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Internal forces and stability in multi-finger grasps

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Abstract

This paper deals with the rotational stability of a rigid body under constant contact forces. For this system, the stiffness tensor is derived, and its basic properties are analyzed. For the gravity-induced stiffness, one condition for stability, formulated in terms of geometric and gravity centers, is obtained. The internal forces are introduced with the use of a virtual linkage model. Within this representation, two conditions for stability under internal force loading are formulated in an analytical form. The conditions obtained are applied to the synthesis of a three-fingered grasp. © *1999 Elsevier Science Ltd. All rights reserved.*

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1. Introduction

One of the fundamental problems in controlling multi-fingered hands is the stability of the resulting grasp. In recent years, the problem has been addressed from different points of view, and a number of approaches to define grasping stability and its relation to concepts such as grasping form and force closure, have been proposed in the literature. Good surveys on this topic can be found in Shimoga (1996) and Walker (1998). This paper addresses the problem in a somewhat simplified way, dealing only with the rotational stability of the grasped object.

Basically, the total compliance of an object, grasped by multiple fingers, $\mathbf{C}_{\text{object}}$, has two sources:

$$\mathbf{C}_{\text{object}} = \mathbf{C}_{\text{fingers}} + \mathbf{C}_{\text{loading}} \tag{1}$$

The first one is due to the compliance of the fingers, while the second is due to the contact force interaction between the fingers and the object. Roughly, the first term in Eq. (1) is defined by the transformation of the joint compliance $\mathbf{C}_{\text{joints}}$ to the Cartesian level through the finger Jacobian $\mathbf{J}$. The Cartesian compliance of the fingers, $\mathbf{C}_{\text{fingers}} = \mathbf{J}\mathbf{C}_{\text{joints}}\mathbf{J}^{\text{T}}$, is symmetric and positive definite (and therefore stable) as long as the joint compliance matrix is stable. On the other hand, the compliance due to the finger interactions, $\mathbf{C}_{\text{loading}}$, is not necessarily positive definite. It depends on the contact force distribution, and is often the source of grasping instability. This phenomenon is reported by Nguen (1989), Cutkosky and Kao (1989), and Kaneko et al. (1990). It should be noted that a similar subject – stability due to internal forces in mechanisms with closed kinematic chains – is analyzed by Hanafusa and Adli (1991), and Yi et al. (1991).

One possible approach to provide stable grasping can be formulated as follows: for a given matrix $\mathbf{C}_{\text{loading}}$ find out the total finger compliance $\mathbf{C}_{\text{fingers}}$ so that the object compliance matrix $\mathbf{C}_{\text{object}}$ is positive definite. Theoretically, this approach can work nicely. However, this is a case-dependent approach, and there is no systematic procedure for adjusting the compliance of the fingers to that of the object.

Another possible solution to the stability problem is based on decomposition of the total compliance, and designing the corresponding matrices, $\mathbf{C}_{\text{fingers}}$ and $\mathbf{C}_{\text{loading}}$, separately. Indeed, if they are both stable, then the resulting compliance will also be stable. Conceptually, this approach is taken in this paper. The compliance of the fingers is not considered at all. The grasp is considered to be stable if its stiffness matrix is not negative definite. The reason for taking this view is simple – even if the contact-force-induced compliance is positive semi-definite, the resulting compliance of the system can

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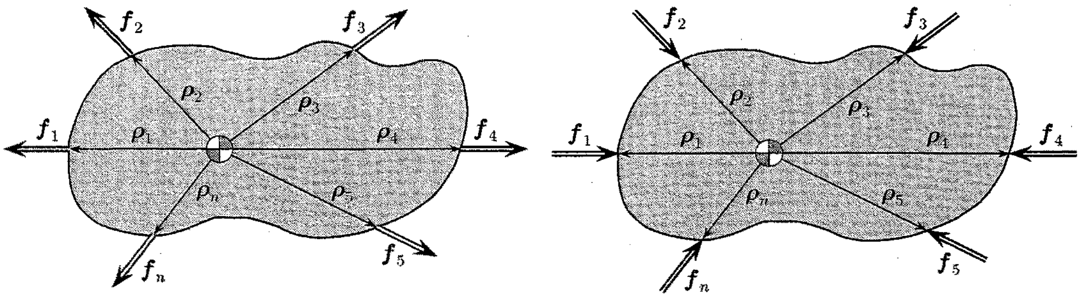

Fig. 3. Stretching and compressive forces.

$\mathbf{\Omega}(\mathbf{a}) \cdot \mathbf{b} \equiv \mathbf{a} \times \mathbf{b}$. If, however, the object is not at equilibrium, $\mathbf{K}$ is always asymmetric.

Next, even though $\mathbf{K}$ is symmetric for equilibrium, it is not always and not necessarily positive definite. The judgment on the positive definiteness of $\mathbf{K}$ can be done easily only for some simple cases of force loading. Consider, for example, the case when all the applied forces are coplanar to the correspondent vectors $\boldsymbol{\rho}_i$, as shown in Fig. 3. Here, $\mathbf{f}_i = k_i \boldsymbol{\rho}_i$ and formula (5) gives

$$\mathbf{K} = \sum_{i=1}^{n} k_i \{ (\boldsymbol{\rho}_i^{\mathrm{T}} \boldsymbol{\rho}_i) \mathbf{I} - \boldsymbol{\rho}_i \boldsymbol{\rho}_i^{\mathrm{T}} \} \tag{7}$$

As can be seen, $\mathbf{K}$ has the structure of the inertia tensor of a system of points built on the vectors $\boldsymbol{\rho}_i$, with k_i playing the role of masses. Therefore, if all $k_i \geqslant 0$, i.e., all the forces are stretching, $\mathbf{K}$ is positive definite and the equilibrium is stable. In the opposite case, when all $k_i \leqslant 0$, i.e., all the forces are compressive, $\mathbf{K}$ is negative definite and the equilibrium is unstable.

However, in the general case, when k_i have different signs or when the applied forces $\mathbf{f}_i$ are not coplanar to $\boldsymbol{\rho}_i$, it is not that easy to make judgment on the properties of $\mathbf{K}$ without direct computations. Hence, an additional study of the force structure is required. Finally, please note that the forces dealt with in this paper are assumed to be constant in the inertial frame. If they are constant in the body frame, one can show that $\mathbf{K} = \mathbf{\Omega}(\sum_{i=1}^{n} \boldsymbol{\rho}_i \times \mathbf{f}_i) = 0$. Such forces do not contribute to the rotational stiffness as long as the body is at equilibrium.

3. Force decomposition

To establish relationships between the stability and the force structure, a proper decomposition of the applied forces is necessary. One possible decomposition is based on the pseudo-inversion of the grasp matrix. Such a decomposition, interpreted in terms of the screw theory, has been given by Kumar and Waldron (1989). Here, the pseudo-inverse decomposition is presented in a different form, based on the classical vectorial notation and changing the reference point O.

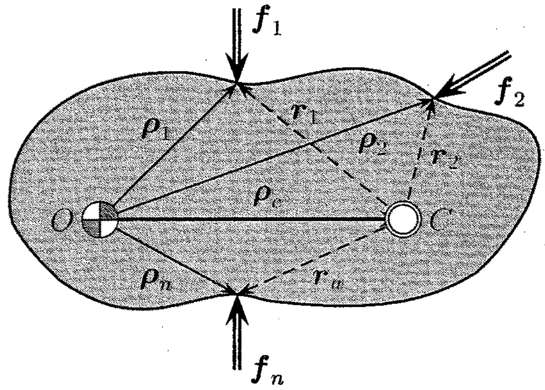

Fig. 4. Shift to the geometric center.

To facilitate the calculations, the reference point O is shifted to the geometric center C (Fig. 4) defined by

$$\boldsymbol{\rho}_c = \frac{1}{n} \sum_{i=1}^{n} \boldsymbol{\rho}_i \tag{8}$$

Introducing the block vectors $\mathbf{\Phi}_o = \{ -m\mathbf{g}, \mathbf{0} \}^{\mathrm{T}}$ and $\mathbf{f} = \{ \mathbf{f}_1^{\mathrm{T}}, \ldots, \mathbf{f}_n^{\mathrm{T}} \}^{\mathrm{T}}$, one can rewrite the static equations (2) in the following form:

$$\mathbf{\Phi}_o = \mathbf{B}_o \mathbf{f} = \mathbf{B}_{oc} \mathbf{B}_c \mathbf{f} \tag{9}$$

where

$$\mathbf{B}_{oc} = \begin{Vmatrix} \mathbf{I} & \mathrm{O} \\ \mathbf{\Omega}(\boldsymbol{\rho}_c) & \mathbf{I} \end{Vmatrix}, \quad \mathbf{B}_c = \begin{Vmatrix} \mathbf{I} & \ldots & \mathbf{I} \\ \mathbf{\Omega}(\mathbf{r}_1) & \ldots & \mathbf{\Omega}(\mathbf{r}_n) \end{Vmatrix} \tag{10}$$

and $\mathbf{r}_i = \boldsymbol{\rho}_i - \boldsymbol{\rho}_c$. If $n \geqslant 3$ ($n \geqslant 2$ in the planar case) and the contact points are not coplanar, $\mathbf{B}_{oc}\mathbf{B}_c$ is a full-rank decomposition of the grasp matrix $\mathbf{B}_o$ and, therefore, $\mathbf{B}_o^+ = \mathbf{B}_c^+ \mathbf{B}_{oc}^{-1}$.

Note that the symbolic computation of the pseudo-inverse $\mathbf{B}_c^+ = \mathbf{B}_c^{\mathrm{T}} (\mathbf{B}_c \mathbf{B}_c^{\mathrm{T}})^{-1}$ is much easier than that for the original, non-decomposed matrix $\mathbf{B}_o$. It is due to the fact

4. Stability analysis

Having decomposed the applied forces **f**, one can decompose the stiffness tensor $\mathbf{K} = \mathbf{K}_G + \mathbf{K}_I$ into the gravity-inducing component, $\mathbf{K}_G$, and the internal-force-inducing component, $\mathbf{K}_I$. In what follows, the conditions under which the matrices $\mathbf{K}_G$ and $\mathbf{K}_I$ become positive semi-definite, are established.

4.1. Gravity-induced stiffness

The matrix $\mathbf{K}_G$ can be represented through the geometric invariants of the grasp, $\boldsymbol{\rho}_c$ and $\mathbf{J}_c$. Substituting Eq. (13) into Eq. (5) and making use of Jabobi's identity gives

$$\mathbf{K}_G = \boldsymbol{\Omega}^{\mathrm{T}}(\boldsymbol{\rho}_c)\,\boldsymbol{\Omega}(\mathbf{f}_c) + \boldsymbol{\Omega}(\boldsymbol{\mu}_c)\{\mathbf{J}_c - \sigma\mathbf{I}\} \quad (20)$$

where $\boldsymbol{\mu}_c = \mathbf{J}_c^{-1}\mathbf{m}_c$, $\mathbf{m}_c = -\boldsymbol{\rho}_c \times \mathbf{f}_c$, $\mathbf{f}_c = -m\mathbf{g}$ and $\sigma = \frac{1}{2}\mathbf{J}_c$. If the geometric center of the grasp coincides with the center of mass ($\boldsymbol{\rho}_c = \mathbf{0}$), $\mathbf{K}_G$ does not contribute to the total stiffness. Another particular case of the all-zero eigenvalues of the matrix $\mathbf{K}_G$ is the one where the object is planar and its plane is orthogonal to the gravity force.

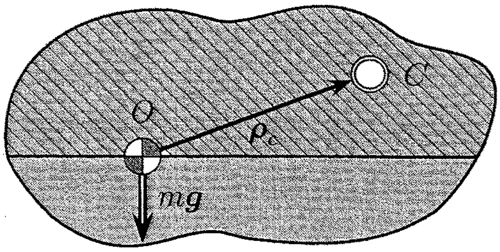

Fig. 6. Gravitational stability.

In the planar case, the stability condition is

$$\mathbf{K}_G = \boldsymbol{\rho}_c^{\mathrm{T}}\mathbf{f}_c \geqslant 0 \quad (21)$$

It has the following geometric interpretation: for the no-internal-force grasp to be stable, the geometric center $\boldsymbol{\rho}_c$ must be placed above the center of mass of the object (Fig. 6).

In the spatial case, however, the judgment on the eigenvalues of $\mathbf{K}_G$ is not that simple. Inspecting the structure of $\mathbf{K}_G$, one can show that $\mathrm{tr}\,\mathbf{K}_G = 2\boldsymbol{\rho}_c^{\mathrm{T}}\mathbf{f}_c$. Hence, by the Routh–Hurwicz criterion the condition (21) is also necessary for the stability in the spatial case. However, taken alone it is not sufficient, since the other two conditions of positive definiteness of $\mathbf{K}_G$ must be established and analyzed.

In the general case, to find an interpretation of the stability conditions for the matrix $\mathbf{K}_G$ in terms of the geometric invariants of the grasp is a difficult problem. Here, only two particular cases, when the eigenvalues of $\mathbf{K}_G$ can be identified easily, are considered.

In the first particular case the vectors $\boldsymbol{\rho}_c$ and $\mathbf{f}_c$ are coplanar, i.e., $\boldsymbol{\rho}_c = k\mathbf{f}_c$. Under this assumption the second term in Eq. (20) vanishes and the eigenvalues of $\mathbf{K}_G$ are defined as follows: $\lambda_1 = 0$, $\lambda_{2,3} = \boldsymbol{\rho}_c^{\mathrm{T}}\mathbf{f}_c$. Hence, Eq. (21) is

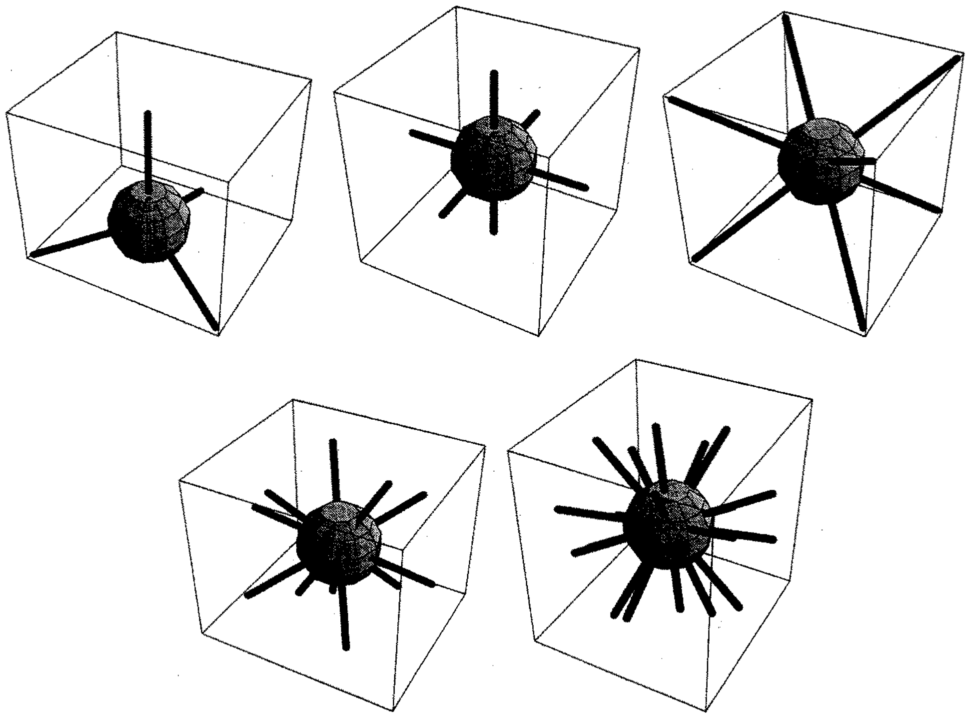

Fig. 7. Regular-polyhedron-type configurations.

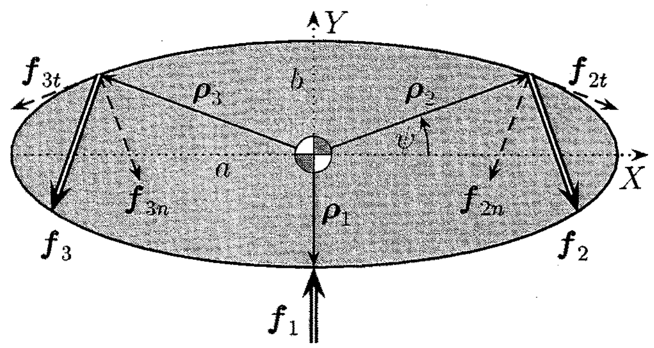

Fig. 8. Three-fingered grasp.

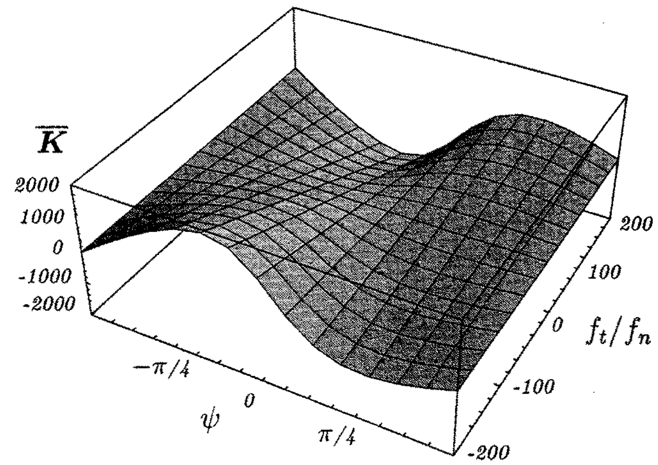

Fig. 9. Normalized rotational stiffness.

5. Synthesis of a stable grasp

Consider a planar, elliptic object, grasped by a three-fingered hand. Assume the symmetrical placement of the second and the third contact points on the object as shown in Fig. 8. The contact points are defined as follows: $\boldsymbol{\rho}_1 = \{0, -b\}^{\mathrm{T}}$, $\boldsymbol{\rho}_2 = \{a\cos\psi, b\sin\psi\}^{\mathrm{T}}$, $\boldsymbol{\rho}_3 = \{-a\cos\psi, b\sin\psi\}^{\mathrm{T}}$, where the grasping angle $\psi t[-\pi/2, \pi/2]$, and a and b are the lengths of the semi-axes of the ellipse.

Assume that the gravity center is at the center of the ellipse. The geometric center of the system of the contact points is

$$\boldsymbol{\rho}_c = \frac{b}{3}\{0, 2\sin\psi - 1\}^{\mathrm{T}} \tag{33}$$

To satisfy the stability condition (21), the geometric center should be above the gravity center. This leads to the following simple condition on the grasping angle: $\psi > \pi/6$.

Next, consider stability due to the internal forces. The normal contact forces, directed along the inward normals, are defined as $\mathbf{f}_{1n} = f_y\{0, 1\}^{\mathrm{T}}$, $\mathbf{f}_{2n} = f_n^*\{-b\cos\psi, -a\sin\psi\}^{\mathrm{T}}$, $\mathbf{f}_{3n} = f_n^*\{b\cos\psi, -a\sin\psi\}^{\mathrm{T}}$, where $f_n^* = f_n/\lambda(\psi)$, and $\lambda(\psi) = \sqrt{a^2\sin^2\psi + b^2\cos^2\psi}$.

Defining the clockwise (at the 2nd point) and counter-clockwise (at the 3rd point) tangential vectors, the friction forces can be represented in the following form: $\mathbf{f}_{1t} = \{0, 0\}^{\mathrm{T}}$, $\mathbf{f}_{2t} = f_t^*\{a\sin\psi, -b\cos\psi\}^{\mathrm{T}}$, $\mathbf{f}_{3t} = f_t^*\{-a\sin\psi, -b\cos\psi\}^{\mathrm{T}}$, where $f_t^* = f_t/\lambda(\psi)$ is the normalized tangential force.

Under the specified contact forces, the moment balance and the horizontal force balance are always satisfied. The static equation for the vertical force balance is given as

$$f_y = 2f_n^* a\sin\psi + 2f_t^* b\cos\psi \tag{34}$$

The unilateral constraints on the normal forces are given by $f_y \geqslant 0$ and $f_n \geqslant 0$. Hence, the equilibrium region in the plane f_t/f_n, ψ is defined as

$$f_t/f_n \geqslant \mu_y(\psi) = -a\sin\psi/b\cos\psi \tag{35}$$

with $f_t/f_n = \mu_y(\psi)$ defining the line of zero internal force f_y. The Coulomb friction constraints are given as

$$-\mu_c \leqslant f_t/f_n \leqslant \mu_c \tag{36}$$

where μ_c stands for the friction coefficient.

The rotational stiffness of the object can be calculated by Eq. (6). It is obtained as

$$\mathbf{K} = -bf_y - 2abf_n^* + 2(a^2 - b^2)\sin\psi\cos\psi f_t^*. \tag{37}$$

Since $f_y \geqslant 0$ and $f_n \geqslant 0$, the rotational stiffness is not positive if the object is a sphere ($a = b$) or if there is no friction ($f_t^* = 0$). Substituting Eq. (34) into Eq. (37) defines

$$\mathbf{K} = -2ab(1 + \sin\psi)f_n^* + 2\cos\psi\{\sin\psi(a^2 - b^2) - b^2\}f_t^* \tag{38}$$

as a function of three variables f_n^*, f_t^*, and ψ. Basic features of this function can be inspected by plotting (for some fixed values of a and b) the normalized stiffness $\bar{\mathbf{K}} = \mathbf{K}/f_n$ as a function of the grasping angle ψ and the normalized friction force f_t/f_n. As shown in Fig. 9, it is a sign-indefinite function having a saddle point in the origin.

To obtain the stability conditions in the analytical form, the following function is introduced:

$$\mu_s(\psi) = \frac{ab(1 + \sin\psi)}{\cos\psi\{\sin\psi(a^2 - b^2) - b^2\}} \tag{39}$$

Note that $f_t/f_n = \mu_s(\psi)$ defines the line of zero stiffness in the plane f_t/f_n, ψ, which is the zero level curve of the surface $\bar{\mathbf{K}}(f_t/f_n, \psi)$.

It follows from Eqs. (35) and (37)–(39) that for $a < \sqrt{2}b$ the stability area and the equilibrium area have no

Now, with f_t/f_n being fixed, one can choose the normal reaction f_n from the desired stiffness $\mathbf{K}_{des}$. It defines

$$f_n = \frac{\mathbf{K}_{des}\lambda(\psi)}{-ab(1+\sin\psi)+\mu_c\cos\psi\{(a^2-b^2)\sin\psi-b^2\}} \tag{46}$$

as a function of the grasping angle, the desired rotational stiffness, and the friction coefficient.

To complete the synthesis of a stable grasp, it is necessary to define an optimal value of the grasping angle ψ. The minimum and the maximum grasping angles for a given friction coefficient μ_c and a given object shape a, b, are defined from equation $\mu_s(\psi) = \mu_c$. Due to possible errors in realization of the force control schemes, it is reasonable to set the interval (41) as large as possible. This corresponds to setting such ψ that gives minimum to $\mu_s(\psi)$.

Solving equation $\mathrm{d}\mu_s(\psi)/\mathrm{d}\psi = 0$, one can show that this minimum is attained under the following optimal grasping angle:

$$\sin\psi_{opt} = \frac{-1+\sqrt{(5a^2-b^2)/(a^2-b^2)}}{2} \tag{47}$$

It is interesting to see how the shape of the ellipse affects ψ_{opt}. Assume a dimensionless parameter $z = b/a$. It can be shown that $\mu_s(\psi_{opt})$ is a monotonic function of z on the interval $0 \leqslant z \leqslant 1/\sqrt{2}$. It goes to infinity as $z \to 1/\sqrt{2}$, i.e., $\psi_{opt} \to \pi/2$. In the other limiting case of $z \to 0$, when $a \gg b$, $\mu_s(\psi_{opt}) \to 0$. Here, $\sin\psi_{opt} = (-1+\sqrt{5})/2$. It is remarkable that in this case the y coordinate of the 2nd and 3rd contact points divides the semi-axis b of the ellipse in the golden section ratio $(1+\sqrt{5})/2$, and the x coordinate of those points divides the semi-axis a in square of the golden section ratio. The optimal grasping angle is plotted in Fig. 14. Grasping configurations corresponding to ψ_{opt} are shown in Fig. 15.

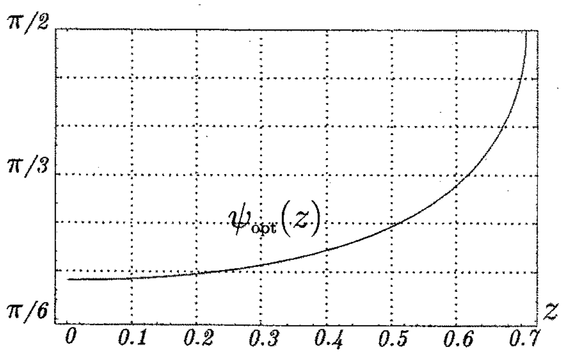

Fig. 14. Optimal grasping angle.

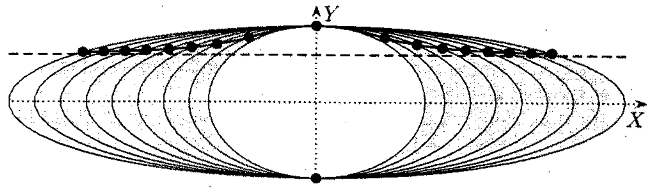

Fig. 15. Optimal grasping configurations.

6. Conclusions

The problem of the rotational stability of a rigid body under constant contact forces has been considered in this paper. The stiffness tensor of the system has been derived, and its basic properties has been established. One condition for stability, formulated in terms of the geometric and gravity centers, has been established for the gravity-induced stiffness. The internal forces have been introduced with the use of a virtual linkage model. Within this representation, two conditions of stable grasping under the internal forces have been derived in analytical form. The conditions obtained have been applied to the synthesis of a three-fingered grasp, for which the optimal grasping configurations and the optimal force distribution have been found.

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