MOTOR IMPEDANCE TRANSFORMATION IN MULTI-JOINT ARM MOVEMENTS

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ABSTRACT - In this paper, we propose a new method to transform the motor impedance from the end-point into the joint level. The CNS regulates the mechanical impedance of the end-point through the muscle and joint, which requires this kind of transformation about the motor impedance. Then it is shown that the redundancy of musculoskeletal system allows to select the impedance of each joint maintaining a specified end-point impedance.

INTRODUCTION

Motor impedance provides the static and dynamic relations between force and motion. Fine regulation of the motor impedance in multi-joint arms is one of the most important issues to perform tasks which require the dynamic interactions to their environments. For example, some frequently cited tasks such as turning a crank, inserting a peg and driving a screw , require stiffness regulation in terms of the task space coordinates and assign appropriate stiffness along each degrees of freedom[1].

Hogan claimed that the position control and force control were simply degenerate or extreme cases of impedance control and showed that biarticular muscles and redundancy of joint degrees of freedom in human arm played important roles in fine regulation of its end-point impedance[2]. Mussa Ivaldi showed that the joint stiffness could act as a constraint condition to the inverse kinematics problem of redundant arm[3]. However, we pay attention to the fact that the CNS is able to regulate the end-point impedance only through the muscle and joint[4][5]. This means that the CNS has to find some way to transform the desired end-point impedance into the joint and muscle impedances.

In this paper, we propose a new method to transform the motor impedance from the end-point level to the joint level. And it is shown that the redundancy of musculoskeletal system allows to select the impedance of each joint maintaining a specified end-point impedance.

THE MOTOR IMPEDANCE

We consider an upper limb model in Fig.1. Let the position vectors described in the joint and end-point coordinates be denoted as $\theta \in R^n$ and $X \in R^r$ respectively. Let also the corresponding force vectors be denoted as $\tau \in R^n$ and $F \in R^r$. n and r are the dimensions of the joint and end-point coordinates, respectively.

The transformation from θ to X is nonlinear. The Jacobian matrix J is the locally linearized transformation matrix which is defined by[6],

$$dX = J(\theta)d\theta . \quad (1)$$

Using the Jacobian matrix J, the transformation from F to τ is given by

$$\tau = J^T F . \quad (2)$$

The motor impedance is a general term for stiffness, viscosity and inertia. Here, the stiffness transformation between the joint coordinates and the end-point coordinates is discussed. Note that the same holds for the viscosity relationship.

The stiffness matrices are defined by,

1) end-point level ; $F = -K_e dX$ (3)

2) joint level ; $\tau = -K_j d\theta$ (4)

where $dX = X - X^e$ and $d\theta = \theta - \theta^e$. X^e and θ^e are equilibrium points of the corresponding vectors.

Using (1)-(4), we can see,

$$K_j = J^T K_e J . \quad (5)$$

The Jacobian matrix J represents the link structures of the human arm.

The corresponding transformation of compliance matrices is given by

$$C_e = J C_j J^T \quad (6)$$

where,

$$C_e = K_e^{-1} \quad (7)$$

$$C_j = K_j^{-1} . \quad (8)$$

If the stiffness matrix K_e is nonsingular in the end-point coordinates, the joint compliance matrix C_j seems to be computed from (5) and (8). However, if the arm has redundant degrees of freedom such as human arm, the transformation of motor impedance occurs a delicate issue. That is, even if K_e is nonsingular and Jacobian matrix J is of full row rank, K_j may not be nonsingular matrix. Then C_j can not be computed from (8). However, it should

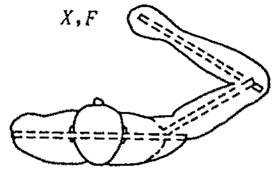

Fig.1 A Model of Human Upper Limb

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be noted that the excess of degrees of freedom of the joint coordinates allows to select the joint impedances in the redundant arm as the needs of the task.

IMPEDANCE TRANSFORMATION IN REDUNDANT ARM

When the end-point stiffness matrix K_e is given and is nonsingular, let's consider the method to transform K_e into the joint stiffness.

The end-point compliance matrix is given by

$$C_e = K_e^{-1} = JC_jJ^T \tag{9}$$

from (6) and (7). This implies that the transformation problem of stiffness matrices is equivalent to solving the matrix equation (9) with respect to the joint compliance matrix C_j.

The general solution of the matrix equation (9) is given by

$$C_j = J^+C_e(J^T)^+ + [Z-J^+JZ(J^+J)^T] \tag{10}$$

where $Z \in R^{n\times n}$ is an arbitrary matrix and the superscript $^+$ denotes the Moore-Penrose generalized inverse[6].

<u>proof.</u> Substituting (10) into (9), we have

$$JC_jJ^T = JJ^+C_e(JJ^+)^T + J[Z-J^+JZ(J^+J)^T]J^T$$
$$= C_e \,. \tag{11}$$

Hence, it is clear that C_j in (10) is a solution of the matrix equation (9). On the other hand, let an arbitrary solution be denoted as S, then we can write

$$S = J^+C_e(J^T)^+ + [Z_1-J^+JZ_1(J^+J)^T] \,, \tag{12}$$

where Z_1 is defined by

$$Z_1 = S - J^+C_e(J^+)^T \,. \tag{13}$$

This completes the proof. [ ]

(10) means that the compliance of each joint can be arbitrarily specified through the matrix Z, while maintaining the desired end-point impedance. Note that the same may hold in terms of the viscosity transformation.

Table 1 shows the simulation results of stiffness transformation with a three-links planar arm (Fig.2). Let the end-point stiffness K_e be given as an identity matrix. In case 2, the compliance of wrist joint is larger than that of the shoulder and elbow joint. Conversely, in case 3, the compliance of shoulder joint is larger than the others. Note that the end-point stiffness in all cases is the identity matrix and that C_j in case 2 and 3 are non-singular matrices.

CONCLUSION

In this paper, we proposed the transformation method of the end-point impedance into the joint impedance. It was shown that the redundancy of the musculoskeletal system allowed to select the impedance of each joint maintaining a specified end-point impedance, which explains a part of the flexibility of human movements.

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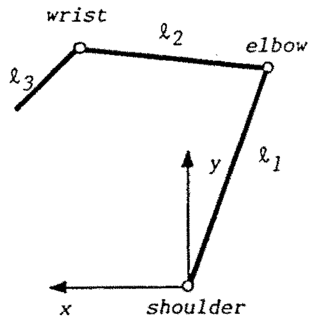

Fig.2 A Three-Link Planar Arm

Table 1 Joint Compliance Matrices with the Same End-Point Compliance

	Case 1	Case 2	Case 3
The Arbitrary Transformation Matrices, Z	$\begin{bmatrix} 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 \end{bmatrix}$	$\begin{bmatrix} 10.0 & 0.0 & 0.0 \\ 0.0 & 10.0 & 0.0 \\ 0.0 & 0.0 & 100.0 \end{bmatrix}$	$\begin{bmatrix} 100.0 & 0.0 & 0.0 \\ 0.0 & 80.0 & 0.0 \\ 0.0 & 0.0 & 10.0 \end{bmatrix}$
The Joint Compliance Matrices, C_j	$\begin{bmatrix} 12.4 & -4.4 & -4.8 \\ -4.4 & 9.3 & 4.9 \\ -4.8 & 4.9 & 3.2 \end{bmatrix}$	$\begin{bmatrix} 9.3 & 0.69 & 0.45 \\ 0.69 & 0.66 & -4.0 \\ 0.45 & -4.0 & 98.3 \end{bmatrix}$	$\begin{bmatrix} 20.8 & -17.0 & 12.2 \\ -17.0 & 27.9 & -17.0 \\ 12.2 & -17.0 & 0.01 \end{bmatrix}$

nominal posture : $\theta_1 = -20$, $\theta_2 = 15$, $\theta_3 = 50$ (deg.)