ACTIVE FORCE CLOSURE FOR MULTIPLE OBJECTS

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Abstract. This paper discusses the active force closure (AFC) for manipulation of multiple objects. AFC for multiple objects is defined in such a way that the finger can generate an arbitrary acceleration onto a certain point of multiple objects. We define two kinds of AFC, i.e., the first one where an arbitrary acceleration can be generated onto each one of the objects, and the second one where an arbitrary acceleration can be generated onto the center of balance of multiple objects without changing the relative position among objects. We show that the grasped object cannot always be manipulated arbitrarily even if the first kind of AFC is satisfied. We also show that the grasped objects are manipulated just like a single rigid body if the second kind of AFC is satisfied. To explain these features of AFCs, numerical examples are shown for the grasp of three objects.

1. Introduction

Robot hand is a typical end-effector for robot arms. A potential advantage for the utilization of a multi-fingered robot hand is that it can manipulate an object within the hand in addition to grasping an object firmly as a simple gripper can do.

So far, although much research has been done on the manipulation by a multi-fingered robot hand, it implicitly assumed to treat a single object. Recently, some researchers (Dauchez and Delbarre, 1991; Kusuge et al., 1995; Aiyama et al., 1998; Mattikalli et al., 1995) researched the grasp of multiple objects. However, they have not considered the manipulation of objects within the hand. The authors(Harada and Kaneko, 1998) have first studied the enveloping grasp for multiple objects. They have shown a condition for judging the rolling contact at each contact point and showed the rolling up condition.

As a concept regarding the manipulation of object, the force closure has been studied (Salisbury and Roth, 1982; Nguyen, 1988; Ponce and Faverjon,

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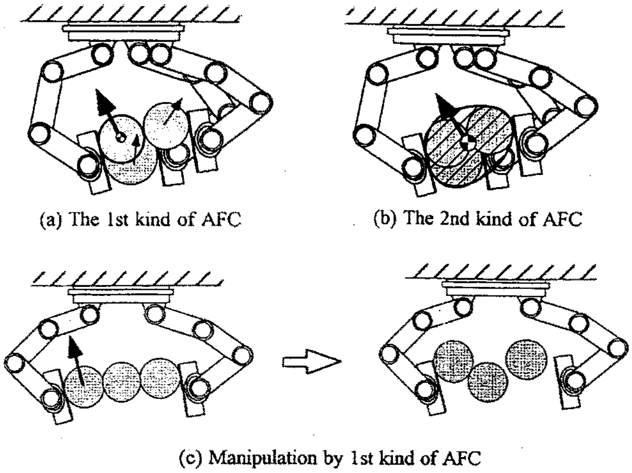

Figure 1. Some issues discussed in the paper

1995; Nakamura et al., 1989; Bicchi, 1995). However, two interpretations for the force closure were given(Trinkle, 1992), where one is that "a finger can generate an arbitrary linear and angular acceleration on the object" and the other is that "the grasped object can structurally oppose the external force and moment without changing the joint torque". For a single object, the former definition becomes the necessary and sufficient condition for arbitrary manipulation since the finger can continuously generate an arbitrary acceleration unless the finger is in a singular posture. On the other hand, the latter definition does not always relate to the manipulation of an object. For example, let us consider the power grasp where each finger is allowed to have multiple contacts with an object. Although the grasped object can resist all the directions of external force and moment without changing the joint torque, the object cannot be manipulated arbitrarily. To overcome this confusion, the force closure was redefined and classified into the active force closure (AFC) and the passive force closure (PFC) (Yoshikawa, 1996). AFC corresponds to the former definition, and PFC corresponds to the latter.

Now, let us consider extending AFC for a single object to that for multiple objects. As shown in Figure 1 (a) and (b), we can define two kinds of AFC for multiple objects, i.e., the first kind of AFC and the second kind of AFC. As shown in Figure 1 (a), the first kind of AFC focuses on one of the grasped objects, and examine whether a robot hand can generate an arbitrary acceleration onto each one object or not. As shown in Figure 1(c), however, when robot hand manipulates multiple objects based on the first kind of AFC, the grasp might be collapsed at the next moment even if an arbitrary acceleration can be generated by fingers at the initial phase. This is because we cannot regulate the relative motion between objects but can only assign the direction of motion of the designated object. Therefore, for the first kind of AFC, we can see that exerting an arbitrary acceleration on object does not always correspond to manipulating an object arbitrarily. In the second kind of AFC, we consider exerting an arbitrary acceleration at the center of balance of multiple objects, and deal with multiple objects just like a single rigid body as shown in Figure 1(b). Since we have to assign the desired acceleration for all objects not to cause the relative motion among objects, the second kind of AFC becomes the stronger condition for multiple-object manipulation than the 1st one.

This paper is organized as follows: We begin by showing the analytical model and assumptions. We formulate the contact force among objects as a function of contact force imparted by each finger. We define two kinds of AFC with their characteristics and show that whether each AFC is satisfied or not is equivalent to solving a proper linear programming problem. Finally, we show a couple of simulations for demonstrating each AFC.

2. Modeling

Figure 2 shows the grasp of m objects by n fingers, where the finger j contacts with the object i, and additionally the object i has a common contact point with the object l. Let Σ_R, Σ_{Bi} $(i = 1, \cdots, m)$ and Σ_{Fj} $(j = 1, \cdots, n)$ be the coordinate systems fixed at the base, at the center of gravity of the object i, and at the tip link of the finger j, respectively. Let $\boldsymbol{f}_{Bi}$ and $\boldsymbol{n}_{Bi}$ $(i = 1, \cdots, m)$ be the total force and moment at the center of gravity of the object i, respectively. Let $\boldsymbol{f}_{Cij}$ $(i = 1, \cdots, m,\ j = 1, \cdots, n)$ and $\boldsymbol{f}_{COt}$ $(t = 1, \cdots, r)$ be the contact force applied by the finger j, and the contact force at the t-th contact between objects where we assume that the object l can apply the contact force to the object i when $i < l$, respectively. We assume that all the fingers have enough degrees of freedom to exert an arbitrary contact force($s_j \geq 3$) where s_j denotes the number of joints of the finger j. By using the above notation, we can obtain the force balance equation for multiple objects as follows(Harada and Kaneko, 1998):

$$\boldsymbol{f}_B = \boldsymbol{D}_B^T \boldsymbol{f}_C, \tag{1}$$

where $\boldsymbol{f}_B = [\boldsymbol{f}_{B1}^T\ \boldsymbol{n}_{B1}^T\ \cdots\ \boldsymbol{f}_{Bm}^T\ \boldsymbol{n}_{Bm}^T]^T \in R^{6m}$, $\boldsymbol{D}_B = [\boldsymbol{D}_{CB}^T\ \boldsymbol{D}_O^T]^T$, $\boldsymbol{f}_C = [\boldsymbol{f}_{CB}^T\ \boldsymbol{f}_O^T]^T \in R^{3n+3r}$, $\boldsymbol{f}_{CB} = [\boldsymbol{f}_{C11}^T\ \cdots \boldsymbol{f}_{Cmn}^T]^T$ is the vector of contact force applied by the fingers, and $\boldsymbol{f}_O = [\boldsymbol{f}_{CO1}^T\ \cdots \boldsymbol{f}_{CO\gamma}^T]^T$. is the vector of contact force caused at the contact points among objects. In order to release us from the nonlinear constraint, we approximate the friction cone by the polyhedral convex cone as follows:

$$\boldsymbol{f}_C = \boldsymbol{V}\boldsymbol{\lambda}, \qquad \boldsymbol{\lambda} \geq 0, \tag{2}$$

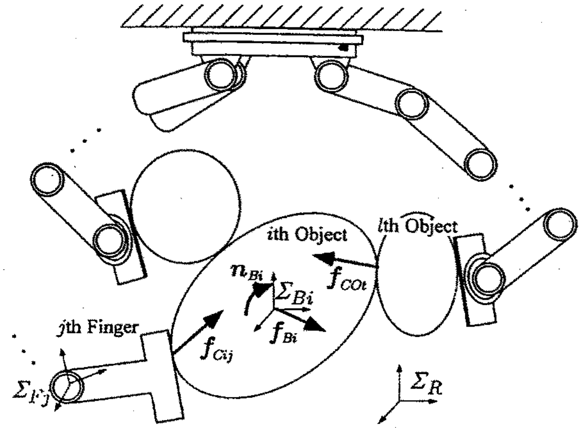

Figure 2. Model of the system

where $\boldsymbol{V}$ denote the matrix consisting of the span vectors of the convex cones, and $\boldsymbol{\lambda}$ denotes the corresponding magnitude of contact forces. This approximation enables us to treat the nonlinear friction constraint as a linear one. We further choose $\boldsymbol{V}$ so that they may coincide with the boundary surface of the actual friction cone. Such an approximation of friction cone enables us to conservatively evaluate the contact force from the viewpoint of causing a slipping motion at the contact point.

2.1. DEPEDENCY OF CONTACT FORCE

To confirm whether an arbitrary acceleration can be generated or not, we have to know the contact force among objects in addition to the contact force given by each finger. For this purpose, we now make clear the dependency of contact force among objects. The equation of motion of the grasped objects is given by

$$\boldsymbol{M}_B\ddot{\boldsymbol{p}}_B + \boldsymbol{h}_B = \boldsymbol{D}_{CB}^T\boldsymbol{f}_{CB} + \boldsymbol{D}_O^T\boldsymbol{f}_O, \tag{3}$$

where $\boldsymbol{M}_B = \mathrm{diag}[m_{B1}\boldsymbol{I}_3\ \boldsymbol{H}_{B1}\ \cdots m_{Bm}\boldsymbol{I}_3\ \boldsymbol{H}_{Bm}]$, $\ddot{\boldsymbol{p}}_B = [\ddot{\boldsymbol{p}}_{B1}^T\ \dot{\boldsymbol{\omega}}_{B1}^T\ \cdots\ \ddot{\boldsymbol{p}}_{Bm}^T\ \dot{\boldsymbol{\omega}}_{Bm}^T]^T$, and $\boldsymbol{h}_B$ are the inertia matrix, the acceleration vector at the center of mass of each object, and the vector with respect to the centrifugal and the Coriolis force, respectively, m_{Bi}, $\boldsymbol{H}_{Bi}$, $\ddot{\boldsymbol{p}}_{Bi}$, and $\dot{\boldsymbol{\omega}}_{Bi}$ denote the mass of the object i, the inertia tensor of the object i, the acceleration and the angular acceleration at the center of gravity of the object i, respectively. The constraint condition among objects is expressed as

$$\boldsymbol{D}_O\ddot{\boldsymbol{p}}_B = \boldsymbol{0}. \tag{4}$$

By using eq.(3) and the differentiation of eq.(4), the following relation is derived:

$$Af_C = b, \tag{5}$$

where $A = [D_O M_B^{-1} D_{CB}^T \quad D_O M_B^{-1} D_O^T]$ and $b = D_O M_B^{-1} h_B - \dot{D}_O \dot{p}_B$. Eqs.(5) shows the dependency of the contact force, namely f_O is determined dependently according to f_{CB}.

3. Definition of AFCs

Let us construct the mathematical formulation of AFC for multiple objects. We define AFC for multiple objects as follows:

AFC for multiple objects: Arbitrary translational and angular acceleration can be exerted at a reference point of multiple objects.

We define two kinds of AFCs for multiple objects as an extension of AFC of a single object, i.e., the 1st kind of AFC and the 2nd kind of AFC. These AFCs are defined as follows:

The 1st kind of AFC: We focus on one object. If an arbitrary acceleration can be exerted on each object, the grasp is termed as the 1st kind of AFC.

The 2nd kind of AFC: We focus on the center of mass of multiple objects. If an arbitrary acceleration can be exerted at the center of mass of the multiple objects without causing the relative motion among objects, the grasp is termed as the 2nd kind of AFC.

As described in Section 1, the object can not always be manipulated arbitrarily, even if an arbitrary acceleration can be exerted on one of each object. The 2nd kind of AFC ensures that multiple objects can be manipulated just like a single object. Now, we formulate the condition for satisfying each definition of AFCs.

The 1st kind of AFC : Assuming that all objects are stationary ($\dot{p}_B = 0$), we focus on the center of gravity of the i-th object. By using eq.(1) and eq.(3), the relation of acceleration exerted by fingers is derived as follows:

$$M_B \ddot{p}_B = f_B. \tag{6}$$

From eq.(furu:1), we extract the the equation for the i-th object to examine whether an arbitrary acceleration can be applied or not on the i-th object where it is given by

$$M_{Bi} \begin{bmatrix} \ddot{p}_{Bi} \\ \dot{\omega}_{Bi} \end{bmatrix} = \begin{bmatrix} f_{Bi} \\ n_{Bi} \end{bmatrix}, \tag{7}$$

where $M_{Bi} = \mathrm{diag}[m_{Bi}I_3\ H_{Bi}]$. The total force and moment on i-th object is expressed as follows:

$$\begin{bmatrix} f_{Bi} \\ n_{Bi} \end{bmatrix} = D_{Bi}^T f_C, \tag{8}$$

where D_{Bi}^T is derived by extracting the lines of D_B^T from the $(6i+1)$th to the $(6i+6)$th. By using eqs.(8), (7), (2) and (5), we can formulate a linear programming problem to examine the existence of contact force for a given acceleration as follows:

$$\begin{aligned} \text{Minimize} \qquad & z = a^T\lambda, \qquad a = [1\cdots 1]^T, \\ \text{Subject to} \qquad & M_{Bi}\begin{bmatrix} \ddot{p}_{Bi} \\ \dot{\omega}_{Bi} \end{bmatrix} = D_{Bi}^T V\lambda, \\ & AV\lambda = b, \\ & \lambda \geq 0. \end{aligned} \tag{9}$$

For a given set of accelerations, we now examine whether there exists a set of contact force within the approximated friction cone or not. We consider the twelve sets of unit accelerations such as $[\ddot{p}_{Bi}^T\ \dot{\omega}_{Bi}^T]^T = e_1, e_2, \cdots, e_6, -e_1, -e_2, \cdots, -e_6$, where $e_k \in R^6$ $(k = 1, \cdots, 6)$ denotes the k-th unit vector. Substituting these unit accelerations into the linear programming problem, if all the linear programming problems have solutions, it is guaranteed that an arbitrary acceleration can be exerted on the i-th object. We can prove that an arbitrary acceleration can be applied if the linear programing problem has a solution for the twelve sets of accelerations by following the paper(Nakamura et al., 1989). If all objects satisfy this condition, the grasp is termed as the 1st kind of AFC. It should be noted that, through the linear programming problem, we are not interesting to obtain the optimal solution but to confirm whether there exists a set of contact forces or not making the focused object move to the desired direction.

The 2nd kind of AFC: Supposing that all objects are stationary, we focus on the center of mass of multiple objects. The relation between the acceleration at the center of mass and the acceleration of each object is derived as follows:

$$\ddot{p}_B = D_G \begin{bmatrix} \ddot{p}_G \\ \dot{\omega}_G \end{bmatrix}, \tag{10}$$

where $\ddot{p}_G$, $\dot{\omega}_G$ is the translational and angular acceleration at the center of mass of the grasp,

$$D_G = \begin{bmatrix} I_3 & 0 & \cdots & I_3 & 0 \\ ((p_{B1} - p_G)\times) & I_3 & \cdots & ((p_{Bm} - p_G)\times) & I_3 \end{bmatrix}^T.$$

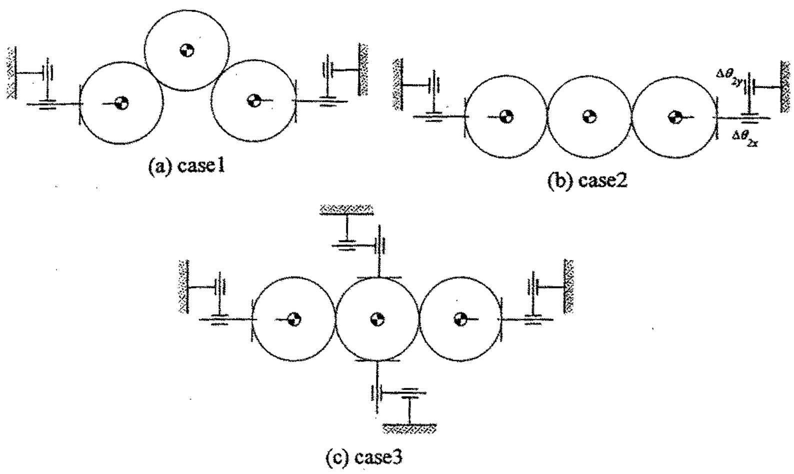

Figure 3. Three-object grasps in numerical example

where $*\times$ denotes the skew-symmetric matrix equivalent to the vector product of $*$. By using eqs.(2), (5), (6) and (10), we can formulate the following linear programming:

$$\text{Minimize} \qquad z = \boldsymbol{a}^T \boldsymbol{\lambda}, \qquad \boldsymbol{a} = [1 \cdots 1]^T,$$

$$\text{Subject to} \qquad \begin{aligned} \boldsymbol{M}_B \boldsymbol{D}_G \begin{bmatrix} \ddot{\boldsymbol{p}}_G \\ \dot{\boldsymbol{\omega}}_G \end{bmatrix} &= \boldsymbol{D}_B^T \boldsymbol{V} \boldsymbol{\lambda}, \\ \boldsymbol{A}\boldsymbol{V}\boldsymbol{\lambda} &= \boldsymbol{b}, \\ \boldsymbol{\lambda} &\geq 0. \end{aligned} \tag{11}$$

Considering the twelve sets of unit accelerations such as $[\ddot{\boldsymbol{p}}_G^T \ \dot{\boldsymbol{\omega}}_G^T]^T = \boldsymbol{e}_1$, $\boldsymbol{e}_2, \cdots, \boldsymbol{e}_6, -\boldsymbol{e}_1, -\boldsymbol{e}_2, \cdots, -\boldsymbol{e}_6$, if all the corresponding linear programming problems have solutions, it is guaranteed that an arbitrary acceleration can be exerted on the center of balance of multiple objects(Nakamura et al., 1989). Such a grasp is termed as the 2nd kind of AFC.

Note that, in the formulation of the 2nd kind of AFC, the acceleration at the center of balance is assigned to each object. Therefore, the objects move just like a single rigid body without causing the relative motion.

4. Examples

We performed numerical examples for three grasp configurations as shown in Figure 3. For simplicity, we consider 2D examples where the mass and the radius of all objects are assumed to be unity, and the frictional angle is set as $\pi/4$.

We first examine the 1st kind of AFC for case 1 and 2. Let us focus on the center object of case1. By solving the linear programming problem

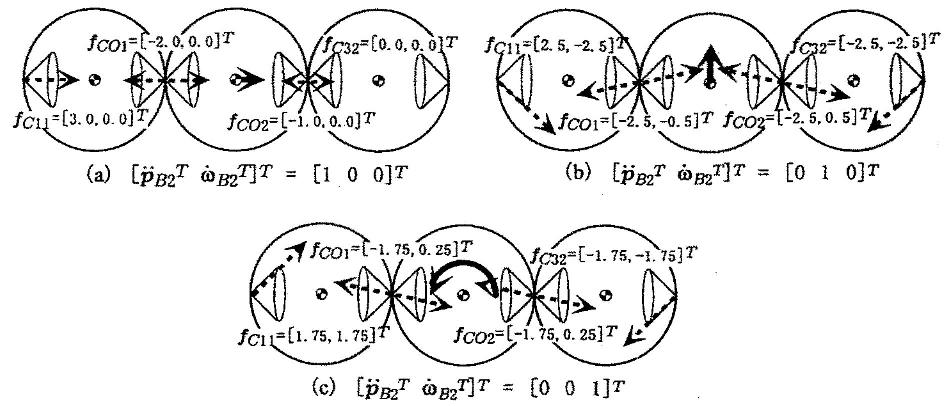

Figure 4. Numerical solutions (1st kind of AFC for the 2nd object)

(eq.(9)), we find that it provides no solution of acceleration in a downward direction. Therefore, case1 does not satisfy the 1st kind of AFC. Now, let us consider case2. We again focus on the center object where the solutions of the linear programming problem are shown in Figure 4. Note that, for a 2D model, the number of linear independent acceleration is three. Although we have to examine six linear programming problems corresponding to this acceleration, we show the result of only the three of them since the grasp is symmetry with respect to the center object. In Figure 4, the dotted lines denote the contact force corresponding to the acceleration expressed by the solid line. From the result shown in Figure 4, we can see that an arbitrary acceleration can be exerted on the center object. We further focus on the left-hand object where an arbitrary acceleration can also be exerted on the left-hand object as shown in Figure 5. We note that the result for the right-hand object is same as the one for the left-hand object. Therefore, since an arbitrary acceleration can be exerted on all objects, we can see that case2 satisfies the 1st kind of AFC.

Based on the above discussions, we consider manipulating multiple objects satisfying the 1st kind of AFC. Let us consider manipulating each one object in case2. Even if an arbitrary acceleration can be generated on the focused object, the direction of acceleration generated on the other objects is unknown. Therefore, as soon as the acceleration is exerted on the object, the grasp configuration will change to the one as shown in case1. Since case1 does not satisfy the 1st kind of AFC, the grasp no more satisfies the 1st kind of AFC. Therefore, although case2 satisfies the 1st kind of AFC, an arbitrary manipulation is impossible. However, since an arbitrary object can be moved to a desired direction, the grasp satisfying the 1st kind of AFC is useful when we want to move one object to a desired direction at a

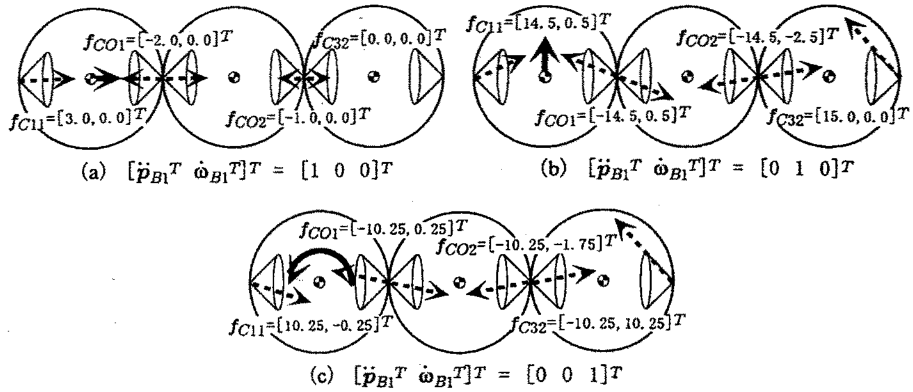

Figure 5. Numerical solutions (1st kind of AFC for the 1st object)

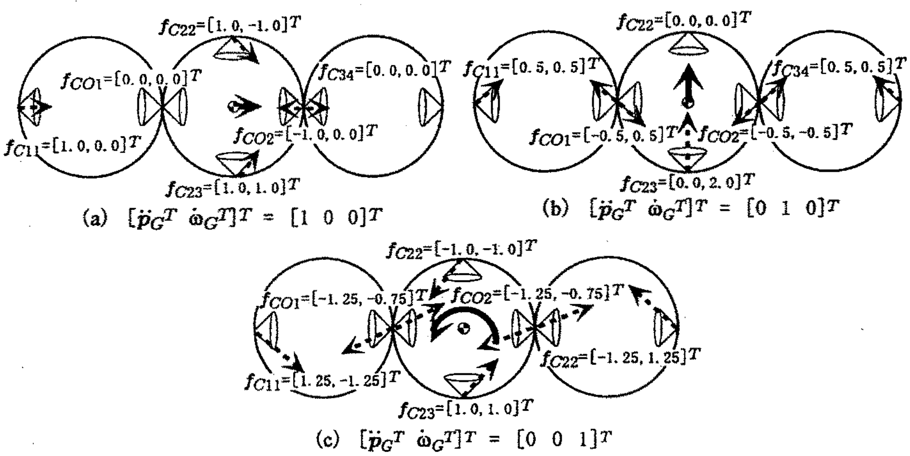

Figure 6. Numerical solutions (2nd kind of AFC for case 3)

given instant of time.

Next, let us examine the 2nd kind of AFC by comparing case2 and case3. By solving the linear programming problem (eq.(11)), it can be found that case2 does not satisfy the 2nd kind of AFC, while case3 satisfies it. The solutions of the linear programming problem (eq.(11)) for case3 are illustrated in Figure 6, where we see that the grasped objects can be manipulated without changing the relative position.

5. Conclusions

This paper discussed the Active Force Closure (AFC) for multiple objects. We provided the relationship between the force at the center of gravity of the objects and the contact force. Without taking the relative motion into account, we defined the first kind of AFC focused one of each object and the second kind of AFC focused the center of mass of the grasp. By numerical examples, we showed that the concept of exerting the arbitrary acceleration on the objects did not always correspond to the concept manipulating the objects arbitrarily, when multiple objects are manipulated simultaneously.

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