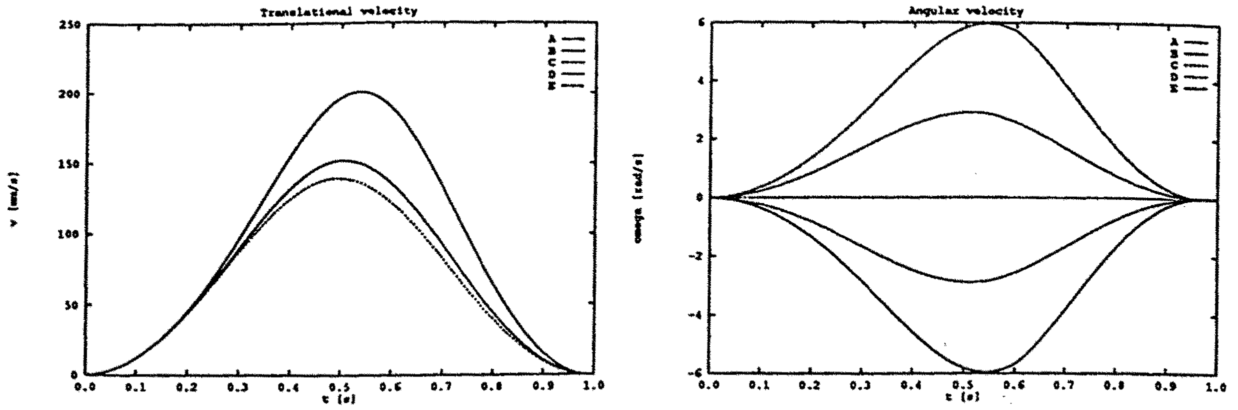

Figure 2: Simulated translational velocity (left) and angular velocity (right)

It can be seen that while the error term for symmetry is large, the equation reduces to $\Delta\theta = \Delta\theta_{sym}$. This equation works just like a feedback control law to attain a symmetric configuration where the target and present directions of the trajectory are symmetric with respect to the line between the initial and target points. Once the symmetric configuration is attained, the symmetry error vanishes and the motion equation of the angular velocity ω reduces to $\Delta\theta = 2\sin\theta$.

Consequently, once the symmetric configuration is attained, the ξ-model can generate the circular trajectory. The trajectory terminates at the target point where the translational velocity becomes zero. This guarantees the convergency of the ξ-model for curved movements. The value of σ determines the relative weights of the two components of the error function.

A number of trajectories were simulated by the ξ-model; the initial and target points were fixed, while several initial and target directions satisfying the symmetric configuration condition were used. The corresponding translational and angular velocities of the generated trajectories are indicated in Fig. 2. The spatial profiles of the generated trajectories are almost circular and the velocity profiles are almost bell-shaped, in accordance with some preliminary experimental data. The main difference with real drawing trajectories is that the ξ-model can generate the curved trajectory as well as the straight trajectory as a single stroke movement, whereas a segmentation phenomenon takes place for complex real trajectories. In a preliminary set of experiments, in which we recorded hand movements with a digitizing tablet, we estimated the range of values of the angular differences that produce trajectories with unimodal speed profiles.

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