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Chapter 15

Robust Static and Dynamic Output Feedback Suboptimal Control of Uncertain Discrete-Time Systems Using Additive Gain Perturbations

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Abstract

This paper provides a novel design method in the case of an output feedback suboptimal control problem for a class of uncertain discrete-time system using additive gain perturbations. Based on the linear matrix inequality (LMI), a class of the fixed output feedback controller is established, and some sufficient conditions for the existence of the suboptimal controller are derived. The novel contribution is that time-variant additive gain perturbations are included in the feedback systems. Although the additive gain perturbations work using the feedback systems, both stability of closed-loop systems and adequate suboptimal cost are attained. The numerical example demonstrates that the large cost due to the LMI design can be reduced by using additive gain perturbations.

1. Introduction

It is well known that uncertainty occurs in many dynamic systems, and is frequently a source of instability and performance degradation of systems. In recent years, the problem of designing robust controllers for linear systems with parameter uncertainty has received considerable attention in control system literature (see e.g., [32] and reference therein).

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Although there have been numerous results on the robust control of discrete-time uncertain systems, there have been considerable efforts to find a controller in order to guarantee robust stability. However, it is also desirable to design a control system that is not only asymptotically stable but also guarantees an adequate cost performance level. One design approach to this problem is the so-called guaranteed cost control [25]. This approach has the advantage of placing an upper bound on a given performance index, and it is guaranteed that the system performance degradation due to the uncertainty is smaller than the cost bound.

When controlling a practical system, it is not always possible to have access to the state vector, and only partial information from a measured output vector is available. Therefore, the output feedback problem for uncertain systems is an important problem. For example, the guaranteed cost control for an uncertain discrete time system by using static output feedback control that is based on Riccati equation has been discussed in [4]. However, for the existing static output feedback control system design, the implementation of the controller appears to be difficult because a conservative condition is assumed. In recent years, linear matrix inequality (LMI) has gained considerable attention for its computational efficiency and usefulness in control theory. For example, the necessary and sufficient condition for stabilizability via the static output feedback is proposed in terms of two LMIs under a coupling condition [13]. On the other hand, the guaranteed cost control problem for a class of the uncertain system with delay that is based on the LMI design approach was solved by using the output feedback [17]. Furthermore, the output feedback guaranteed cost control problems for the uncertain discrete delay and discrete-time systems through the LMI optimization technique have been tackled, respectively [24, 34]. However, due to the presence of the design parameter that is included in the LMI technique, it is known that the cost bound becomes fairly large. Moreover, the controller gain perturbations have not been considered.

In the past decade, several stable adaptive neural control approaches have been introduced [3, 23]. Moreover, closely related fuzzy control schemes have been studied [22, 29]. Later, the theoretical foundations for the efficient design of NN controllers based on inverse control have been reported [2]. The stability properties of the learning scheme using neural networks were investigated for the restricted case [27]. In [28], the feedback control law that guarantees semiglobal uniform ultimate boundedness has been proposed. On the other hand, several good NN control approaches have been proposed based on Lyapunovfs stability theory [5, 6, 15]. However, these researches have focused mainly on the analysis of the stability.

As another important study, the linear quadratic regulator (LQR) problem using the NN or fuzzy logic has been investigated [1,9,10,30]. It is the advantage of these approaches that the controllers can be implemented even without an exact knowledge of the plant dynamics. However, the stability may not be guaranteed because in these researches the stability of the original overall closed-loop system that includes the neurocontroller or fuzzy controller has not been considered. In fact, it has been shown that the system stability is destroyed when the degree of the system nonlinearity is high [9]. In [16], a nonlinear optimal design method that integrates linear optimal control techniques and neural network has been investigated. The global asymptotic stability is guaranteed under the assumption that the nonlinear function is known completely [16]. However, if such an assumption is not met, only a uniformly ultimate boundedness is attained. Moreover, an output feedback scheme has not been considered. Although the stability of the closed-loop system with the neurocontroller have been studied via the LMI-based design approach [11, 18–20], the output feedback system that has uncertainty in the input matrix has also not been considered.

In this paper, the output feedback suboptimal control problem of the discrete-time uncertain system that has uncertainty in both the state and input matrices is discussed. The new contributions of our study are as follows. First, it is newly shown that the output feedback control can be designed by adapting an additive control input, such as a neurocontroller or fuzzy controller. Second, although the neurocontroller or fuzzy controller is included in the discrete-time uncertain system, the closed-loop system is guaranteed robust stability, and there is a reduction in the cost. Another important feature is that a class of the fixed output feedback controller of the discrete-time uncertain system with additive gain perturbations [33] is newly established by means of the LMI. Furthermore, in order to reduce the large cost incurred by the LMI approach, the fuzzy controller is substituted into the additive gain perturbations. As a result, although additive gain perturbations such as the fuzzy controller are included in the discrete-time uncertain system, robust stability of the closed-loop system and reduction in the cost are both attained. It is noteworthy that the concept of such a novel controller synthesis has not existed till now. Finally, in order to demonstrate the efficiency of our design approach, a numerical example is given when the fuzzy controller is used.

2. Novel Concept

First, in order to show the effectiveness of the novel suboptimal control concept via the additive control gain, one example is demonstrated. The example is a scalar dimension, and the controller is designed to trace the required trajectory with minimum energy.

Let us consider the following system.

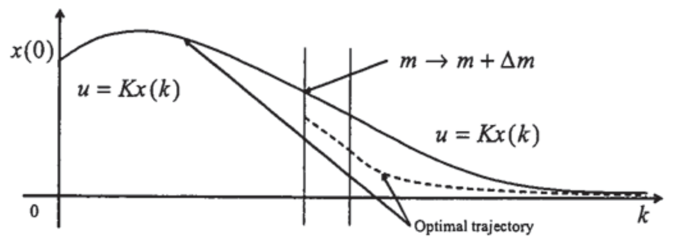
$$\begin{aligned} x(k+1) &= [A + DF(k)E_a]\,x(k) + [B + DF(k)E_b]\,u(k) \\ &= F(k)x(k) + u(k), \quad 0 < F(k) \le 0.5, && \text{(1a)} \\ u(k) &= [K + D_k N(k) E_k]\,x(k) = [-1 + N(k)]\,x(k), \quad 0 \le N(k) \le 1, && \text{(1b)} \\ J &= \sum_{k=0}^{\infty}\Big[x^T(k)Qx(k) + u^T(k)Ru(k)\Big] = \sum_{k=0}^{\infty}\Big[x^2(k) + u^2(k)\Big]. && \text{(1c)} \end{aligned}$$

Then, taking $0 < F(k) \le 0.5$ and $0 \le N(k) \le 1$ into account, the closed-loop system (2) is stable.

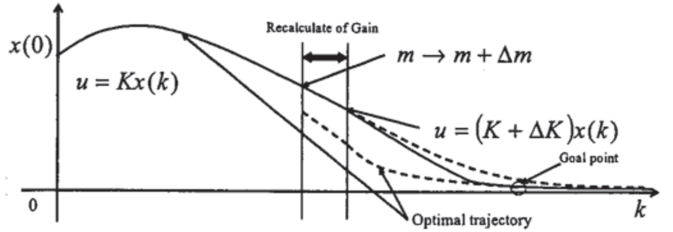
$$x(k+1) = [-1 + F(k) + N(k)]\,x(k), \quad -1 < -1 + F(k) + N(k) \le 0.5. \tag{2}$$

It is assumed that the uncertainty of $F(k)$ should be changed as the step response, e.g., the uncertain parameter such as mass jumps from the original value to the update value due to the reduction of mass for any time. It should be noted that $N(k)$ is the proposed novel time-varying additive control gain.

In this situation, the LQR technique is used to minimize the cost function (1c). For using the existing LQR theory cannot be applied to this problem because the system has the



(a) Without additive control gain



(b) With additive control gain

Figure 1. New concept using additive control gain.

uncertainty with the change of the step response. On the other hand, it is possible to solve this optimization problem by introducing the additive control gain $N(k)$. Furthermore, the closed-loop system is stable because $-1 \leq -1 + N(k) \leq 0$ holds.

In fact, when the uncertainty of $F(k)$ has changed as follows for any time $k = \bar{k}$,

$$F(k) = 0.5, \quad 0 \leq k \leq \bar{k} \rightarrow F(k) = 0.25, \quad \bar{k} \leq k \tag{3}$$

the optimal gains can be computed by applying the LQR theory, respectively.

$$F(k) = 0.5 \Rightarrow -1 + N(k) = -0.26556, \quad 0 \leq k \leq \bar{k},$$
$$F(k) = 0.25 \Rightarrow -1 + N(k) = -0.12695, \quad \bar{k} \leq k.$$

It should be noted that these values satisfy the range of stability margin for $N(k)$. Finally, the suboptimal trajectory can also be attained by recalculating and letting the new gain after the change of mass. The concept of the novel control synthesis is illustrated in Figure 1.

The above concept seems to be natural and reliable. In this paper, the adaptation of the additive control gain will be carried out artificially by using the fuzzy logic control, while the fixed control gain can be computed by applying the guaranteed cost control technique via the LMIs.

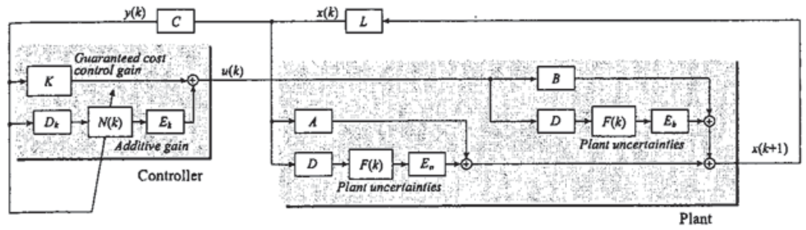


Figure 2. Block diagram of a new proposed system.

3. Preliminary

Consider the following class of the uncertain discrete-time linear system.

$$x(k+1) = [A + \Delta A(k)]\, x(k) + [B + \Delta B(k)]\, u(k), \tag{4a}$$
$$y(k) = Cx(k), \tag{4b}$$
$$u(k) = [K + \Delta K(k)]\, y(k), \tag{4c}$$

where $x(k) \in \Re^n$ is the state, $y(k) \in \Re^l$ is the output and $u(k) \in \Re^m$ is the control input. A, B and C are known constant matrices, $K \in \Re^{m \times l}$ is the fixed control matrix of the controller (4c). $\Delta A(k)$ and $\Delta B(k)$ are parameter uncertainties, and $\Delta K(k)$ is the additive gain perturbations. The parameter uncertainties and the additive gain perturbations considered here are assumed to be of the following form

$$\left[\begin{array}{cc} \Delta A(k) & \Delta B(k) \end{array}\right] = DF(k) \left[\begin{array}{cc} E_a & E_b \end{array}\right], \quad \Delta K(k) = D_k N(k) E_k, \tag{5}$$

where D, D_k, E_a, E_b and E_k are known constant matrices, $F(k) \in \Re^{p_a \times q_a}$ is an unknown matrix function and $N(k) \in \Re^{p_n \times q_n}$ is an arbitrary function. It is assumed that $F(k)$ and $N(k)$ satisfy (6).

$$F^T(k)F(k) \leq I_{q_a}, \quad N^T(k)N(k) \leq I_{q_n}. \tag{6}$$

Although these assumptions (6) appear to be a conservative condition, they are necessary to establish the LMI condition. It should be noted that the assumptions are based on the control-oriented assumption from the existing results [4, 31, 32]. Moreover, $N(k)$ will be used as the additive gain perturbations such as a neurocontroller or fuzzy controller.

The block diagram of the new proposed method is shown in Figure 2, where L is a time lag. It should be noted that the controller (4c) has additive gain perturbations as the matrix function $\Delta K(k)$, as compared to the existing results [9, 10]. In this paper, it is considered that the suboptimal control can be achieved by adapting the arbitrary function $N(k)$ with a NN or fuzzy logic.

The quadratic performance index (7) is associated with the system (1),

$$J = \sum_{k=0}^{\infty} \left[x^T(k)Qx(k) + u^T(k)Ru(k) \right], \tag{7}$$

where $Q \in \Re^{n\times n}$ and $R \in \Re^{m\times m}$ are given as the positive definite symmetric matrices.

It should be noted that the transient response can be improved appropriately by changing the weight matrices $Q \in \Re^{n\times n}$ and $R \in \Re^{m\times m}$. In this situation, the definition of the suboptimal control with additive gain perturbations is given below.

Definition 3..1. For the uncertain discrete-time system (1) and cost function (7), if there exists a fixed control matrix K and a positive scalar J^* such that for the admissible uncertainties and additive control gain (5), the closed-loop system is asymptotically stable and the closed-loop value of the cost function (7) satisfies $J < J^*$, then J^* and K are said to be the suboptimal cost and suboptimal control gain matrix, respectively.

The above definition is very popular for dealing with the time-varying uncertainties and is also used in [25].

The following theorem gives the sufficient condition for the existence of the suboptimal control.

Lemma 3..2. *Suppose that the following matrix inequality holds for the uncertain discrete-time system* (1) *with the cost function* (7) *for all* $x(k) \neq 0$.

$$x^T(k+1)Px(k+1) - x^T(k)Px(k) + x^T(k)\left[Q + C^T\tilde{K}^T R\tilde{K}C\right]x(k) < 0, \tag{8}$$

where $\tilde{K} := K + D_k N(k) E_k$.

If such a condition is met, the matrix K *of the controller* (4c) *is the suboptimal control matrix associated with the cost function* (7). *That is, the closed-loop uncertain system*

$$x(k+1) \quad = \quad [(A + DF(k)E_a) + (B + DF(k)E_b)\tilde{K}C]x(k), \tag{9}$$

is stable and achieves the following inequality

$$J < J^* = x^T(0)Px(0), \tag{10}$$

where $x(0) \neq 0$.

Proof. Let us define the following Lyapunov function candidate

$$V(x(k)) = x^T(k)Px(k), \tag{11}$$

where P is the positive definite matrix. By considering (8), it follows that $V(x(k+1)) - V(x(k)) = x^T(k+1)Px(k+1) - x^T(k)Px(k) < 0$ holds. Thus, the closed-loop uncertain system is stable. Moreover, summing the inequality (8) from zero to N results in

$$x^T(N+1)Px(N+1) - x^T(0)Px(0) + \sum_{k=0}^{N} x^T(k)[Q + C^T\tilde{K}^T R\tilde{K}C]x(k) < 0.$$

Since the closed-loop uncertain system is stable, $x(N+1) \to 0$ as $N \to \infty$. Finally, (10) holds. This is the desired result. □

The objective of this section is to design a fixed suboptimal control gain matrix K for the uncertain system (1) via the LMI design approach.

$$\begin{bmatrix} 0 \\ B \\ I_m \\ 0 \\ 0 \\ E_b \\ 0 \end{bmatrix}^{\perp} \begin{bmatrix} -Y & A^T & 0 & 0 & E_a^T & 0 & I_n \\ A & -X + (\mu_1 + \mu_2)DD^T & BD_kD_k^T & BD_k & 0 & 0 & 0 \\ 0 & [BD_kD_k^T]^T & -R^{-1} + D_kD_k^T & 0 & [E_bD_kD_k^T]^T & 0 & 0 \\ 0 & [BD_k]^T & 0 & -I_m & [E_bD_k]^T & 0 & 0 \\ E_a & 0 & E_bD_kD_k^T & E_bD_k & -\mu_1 I_{q_a} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\mu_2 I_{q_n} & 0 \\ I_n & 0 & 0 & 0 & 0 & 0 & -(Q + E_k^T E_k)^{-1} \end{bmatrix} \begin{bmatrix} 0 \\ B \\ I_m \\ 0 \\ 0 \\ E_b \\ 0 \end{bmatrix}^{\perp T} < 0, \tag{12a}$$

$$\begin{bmatrix} C^T \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^{\perp} \begin{bmatrix} -Y & A^T & 0 & 0 & E_a^T & 0 & I_n \\ A & -X + (\mu_1 + \mu_2)DD^T & BD_kD_k^T & BD_k & 0 & 0 & 0 \\ 0 & [BD_kD_k^T]^T & -R^{-1} + D_kD_k^T & 0 & [E_bD_kD_k^T]^T & 0 & 0 \\ 0 & [BD_k]^T & 0 & -I_m & [E_bD_k]^T & 0 & 0 \\ E_a & 0 & E_bD_kD_k^T & E_bD_k & -\mu_1 I_{q_a} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\mu_2 I_{q_n} & 0 \\ I_n & 0 & 0 & 0 & 0 & 0 & -(Q + E_k^T E_k)^{-1} \end{bmatrix} \begin{bmatrix} C^T \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^{\perp T} < 0, \tag{12b}$$

$$\begin{bmatrix} X & I_n \\ I_n & Y \end{bmatrix} \geq 0, \ \mathrm{rank}\begin{bmatrix} X & I_n \\ I_n & Y \end{bmatrix} \leq n. \tag{12c}$$

Theorem 3..3. *Consider the uncertain discrete-time system* (1) *and cost function* (7). *For the unknown matrix function* $F(k)$ *and arbitrary function* $N(k)$, *if the LMIs* (9) *have a feasible solution such as the symmetric positive definite matrices* $X \in \Re^{n\times n}$ *and* $Y \in \Re^{n\times n}$, *and the positive scalars* $\mu_1 > 0$ *and* $\mu_2 > 0$, *then* K *is a fixed suboptimal control matrix gain.*

Furthermore, the corresponding value of the cost function (7) *satisfies the following inequality* (13) *for all admissible uncertainties* $F(k)$ *and the arbitrary function* $N(k)$

$$J < J^* = x^T(0)X^{-1}x(0). \tag{13}$$

In order to prove Theorem 3, the following Lemmas will be used [12, 14].

Lemma 3..4 ([12]). *Let matrices* $\mathtt{U} \in \Re^{d_n\times d_m}$, $\mathtt{V} \in \Re^{d_k\times d_n}$, *and* $\mathtt{W} = \mathtt{W}^T \in \Re^{d_n\times d_n}$ *be given. Suppose* $\mathrm{rank}\,(\mathtt{U}) = d_m < d_n$ *and* $\mathrm{rank}\,(\mathtt{V}) = d_k < d_n$. *Then there exists a matrix* $\mathtt{K} \in \Re^{d_m\times d_k}$ *satisfying* $\mathtt{UKV} + (\mathtt{UKV})^T + \mathtt{W} < 0$ *if and only if the matrices* U, V *and* W *satisfy* $\mathtt{U}^{\perp}\mathtt{W}\mathtt{U}^{\perp} < 0$, $\mathtt{V}^{T\perp}\mathtt{W}\mathtt{V}^{T\perp T} < 0$, *where* $\mathcal{M}^{\perp}$ *denotes a left annihilator of* $\mathcal{M}$.

Lemma 3..5 ([14]). *Let* G, H *and* F *be real matrices of appropriate dimensions with* $\mathtt{F}\mathtt{F}^T \leq I_n$. *Then, for any given* $\varphi > 0$, *the inequality* $\mathtt{GFH} + (\mathtt{GFH})^T \leq \varphi\mathtt{G}\mathtt{G}^T + \varphi^{-1}\mathtt{H}^T\mathtt{H}$ *holds.*

Proof. Applying the Schur complement [35] and standard inequality as Lemma 5 to the matrix inequality (8), and using the existing results [13] as Lemma 4 yield the inequalities (9). On the other hand, since the results of the cost bound (10) can be proved by using a similar argument for the proof of Lemma 2, it is omitted. □

It is shown that the overall stability of closed-loop systems is guaranteed in Theorem 3. We propose that a neurocontroller or fuzzy controller can be substituted for the controller based on additive gain perturbations. Based on this proposal, the proof has been completed by regarding these additive gain perturbations as the uncertainties. Finally, although the neurocontroller or fuzzy controller is included in the discrete-time uncertain system, the robust stability of the closed-loop system is attained.

Although the considered problem has the additive gain perturbations in the output feedback gain, it might appear that the obtained results via the LMI condition is not a novel contribution because they can be easily derived by applying the existing results [24]. However, the method has succeeded in avoiding the bilinear matrix inequality (BMI) condition that has been established in [21]. Moreover, the main contribution is to propose that the additive gain perturbations such as a neurocontroller or fuzzy controller instead of an uncertainty output feedback controller can be used for achieving reduction in the enormous cost caused by the conservative LMI conditions. It is worth pointing out that there the concept of such a novel controller synthesis has not previously existed.

The following algorithm [7] can be used to find the matrix pair (X, Y) such that the LMIs (9) with $X = Y^{-1} > 0$ are satisfied.

Algorithm 1 ([7]). To solve such above problem, the linearization algorithm is conceptually described as follows.

1) Find a feasible point $(\varepsilon_1^0,\ \varepsilon_2^0,\ X^0,\ Y^0)$ that satisfy the LMIs (9). If there are no such points, exit. Set $r = 0$.

2) Set $V^r = Y^r, W^r = X^r$, and find X^{r+1}, Y^{r+1} that solve the following LMI problem.

minimize $\mathbf{Trace}(V^r X + W^r Y)$ subject to the LMIs (9).

3) If a stopping criterion is satisfied, exit. Otherwise, set r = r+1 and go to step 2).

Based on [7], it was shown that the algorithm converges to some value. On the other hand, it should be noted that Algorithm 1 may not be able to find the smallest-order controller in all the cases [7]. Moreover, it is possible to verify a vibration phenomenon and a very slow rate of convergence through simulation in some cases [8].

It is easy to acquire a solution set of $(\varepsilon_1,\ \varepsilon_2,\ X,\ Y)$ because the algorithm is simple LMI problem. If the solution set has the relation $X = Y^{-1} > 0$, the suboptimal control gain matrix K is obtained by using the Matlab toolbox with Lemma 4.

For the uncertain discrete-time system (1) associated with the cost function (7), the suboptimal cost J^* can be achieved if the feasible solution exists.

4. Main Idea

The LMI approach for the uncertain discrete-time systems usually results in the conservative controller design due to the existence of the uncertainties ΔA, ΔB and the additive gain perturbations ΔK. As a result, the cost J becomes large. The main contribution of this paper is to apply the NN or fuzzy logic as the additive gain perturbations to improve the cost performance.

It is well known that NNs have found wide potential applications in system control because of their ability to perform nonlinear mapping. Therefore, since a sufficiently accurate model of the system is generally not available, using the nonlinear mapping provided by the neural output with the uncertainty determined will result in a better performance. On the other hand, fuzzy control is a theory that can be the best option when the existing control method is hard to apply to systems due to difficulties with regard to a mathematical model or due to a nonlinear model. Since fuzzy control translates a rule of thumb into certain inputs/outputs, it is easy to design the controller and it is possible to implement it without the knowledge of the exact system model.

It should be noted that the proposed neurocontroller and fuzzy controller regulate their outputs in real time with a robust stability by the LMI approach. Hence, it can be expected that there will be a reduction in the cost when the uncertain discrete-time system performs the nominal closed-loop system. That is, the neurocontroller or fuzzy controller is required to compensate as the nominal system. This idea can be explained as follows. First, it should be noted that the gain for the nominal systems will be derived from the LQR theory. Since the nominal systems attain the minimum cost, if the neurocontroller or fuzzy controller is programmed such that the resulting system response closely approaches that of a nominal system, these controlled systems can be expected to achieve better performances.

Let us consider the following nominal system without uncertainties.

$$\begin{aligned} \hat{x}(k+1) &= A\hat{x}(k) + B\hat{u}(k), && (14a)\\ \hat{y}(k) &= C\hat{x}(k), && (14b)\\ \hat{u}(k) &= \hat{K}\hat{y}(k), && (14c) \end{aligned}$$

where $\hat{x}(k) \in \Re^n$ is the state, $\hat{y}(k) \in \Re^l$ is the output and $\hat{u}(k) \in \Re^m$ is the control input. $\hat{K} \in \Re^{m\times l}$ is the output feedback gain for the nominal system (11). The quadratic cost function (9) is associated with the system (11).

$$\hat{J} = \sum_{k=0}^{\infty} \left[\hat{x}^T(k)Q\hat{x}(k) + \hat{u}^T(k)R\hat{u}(k)\right]. \tag{15}$$

The control gain $\hat{K}$ is derived by means of the existing LMI approach [13] for the nominal system (11) and cost function (15). For the nominal system (11) and the cost function (15), it is known that the cost of the nominal system $\hat{J}^*$ is smaller than that of the uncertain system J^*. As a result, if the behavior of the additive gain perturbation that is obtained from the NN or fuzzy logic is sufficiently close to that of the closed-loop nominal system that is based on the LQR theory, the increase in the cost caused by the LMI design can be reduced.

Remark 4..1. Without loss of generality, using the result in [26], it can be assumed that the uncertainty $F(k)$ is the Gaussian white noise process with zero mean. Moreover, the condition $E[x(0)x^T(0)] = I_n$ is also assumed, where $E[\cdot]$ denotes the expectation value. It may be noted that although these conditions seem to be conservative, they can be checked before control is implemented. In this situation, the cost of the uncertain system under the proposed control without the intelligent control scheme such as NN can be computed as follows.

$$\begin{aligned} J^* &:= E\left[\sum_{k=0}^{\infty}\Big(x^T(k)Qx(k)+u^T(k)Ru(k)\Big)\right] \\ &= \sum_{k=0}^{\infty} E\left[x^T(k)(Q+C^TK^TRKC)x(k)\right] \\ &= \sum_{k=0}^{\infty} \text{Trace}\left\{(Q+C^TK^TRKC)\Big((A+BKC)^k(A+BKC)^{Tk}+\mathcal{E}(k)\Big)\right\}, \end{aligned}$$

where

$$\begin{aligned} x(k) &= \mathcal{A}(k-1)\cdots\mathcal{A}(1)\mathcal{A}(0)x(0), \ \ k\ge 1, \ \ \mathcal{A}(k) := A+BKC+\mathcal{F}(k), \\ \mathcal{F}(k) &:= DF(k)(E_a+E_bKC), \\ E[F(k)] &:= 0, \ E[F^T(k)F(k)] := \sigma I_{q_a}, \\ \mathcal{E}(k) &:= E\big[\mathcal{F}(k-1)\cdots\mathcal{F}(1)\mathcal{F}(0)\mathcal{F}^T(0)\mathcal{F}^T(1)\cdots\mathcal{F}^T(k-1)\big]+\cdots \\ &= k\sigma D(E_a+E_bKC)(E_a+E_bKC)^TD^T+\cdots\ge 0, \ \ k\ge 1, \ \ \mathcal{E}(0)=0 \end{aligned}$$

and K is the output feedback control gain that is based on the proposed LMIs (9).

On the other hand, the cost of the nominal system under the control of (Iwasaki et al 1994) is given below.

$$\begin{aligned} \hat{J}^* &:= E\left[\sum_{k=0}^{\infty}\Big(x^T(k)Qx(k)+u^T(k)Ru(k)\Big)\right] \\ &= \sum_{k=0}^{\infty} E\left[x^T(k)(Q+C^T\hat{K}^TR\hat{K}C)x(k)\right] \\ &= \sum_{k=0}^{\infty} \text{Trace}\Big\{(Q+C^T\hat{K}^TR\hat{K}C)(A+B\hat{K}C)^k(A+B\hat{K}C)^{Tk}\Big\}, \end{aligned}$$

where $\hat{K}$ is the suboptimal gain such that $\hat{J}^*$ is minimized.

Thus, it can be shown that the cost of the nominal system is smaller than that for the uncertain system without NN or fuzzy control.

5. Control Algorithm Using Fuzzy Logic

In order to ensure easy implementation and simple design, fuzzy control is proposed for the reduction in the cost performance. The proposed fuzzy controller regulates the arbitrary function $N(k)$ so that the response of the uncertain system may approach that of the nominal system. It should be noted that although the neurocontroller can also be regulated and this discussion appears to be helpful, it is omitted due to page limitation.

In this paper, the error $E_f(k)$ between the proposed system (1) and the nominal system (11) and the difference of error $\Delta E_f(k)$ are defined as the performance index to decide the fuzzy rules. The error $E_f(k)$ and the difference of error $\Delta E_f(k)$ can be defined as

$$\begin{aligned} E_f(k) &= \hat{y}(k)-y(k), && (16a) \\ \Delta E_f(k) &= E_f(k)-E_f(k-1). && (16b) \end{aligned}$$

$E_f(k)$ and $\Delta E_f(k)$ are defined as the input of the fuzzy controller. Hence, the fuzzy controller outputs arbitrary function $N(k)$. The membership functions and their ranges are shown in Figure 3 and Figure 4. As a symbol that denotes degree and sign of $E_f(k)$, $\Delta E_f(k)$ and $N(k)$, Negative Big (NB), Negative Middle (NM), Negative Small (NS), Zero (ZO), Positive Small (PS), Positive Middle (PM), Positive Big (PB) are defined. The range of the membership functions is selected according to the maximum error and the difference of the error values when $N(k)=0$ for the proposed system (1).

The relationship between the input and output of the fuzzy controller is the most important part. This relationship, which is called if-then rules, must be obtained correctly to improve the performance of the fuzzy logic control system. The fuzzy logic is determined not by a strict value but by a vague expression. Therefore, the proposed fuzzy rules can be achieved by expressions such as "Big" or "Small". The process for determining the rules is whether the arbitrary function $N(k)$ should be increased or decreased by the error $E_f(k)$ and the difference of in the error $\Delta E_f(k)$. As a result, the control rules when the initial condition of the proposed system changes from the positive values to the origin are considered.

In order to determine the amount of increment or decrement fot the arbitrary function $N(k)$, If-then rules are used. These rules are converted into a table as given in Table 1. For example, when $\Delta E_f(k)$ is Zero (ZO) and $E_f(k)$ is Negative Big (NB), then $N(k)$ is should be Positive Big (PB) to increase the absolute value of $\| K+\Delta K(k) \|$. As a result, the convergence (change) will be fast (great). In the other case, when $\Delta E_f(k)$ is Positive Big (PB) and $E_f(k)$ is Zero (ZO), then $N(k)$ is should be Negative Big (NB) so that the absolute value of $\| K+\Delta K(k) \|$ is decreased. Thus, the convergence (change) will be slow (small). In this way, the control rules are set by considering how $E_f(k)$ and $\Delta E_f(k)$ change.

In this paper, fuzzy subsets of the output $H_i(k)$ are given in the following form

$$\begin{aligned} &\text{If } \ E_r(k) \ \text{ is } \ h_{1i}(k) \ \text{ and } \ \Delta E_r(k) \ \text{ is } \ h_{2i}(k), \\ &\text{then } \ N(k) \ \text{ is } \ H_i(k), \ \ i=1,\ldots,M, \end{aligned} \tag{17}$$

where M is the total number of rules, and $h_{1i}(k)$ and $h_{2i}(k)$ are fuzzy subsets of the input at step k. An OR operation is applied to the fuzzy subsets $H_i(k)$, and $N(k)$ can be obtained by calculating its center of gravity. Then, $N(k)$ is given by

$$N(k) = \frac{\sum_{i=1}^{M}\phi_i\,S(\phi_i)}{\sum_{i=1}^{M}S(\phi_i)}, \tag{18}$$

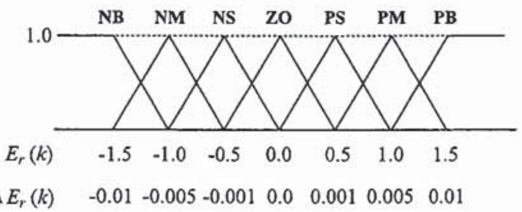


Figure 3. Membership function for input of fuzzy controller.

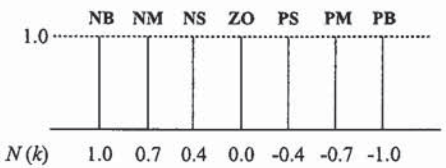


Figure 4. Membership function for output of fuzzy controller.

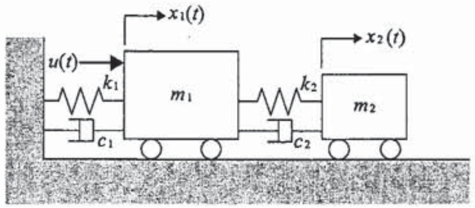


Figure 5. Two cart systems.

where $S(\phi_i)$ is the OR operation set of $H_i(k)$, and ϕ_i is the horizontal axis of the membership function for the output of the fuzzy controller. Using (17), (18) and the proposed If-then rules, the fuzzy controller can regulate $N(k)$ so that the cost J at each step k is decreased.

6. Numerical Example

In order to demonstrate the effectiveness of the proposed fuzzy controller, a numerical example is given. Let us consider a two-cart system that is shown in Figure 5. $x_1(t)$ and $x_2(t)$ are respectively the positions of the cart A and the cart B, and $u(t)$ is the control input, k_1 and k_2 are the spring constants, c_1 and c_2 are the damper constants and m_1 and m_2 are mass of cart A and cart B, respectively. In this system, a frictional force between the floor and wheel of the cart is not considered. By choosing the cart positions and their

Table 1. If-then rules for uncertain system.

		$\Delta E_r(k)$						
		NB	NM	NS	ZO	PS	PM	PB
	NB				PB	PM		
	NM				PM	PS		
	NS				PS	ZO		NM
$E_r(k)$	ZO	PB	PM	PS	ZO	NS	NM	NB
	PS	PM		ZO	NS			
	PM			NS	NM			
	PB			NM	NB			

velocities as the state variables and observing the cart positions as the output variables, the continuous-time state-space model of the two cart system is given by

$$\dot{x}(t) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\dfrac{k_1+k_2}{m_1} & \dfrac{k_2}{m_1} & -\dfrac{c_1+c_2}{m_1} & \dfrac{c_2}{m_1} \\ \dfrac{k_2}{m_2} & -\dfrac{k_2}{m_2} & \dfrac{c_2}{m_2} & -\dfrac{c_2}{m_2} \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0 \\ \dfrac{1}{m_1} \\ 0 \end{bmatrix} u(t), \tag{19a}$$

$$y(t) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} x(t), \tag{19b}$$

where $x(t) = [x_1(t)\ x_2(t)\ \dot{x}_1(t)\ \dot{x}_2(t)]^T$, $y(t) = [x_1(t)\ x_2(t)]^T$. In this paper, the parameters of the cart system (22) are chosen as $m_1 = 2.0$ [kg], $m_2 = 1.0$ [kg], $k_1 = 1.3$ [N/m], $k_2 = 0.7$ [N/m], $c_1 = 0.9$ [Ns/m] and $c_2 = 0.5$ [Ns/m].

Changing continuous-time description into discrete-time description as sampling time $t_s = 0.01$[s], the matrices for the system (1) are given by

$$A = \begin{bmatrix} 1.0000 & 0.0000 & 0.0100 & 0.0000 \\ 0.0000 & 1.0000 & 0.0000 & 0.0010 \\ -0.0100 & 0.0035 & 0.9930 & 0.0025 \\ 0.0070 & -0.0070 & 0.0050 & 0.9950 \end{bmatrix}, \quad B = \begin{bmatrix} 0.0000 \\ 0.0000 \\ 0.0050 \\ 0.0000 \end{bmatrix},$$

$$C = \begin{bmatrix} 1.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 1.0 & 0.0 & 0.0 \end{bmatrix}, \quad D = \begin{bmatrix} 0.0 \\ 0.0 \\ 0.012 \\ 0.0 \end{bmatrix},$$

$$E_a = \begin{bmatrix} 0.15 & -0.052 & 0.1 & -0.037 \end{bmatrix}, \quad E_b = -0.05,$$

$$D_k = \begin{bmatrix} 0.5 & 0.1 \end{bmatrix}, \quad E_k = 1.0, \quad F(k) = f(k), \quad N(k) = \begin{bmatrix} N_1(k) & 0 \\ 0 & N_2(k) \end{bmatrix},$$

where $N_1(k)$ and $N_2(k)$ are the outputs of the fuzzy control.

Table 2. The actual costs. (The cost of the nominal system is $\hat{J} = 1.3941e + 04$.)

$F(k)$	With fuzzy	Without fuzzy
1	1.2669e+04	1.5441e+04
$\exp(-0.002k)$	1.2309e+04	1.4720e+04
$\cos(\pi k/18.0)$	1.1709e+04	1.3990e+04

In this system, it is assumed that the mass m_1 of the cart A can vary from 1.7 to 2.4, and D, E_a and E_b are fixed. The initial system condition is $x(0) = [2.0\ 3.0\ 0.0\ \ 0.0]^T$, and the weighting matrices are chosen as $Q = \mathrm{diag}(4.0,\ 3.0,\ 2.0,\ 2.0)$ and $R = 1.0$, respectively.

The assumption in inequalities (3) is rather restrictive and difficult to verify. Moreover, the partitioning method for the matrices D, E_a and E_b is not unique. It should be noted that a detailed method of identification of these matrices along with many examples has been given by [26].

The output feedback control gain K that is based on the proposed LMIs (9) is given by

$$K = [\ K_1 \quad K_2\ ] = [\ -2.1232 \quad 0.4290\ ]\,. \tag{20}$$

For the nominal system (11), the output feedback control gain $\hat{K}$ which is based on the LMI design method in [13] is given by

$$\hat{K} = [\ \hat{K}_1 \quad \hat{K}_2\ ] = [\ -0.9636 \quad -0.1631\ ]\,. \tag{21}$$

In order to compare with the proposed method, let us consider the following system without the proposed additive gain.

$$\begin{aligned} \bar{x}(k+1) &= [A + \Delta A(k)]\,\bar{x}(k) + [B + \Delta B]\,\bar{u}(k) && (22a)\\ \bar{y}(k) &= C\bar{x}(k) && (22b)\\ \bar{u}(k) &= \bar{K}\bar{y}(k) && (22c) \end{aligned}$$

where $\bar{x}(k) \in \Re^n$ is the state, $\bar{y}(k) \in \Re^l$ is the output and $\bar{u}(k) \in \Re^m$ is the control input. $\bar{K} \in \Re^{m\times l}$ is the output feedback gain for the uncertain system (19). The quadratic cost function (23) is associated with the system (19).

$$\bar{J} = \sum_{k=0}^{\infty} \left[\bar{x}^T(k)Q\bar{x}(k) + \bar{u}^T(k)R\bar{u}(k)\right]. \tag{23}$$

$\bar{K}$ is designed by using the proposed LMI approach for the uncertain system (19) without the additive gain perturbations.

$$\bar{K} = [\ \bar{K}_1 \quad \bar{K}_2\ ] = [\ -1.14359 \quad -0.1719\ ]\,. \tag{24}$$

The results of the cost for the proposed system (1) with the fuzzy controller and the uncertain system (19) without the additive gain perturbations are shown in Table 2. In all cases, the cost J with the fuzzy controller is smaller than the cost $\bar{J}$ without the fuzzy controller. Therefore, it is also shown from Table 2 that it is possible to improve the cost by applying the new proposed fuzzy controller.

The simulation results of $f(k) = 1$ are shown in Figure 6. It is verified from Figure 6 (a), ..., (e) that the response of the proposed fuzzy controller is faster than that of the controller without the fuzzy controller. Figure 6 (f) shows the result for the feedback gain with the additive gain perturbations $K + \Delta K(k)$. It is also verified that the proposed fuzzy rules can reduce the cost and compensate for the uncertainties of each system.

Figure 7 shows the response of the system with the proposed fuzzy controller and the nominal control system under $f(k) = 1$. The state variables x_i, $i = 1, \ldots, 4$ can trace the state variables $\hat{x}_i$, $i = 1, \ldots, 4$ well as shown in Figure 7 (a), ..., (e). Since $K + \Delta K(k)$ changes to compensate the system uncertainties and its response can be close to the nominal response, the proposed fuzzy controller can reduce the cost. Therefore, the control rules are adequate for the fuzzy logic. Moreover, since it is easy to decide the fuzzy logic as compared with the conservative condition of the learning algorithm of the proposed neurocontroller, it is expected that the proposed fuzzy controller is very useful and reliable.

6.1. Comparison between the Static Feedback and Proposed Augmented Controllers

In the classical LQR theory, we first form a mathematical model of the plant dynamics based on the existing information on the plant dynamics. If the model equation is an accurate representation of the plant dynamics, it can generate an suboptimal control input. However, in actual applications, the knowledge of the plant dynamics is rarely exhaustive, and it is difficult to express the actual plant dynamics in terms of mathematical equations precisely. As a result, these above-mentioned factors result in the generation of an suboptimality gap. Therefore, it is preferable to adapt the static output feedback gain by using the time-varying feedback gain to identify the uncertainty and unmodelled nonlinearity. Moreover, such a fixed controller may not satisfy the suboptimality due to the variation in the initial condition; this is because the proposed static output feedback gain is designed under the conservative condition $E[x(0)x^T(0)] = I_n$ [25]. Thus, using the time-varying feedback gain is recommended because suboptimality can be achieved under the various initial conditions.

7. Dynamic Output Feedback

Consider the uncertain systems (4a) and (4b), and suppose that a controller is to be constructed such that the closed-loop system is quadratically stable.

$$\begin{aligned} \xi(k+1) &= A_c\xi(k) + B_c y(k), && (25a)\\ u(k) &= [K + \Delta K(k)]\,\xi(k) + C_c y(k). && (25b) \end{aligned}$$

Here, $\xi(k) \in \Re^{n_c}$ is the state vector of the dynamic controller with $n_c \le n$. Furthermore, the controller is required to minimize a bound on a given quadratic cost function (7).

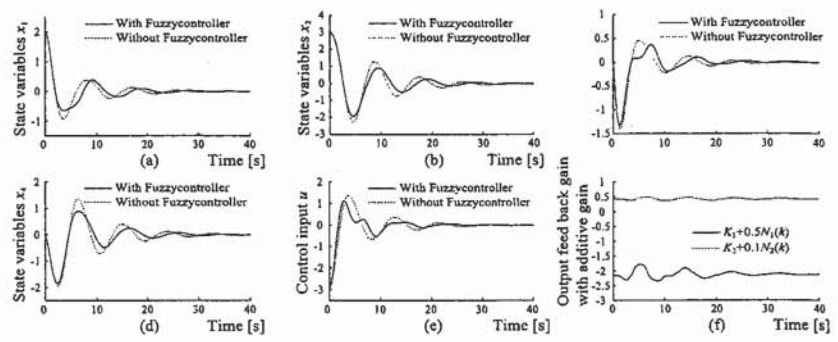


Figure 6. Simulation results of the fuzzy controller when $f(k) = 1.0$. (a), (b), (c), (d): State variables. (e): Control input. (f): Output feedback gain as additive gain perturbations.

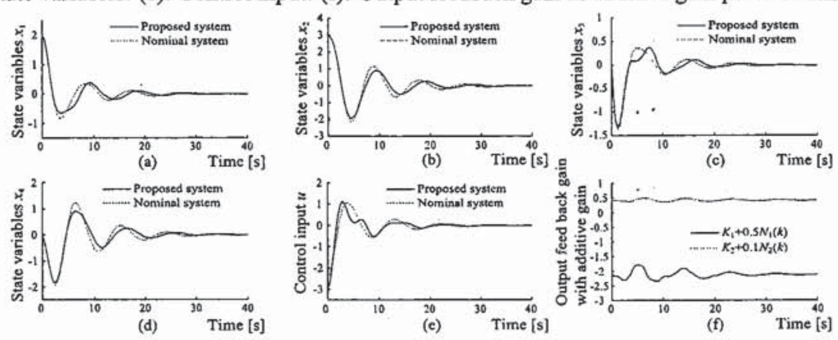


Figure 7. Simulation results of the proposed system versus the nominal control system of the fuzzy controller when $f(k) = 1.0$. (a), (b), (c), (d): State variables. (e): Control input. (f): Output feedback gain as additive gain perturbations.

This problem can be transformed into a problem of designing a static output feedback controller of the form

$$\begin{bmatrix} u(k) \\ z(k) \end{bmatrix} = \begin{bmatrix} C_c & K + \Delta K(k) \\ B_c & A_c \end{bmatrix} \begin{bmatrix} y(k) \\ \xi(k) \end{bmatrix} \tag{26}$$

for the uncertain system

$$\bar{x}(k+1) = \left[\bar{A} + \Delta\bar{A}(k)\right]\bar{x}(k) + \left[\bar{B} + \Delta\bar{B}(k)\right]\bar{u}(k), \tag{27a}$$

$$\bar{y}(k) = \bar{C}\bar{x}(k), \tag{27b}$$

$$\bar{u}(k) = \left[\bar{K} + \Delta\bar{K}(k)\right]\bar{y}(k), \tag{27c}$$

where

$$\bar{x}(k) := \begin{bmatrix} x(k) \\ \xi(k) \end{bmatrix}, \quad \bar{A} := \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix}, \quad \Delta\bar{A}(k) := \begin{bmatrix} \Delta A(k) & 0 \\ 0 & 0 \end{bmatrix},$$

$$\bar{B} := \begin{bmatrix} B & 0 \\ 0 & I_{n_c} \end{bmatrix}, \quad \Delta\bar{B}(k) := \begin{bmatrix} \Delta B(k) & 0 \\ 0 & 0 \end{bmatrix},$$

$$\bar{u}(k) := \begin{bmatrix} u(k) \\ z(k) \end{bmatrix}, \quad \bar{C} := \begin{bmatrix} C & 0 \\ 0 & I_{n_c} \end{bmatrix}, \quad \bar{K} := \begin{bmatrix} C_c & K \\ B_c & A_c \end{bmatrix},$$

$$\Delta\bar{K}(k) := \begin{bmatrix} 0 & \Delta K(k) \\ 0 & 0 \end{bmatrix}, \quad \bar{y}(k) := \begin{bmatrix} y(k) \\ \xi(k) \end{bmatrix}$$

and the cost function

$$J = \sum_{k=0}^{\infty} \left[\bar{x}^T(k)\bar{Q}\bar{x}(k) + \bar{u}^T(k)\bar{R}\bar{u}(k)\right] \tag{28}$$

with

$$\bar{Q} := \begin{bmatrix} Q & 0 \\ 0 & Q_0 \end{bmatrix} > 0, \quad \bar{R} := \begin{bmatrix} R & 0 \\ 0 & R_0 \end{bmatrix} > 0.$$

Finally, since $\bar{C}$ is a full row rank, the static output feedback results can be used to solve the above problem.

8. Conclusions

The applicability of the additive gain perturbations for the output-feedback suboptimal control problem of the discrete-time system that has uncertainties in both state and input matrices has been investigated. Compared with the existing results [11, 18–20], a new LMI condition has been derived. In order to reduce the cost, a fuzzy controller is newly introduced. By substituting the fuzzy controller into the additive gain perturbations, the robust stability and adequate suboptimal cost of the closed-loop system are guaranteed even if such systems include these artificial controllers. The numerical example has shown that the fuzzy controller has succeeded in reducing the large cost caused by the LMI technique.

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