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Paper:

Trajectory Generation of a Multi-Arm Robot Using Virtual Dynamics

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In this paper, with a view to realizing coordinated tasks by multiple manipulators, a method for generating in a decentralized manner the manipulators' space trajectories necessary for cooperative tasks through the cooperation and competition of subsystems corresponding to the manipulators. This method involves the supposition of virtual dynamics for the manipulators and an effort to carry out exchanges of information among these subsystems by means of virtual interactive forces among the manipulators generated by these virtual dynamics. Thus, the method can deal not only with simple cooperative tasks like the movement of a grasped object but also with more complicated cooperative tasks including relative motions among manipulators.

Keywords: Multi-arm robot, Trajectory generation, Decentralized system, Cooperative task

1. Introduction

Research on coordinated tasks by multiple manipulators has recently been conducted actively, but much of the research is concerned with the task of stably grasping an object. These research efforts are aimed at the object-grasping task, and are made by examining the problems of optimal force distribution to each manipulator, methods of planning optimal grasping positions, etc.[1-3]. On the other hand, studies dealing with the problem of trajectory generation for coordinated tasks are concerned, for example, with methods for the time optimal trajectory planning[4,5]. In this case, the end-point trajectory of each manipulator is given beforehand, and the problem is not to generate the trajectory for a coordinated action itself. In addition, the method of generating trajectories by making use of manipulability of multiple manipulators[6] is only applicable to the limited task of grasping a single object with multiple manipulator and, moreover, can merely deal with the case of two manipulators on the basis of the definition of the manipulability.

In contrast to these studies, Yamamoto et al.[7], taking up a task involving one manipulator grasping and moving an object and another manipulator processing the surface of the object, have proposed a method of generating its trajectory. In addition, Tsuji et al.[7] have proposed a method for trajectory generation based on posture control by making positive use of the redundant degree of freedom of a closed link system composed by multiple manipulators. However, since all these methods generate trajectories based on the conditions of geometrical constraints of a closed link structure composed by multiple manipulators, the planning of a trajectory for each manipulator can only be carried out if the information on the actions of all the other manipulators is provided. Thus, the centralized system of planning the actions of all the manipulators by means of a single computer will eventually face problems in terms of failure resistance, flexibility, expandability, etc., as the number of manipulators or the degree of freedom of the joints increases.

One approach that can be taken to overcome the problems possessed by a centralized system is to deal with an autonomous decentralized system which is a system composed by multiple autonomous subsystems in a decentralized manner[9]. When the system has become large-scaled and complicated, then its functions may be handled in part by each of the subsystems so as for the system as whole to effect coordinated control. In this way, various characteristics as described below may be realized:

(1) As no system exists that is to control the entire system, any failure in a subsystem can be handled locally.

(2) By changing interactions among subsystems, it is possible to deal flexibly with various objectives.

(3) The presence of subsystems allows an easy expansion of the system, thereby eliminating the necessity of re-planning the action of the entire system.

Recently, in reference to multi-joint manipulators, a variety of control systems based on the concept of this autonomous decentralized system[10-12] have been proposed. However, these methods attempt to control a single manipulator by decentralizing it into multiple subsystems, but do not deal with multiple manipulators.

On the basis of these observations, in this paper, a method is proposed of generating a trajectory necessary for the cooperative task among multiple manipulators in a decentralized way by the cooperation among subsystems corresponding to the manipulators composing a multi-manipulator system. According to this method, virtual dynamics are imagined for each manipulator, and the virtual interactions between manipulators arising from these virtual dynamics are used to exchange information among subsystems, so that it is possible to deal with not only simple cooperative tasks like the movement of a grasped object but also more complicated cooperative tasks containing relative motions among manipulators.

In what follows, the kinematics of multiple manipulators will be formulated in the chapter 2, a decentralized trajectory generation method using the newly proposed virtual dynamics will be explained in 3, and the effective of this method will be examined by simulation experiments in the chapter 4.

2. Formulation and Kinematics of Multiple Manipulators

2.1. Formulation of Multiple Manipulators

Let us now consider n manipulators performing a task as illustrated in **Fig.1**. The degree of freedom of the joints of each manipulator is denoted by m^i (i=1, ...,n) and the degree of freedom of the task space is expressed by l. Then, a single task point according to the objective of the task is defined.

Here, by using three different coordinate systems, 1) the base coordinate system Σ_0, 2) the task coordinate system Σ_c having its origin at the position of the task point, and 3) the end-point coordinate system Σ_i, i=1,...,n, the position and orientation vector ${}^0X^c = [\,{}^0p^{c^T}, {}^0\Phi^{c^T}]^T \in \boldsymbol{R}^l$ of the origin of the task coordinate system Σ_c represented in the base coordinate system Σ_0, the positional and postural vectors ${}^0X^i = [{}^0p^{i^T}, {}^0\Phi^{i^T}]^T \in \boldsymbol{R}^l$ $(i = 1, \ldots, n)$ of the point of origin of the end-point coordinate system Σ_i as viewed by the base coordinate system Σ_0, and the position and orientation vector ${}^cX^i = [{}^cp^{i^T}, {}^c\Phi^{i^T}]^T \in \boldsymbol{R}^l$ $(i = 1, \ldots, n)$ of the origin of the end-point coordinate system Σ_i represented in the task coordinate system Σ_c are considered (refer to Fig.1). Then, the position and orientation of the task point ${}^0X^c$ can be determined uniquely from ${}^0X^i$ and ${}^0X^i$.

In the case of a task in the three-dimensional space (l=6), for example, the following results can be obtained. if the rotational matrix from Σ_i to Σ_0 is denoted as ${}^0R_i({}^0\Phi^i)$ and the rotational matrix from Σ_i to Σ_c as ${}^cR_i({}^c\phi^i)$, then the relationship among the position vectors ${}^0p^c$, ${}^0p^i$, and ${}^cp^i$ is given as follows:

$${}^0p^c = {}^0p^i - {}^0R_i({}^0\Phi^i)\,{}^cR_i({}^c\Phi^i)^T\,{}^cp^i \quad \ldots\ldots\ldots (1)$$

On the other hand, the use of the Euler angle $\Phi = [\phi, \theta, \psi]^T$ for each orientation vector leads to the following expression for the rotational matrix ${}^0R_c({}^0\Phi^c)$ from Σ_c to Σ_0:

$${}^0R_c({}^0\Phi^c) = {}^0R_i({}^0\Phi^i)\,{}^cR_i({}^c\Phi^i)({}^c\Phi^i)^T$$
$$= \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \quad \ldots\ldots\ldots (2)$$

Then, on the basis of the nature of the Euler angle, the orientation vector ${}^0\Phi^\chi = [{}^0\phi^\chi, {}^0\theta^\chi, {}^0\psi^\chi]^T$ can be given in the following way[13]:

a. When $\sin {}^0\theta^c \neq 0$,

$${}^0\phi^c = \textbf{atan}\ 2(\pm R_{23}, \pm R_{13}) \quad \ldots\ldots\ldots (3)$$

$${}^0\theta^c = \textbf{atan}\ 2(\pm\sqrt{R_{13}^2 + R_{23}^2}\ , R_{33}) \quad \ldots\ldots\ldots (4)$$

$${}^0\psi^c = \textbf{atan}\ 2(\pm R_{32}, \psi R_{31}) \quad \ldots\ldots\ldots (5)$$

b. When $\sin {}^0\theta^c = 0$

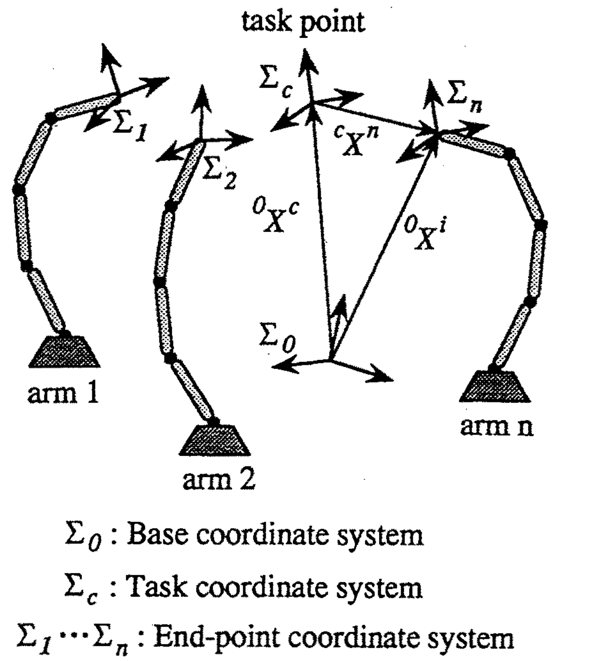

Fig. 1. Coordinated task by multiple manipulators.

$${}^0\phi^c = \text{arbitrary} \quad \ldots\ldots\ldots (6)$$

$${}^0\theta^c = \frac{\pi}{2}(1 - R_{33}) \quad \ldots\ldots\ldots (7)$$

$${}^0\psi^c = \textbf{atan2}\ (R_{21}, R_{22}) - R_{33} \times {}^0\phi^c \quad \ldots\ldots\ldots (8)$$

2.2. Kinematics of Multiple Manipulators

The relationship between the end-point velocity (${}^0\dot X^i$) of the arm i and the joint angular velocity ($\dot g^i \in R^{m^i}$) can be given, as is well known, by

$${}^0\dot X^i = J^i \dot q^i \quad \ldots\ldots\ldots (9)$$

where $J^i \in \boldsymbol{R}^{l\times m^i}$ is the Jacobian matrix of the arm i.

On the other hand, if it is assumed that a rigid link exists between the task point and the end-point of the arm i, then the following relationships are established between the end-point of the arm i and the position of the task point:

$${}^0F^{ci} = G^iH^i\,{}^0F^i = G^i\,{}^0F^{tri} \quad \ldots\ldots\ldots (10)$$

$${}^0\dot X^{tri} = H^i\,{}^0\dot X^i = G^{i^T}\,{}^0\dot X^c \quad \ldots\ldots\ldots (11)$$

Here, ${}^0F^i \in \boldsymbol{R}^l$ expresses the force and moment vector of the end-point of the arm i representatin in Σ_0, and ${}^0\dot X^c$ and ${}^0F^{ci} \in \boldsymbol{R}^l$ represent the velocity of the task point representatin in Σ_0 and the force and moment vector transmitted to the task point by the arm i, respectively. In addition, for the consideration of various contact mechanisms between the end points of the manipulators and the task point, the matrices $H^i \in \boldsymbol{R}^{l_i\times l}$ and $G^i = S^iH^{i^T} \in \boldsymbol{R}^{l\times l_i}$ expressing various contact types are used to denote as ${}^0\dot X^{tri}$ that component of the velocity of the end point of the arm i which can be transmitted from the task point, and as ${}^0F^{tri}$ the component of the force and moment of the end-point of the arm i that can be transmitted to the task point[14,15]. Also l_i is the degree of freedom of the force and moment that can be transmitted between the arm i and the task point. The matrix $S^i \in R^{l\times l}$ expresses the geometric relationship between the task point and the end-point, and can be given as

$$S^i = \begin{bmatrix} I & 0 \\ ({}^cp^i)_{\Sigma_0}\chi & I \end{bmatrix} \quad \ldots\ldots\ldots\ldots\ldots \quad (12)$$

where I is the unit matrix, 0 is a zero matrix, and $({}^cp^i)_{\Sigma_0}$ is ${}^cp^i$ represented in Σ_0. In addition, χ is an operator that satisfies $(\mathbf{a}\chi)\mathbf{b}=\mathbf{a}\times\mathbf{b}$, is defined[16], when $\mathbf{a}=[a,b,c]^T$, as

$$\mathbf{a}\chi = \begin{bmatrix} 0 & -c & b \\ c & 0 & -a \\ -b & a & 0 \end{bmatrix} \quad \ldots\ldots\ldots\ldots\ldots \quad (13)$$

Needless to say, the force and moment vector ${}^0F^c$ acting on the task point is the total sum of all the forces transmitted from the end-points of the manipulators to the task point. As a result, the following relationship is established:

$$^0F^c = \sum_{i=1}^{n} {}^0F^{ci} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (14)$$

All these relationships are summarized in **Fig.2**. In the next chapter, on the basis of the above formulation, a method of decentralized trajectory generation for multiple manipulators using the virtual dynamics to be proposed in this paper will be explained.

3. Decentralized Trajectory Generation Method for Multiple Manipulators Based on Virtual Dynamics

This method involves composing subsystems corresponding to the manipulators and endeavoring to generate the trajectories of the joints, that satisfy the kinematic constraint conditions, in a decentralized way through cooperations among the subsystems. To this end, it is necessary first to express interactions among subsystems. Here, virtual dynamics are supposed for the manipulators and the task point, and the virtual constraint forces generated from these dynamics and the positional constraints arising from the fact of these connecting together are used to express interactions among the subsystems.

3.1. Composition of Subsystems

Let us now consider a case in which among the n manipulators, the first to (n')th manipulators control the position of the task point and the $(n'+1)$st to (n)th manipulators move in relation with the task point.

First, the virtual dynamics of the manipulator i are expressed by using the simplest second order differential equation as follows:

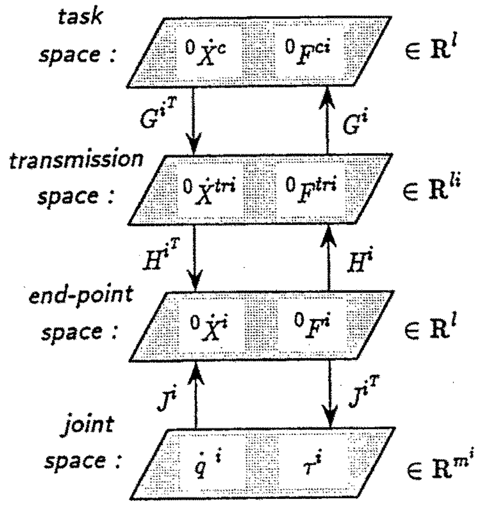

Fig. 2. Kinematic relationships of multiple manipulators.

$$\ddot{q}^i = \tau^i + (H^iJ^i)^T\lambda^i \quad \ldots\ldots\ldots\ldots\ldots \quad (15)$$

where ($\tau^i \in R^{m^i}$) is the virtual control torque of the manipulator i and $\lambda^i \in R^{l_i}$ is the virtual constraint force acting on the manipulator i from the task point.

Then, this virtual control torque τ^i is computed by using the target position ${}^0X^{c^*}$ as follows:

$$\begin{aligned} \tau^i &= (H^iJ^i)(G^i)^+K^i({}^0X^{c^*} - {}^0X^c) - B^i\dot{q}^i \\ &= (H^iJ^i)^T H^i S^{i^{-1}} K^i({}^0X^{c^*} - {}^0X^c) - B^i\dot{q}^i \quad \ldots\ldots \quad (16) \end{aligned}$$

where $(G^i)^+ = (G^{i^T}G^i)^{-1}G^{i^T} = H^iS^{i^{-1}} \in R^{l_i\times l}$ is the pseudo-inverse matrix of G^i, $K^i \in R^{l\times l}$ is the positive definite position feedback gain, and $B^i \in R^{m^i\times m^i}$ is the positive definite viscous friction matrix. Since it is possible to put G^i=0 for the manipulators $i=n'+1, \ldots, n$, namely$(G^i)^+=0$, so Eq.(16) becomes

$$\tau^i = -B^i\dot{q}^i \quad (i = n'+1, \ldots, n) \quad \ldots\ldots\ldots \quad (17)$$

Next, let us consider the motion of the task point. Since a virtual constraint force λ^i is applied to the end-point of the manipulator i from the task point, so inversely a virtual force $-\lambda^i$ is applied to the task point from each end-point. For this reason, the virtual dynamics of the task point is put as

$$^0\ddot{X}^c = M_c^{-1}\sum_{j=1}^{n} G^j(-\lambda^j) = -M_c^{-1}\sum_{j=1}^{n'} G^j\lambda^j \quad \ldots\ldots \quad (18)$$

where $M_c \in R^{l\times l}$ is the virtual inertia matrix of the task point.

Let us now consider the constraint conditions imposed on the end-point of each manipulator. First, the manipulators $i=1,\ldots,n'$ must be constrained by the motion of the task point determined by the virtual dynamic of Eq.(18). In other words, Eq.(11) leads to the relationship

$$ {}^0\dot{X}^{tri} = \dot{G}^{iT}\,{}^0\dot{X}^c + G^{iT}\,{}^0\ddot{X}^c \quad (19)$$

On the other hand, the constraint conditions for the manipulators $i = n' + 1, \ldots, n$ must considered by including in these conditions the motion of the task point but also the relative motion ${}^cX^i \in \boldsymbol{R}^l$ given as the target motion. Then, as it is possible to put l_i=l and H=I, the end-point velocity ${}^0\ddot{X}^{tri}$ can be written as

$$ {}^0\ddot{X}^{tri} = {}^0\ddot{X}^c + \begin{bmatrix} {}^0R_c & 0 \\ 0 & {}^0R_c \end{bmatrix} {}^c\ddot{X}^i + \begin{bmatrix} 2\,{}^0\dot{R}_c & 0 \\ 0 & {}^0\dot{R}_c \end{bmatrix} {}^c\dot{X}^i + \begin{bmatrix} {}^0\ddot{R}_c & 0 \\ 0 & 0 \end{bmatrix} {}^cX^i \quad (20)$$

The end-point velocity ${}^0\ddot{X}^{tri}$ imposed by Eqs.(19) and (20) must be identical with the end-point velocity determined by the joint motion of the manipulators. Hence,

$$ {}^0\ddot{X}^{tri} = \dot{H}^i J^i \dot{q}^i + H^i J^i \ddot{q}^i \quad (21)$$

Then, the joint trajectory of the manipulator i can be obtained, by using Eqs.(15) and (21), as follows:

$$ \begin{bmatrix} \ddot{q}^i \\ \lambda^i \end{bmatrix} = \begin{bmatrix} I & -(H^iJ^i)^T \\ H^iJ^i & 0 \end{bmatrix}^{-1} \begin{bmatrix} \tau^i \\ {}^0\ddot{X}^{tri} - \dot{H}^i J^i \dot{q}^i \end{bmatrix} \quad (22)$$

The trajectory generation method proposed here is now illustrated in **Fig.3.** Each subsystem generates a trajectory in interacting cooperation with the virtual end-point forces λ^i via the virtual dynamics of the task point. In this case, each subsystem can operate independently of other subsystems so that in case a certain manipulator changes from the grasping of an object to a relative motion or a new manipulator is added to the system, for example, it is not necessary to modify its equations of motion. As a next step, the stability of this system will be analyzed.

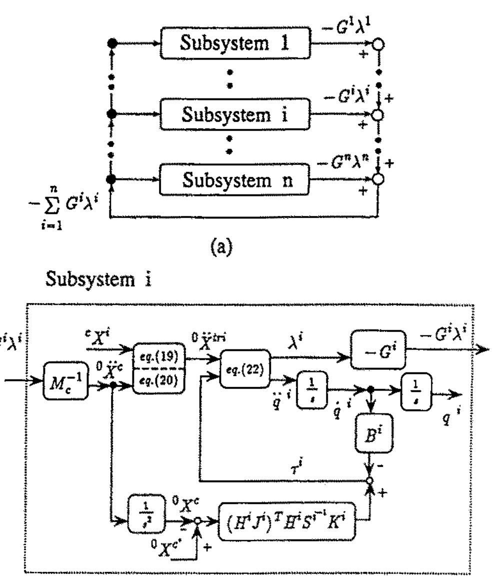

Fig. 3. Composition of a subsystem.

3.2. Stability of the Whole System

Let us now consider two types of energy functions H_1 and H_2 as given below:

$$ H_1 = \sum_{i=1}^{n'} E^i + V_c + \frac{1}{2}\sum_{i=1}^{n'} \dot{q}^{iT}\dot{q}^i \quad (23)$$

$$ E^i = \frac{1}{2}({}^0X^{c^*} - {}^0X^c)^T K^i({}^0X^{c^*} - {}^0X^c) \quad (24)$$

$$ V_c = \frac{1}{2}\,{}^0\dot{X}^{c^T} M_c\,{}^0\dot{X}^c \quad (25)$$

$$ H_2 = \frac{1}{2}\sum_{i=n'+1}^{n} \dot{q}^{iT}\dot{q}^i \quad (26)$$

H_1 and H_2 are energy functions for the motion of the task point and the relative motion of this point, respectively, and E^i represents the error between the position of the task point calculated at each end-point and the target position of the task point, while V_c expresses the virtual motion energy of the task point.

Let us first consider H_1, the motion of the task point. The time derivative $\dot{H}_1$ of the energy function H_1 is given as follows:

$$ \dot{H}_1 = \sum_{i=1}^{n'} \dot{E}^i + \dot{V}_c + \sum_{i=1}^{n'} \dot{q}^{iT}\ddot{q}^i \quad (27)$$

$$ \dot{E}^i = -\,{}^0\dot{X}^{c^T} K^i({}^0X^{c^*} - {}^0X^c) \quad (28)$$

$$ \dot{V}_c = {}^0\dot{X}^{c^T} M_c\,{}^0\ddot{X}^c \quad (29)$$

Substituting Eqs.(15), (16), and (18) into Eqs.(27), (28), and (29) and rearranging, we get

$$ \dot{H}_1 = -\sum_{i=1}^{n'} \dot{q}^{iT} B^i \dot{q}^i \quad (30)$$

Since B^i is a positive definite matrix, it is also true that $\dot{H}_1 \le 0$ and that the energy function H_1 decreases monotonically until $\dot{H}_1 = 0$, namely $\dot{q}^i = 0$ (i=1, …, n'). It is thus possible to assure the stability of the task point and the stability of the manipulators which control the position of the task point.

Let us next consider the relative motion H_2 with the task point. The time derivative $\dot{H}_2$ is given as

$$ \dot{H}_2 = \sum_{i=n'+1}^{n} \dot{q}^{iT}\ddot{q}^i = \sum_{i=n'+1}^{n} [-\dot{q}^{iT} B^i\dot{q}^i + {}^0\dot{X}^{tri\,T}\lambda^i] \quad (31)$$

Let us assume here that the relative motions ${}^{c}X^{i}$ (i=n' + 1, ..., n) as motion targets are given at the final time in such a way as to satisfy ${}^{c}\ddot{X}^{i}$ = ${}^{c}\dot{X}^{i}$ = 0. On the other hand, as ${}^{0}\ddot{X}^{c}$ = ${}^{0}\dot{X}^{c}$ = 0 when the energy function H_1 has converged, it is always the case that at some time point, the relationship ${}^{0}\ddot{X}^{i}$ = ${}^{0}\dot{X}^{i}$ = 0(i=n'+1, ..., n) must be satisfied. In other words, the second term of the right side of Eq.(31) becomes 0. As a result, the energy function H_2, as in the case of H_1, decreases until $\dot{H}_2$ = 0 is reached, namely $\dot{q}^{i}$ = 0 (i=n'+1, ..., n). Thus, the stability of the whole system has just been shown.

4. Simulation Experiments

By applying the decentralized trajectory generation method proposed in this paper to a cooperative task by three 4-joint planar manipulators, computer simulations were carried out (refer to **Fig.4**). In this case, the lengths of all the links of the manipulator were set equal to 0.4m, and the position of the task point was established at the center of gravity of the object (the origin of the task coordinate system: refer to Fig.4). In addition, the parameters used for the simulations were the position feedback gains K^i=*diag.* [100(N/m), 100(N/m) ,100(Nm/rad)] (i=1,2,3) of Eq.(16), the viscous friction matrices B^i=*diag.* [10, 10, 10, 10]($Nm/(rad)$) (i=1,2,3), and the virtual inertia matrix M_c=*diag.* [50(kg), 50(kg), 50(kgm^2).

Figure 5 shows a trajectory generated when the position of the task point was moved to the target position from the initial posture indicated in Fig.4. Figure 5(**a**) shows a stick picture, while Fig.5(**b**) expresses the time trajectory of the position of the task point. In this case, the type of contact between the end-point of each manipulator and the object is assumed to be rigid grasping. In addition, **Fig.6** gives the time variation of the virtual constraint force generated between the arm 1 and the object. On the other hand, **Fig.7** shows the results of the generation of trajectories by using the same initial posture as Fig.5 and the target position of the task point; Fig.7(**a**) indicates the simulation result for the case in which the first joints of the arm 1 and arm 2 are fixed under the assumption of their being faulty, Fig.7(**b**) illustrates another result for the case in which the end-point of every arm is in contact at a point with the object, and Fig.7(**c**) expresses yet another for the case in which the end point of the arm 3 undergoes a relative motion along the surface of the object. It is seen from Fig.7 that in every case, the position of the task point reaches the target position, but its intermediate trajectories and final postures are

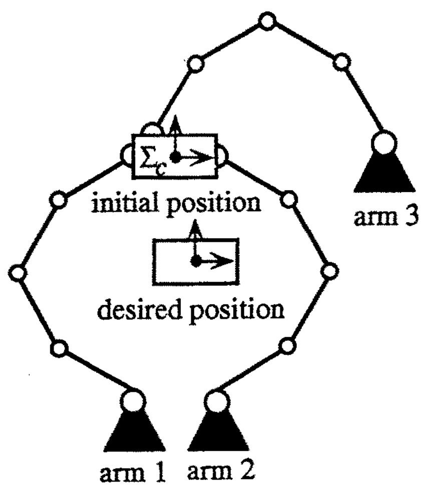

Fig. 4. Three 4-joint planar manipulators and the task coordinate system.

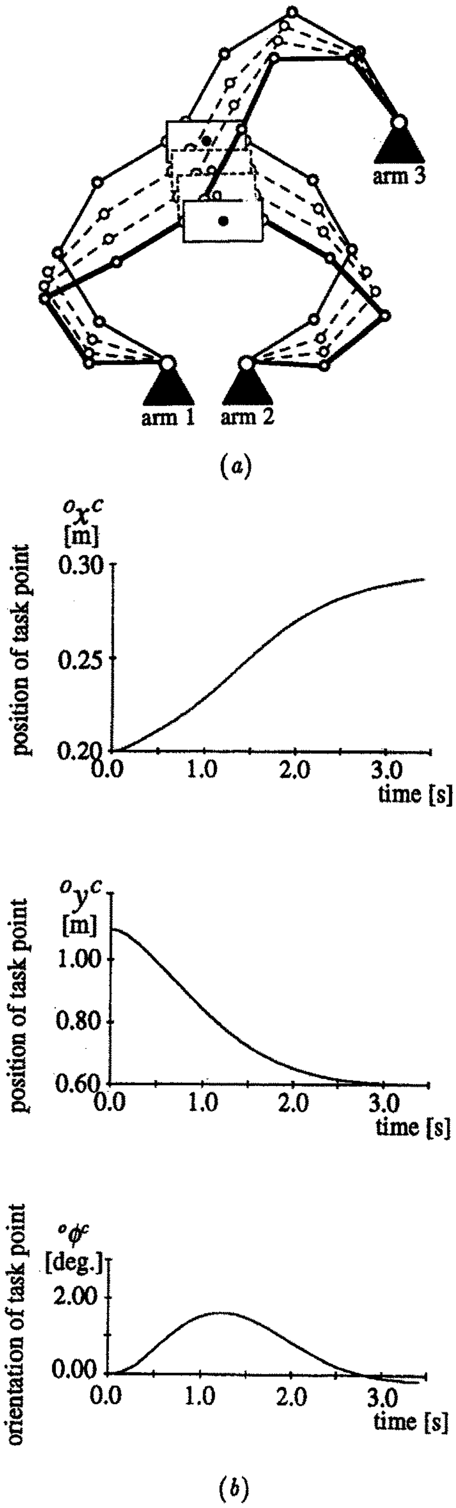

Fig. 5. An example of trajectory generated.

different to a considerable extent among the cases.

Since some joints are fixed in Fig.7(a) in contrast to the situation of Fig.5(a) where all the joints are utilized for moving the object, so both arm 1 and arm 2 act by means of only three joints each. Due to the fact that the fixing of a joint is handled inside its arm and does not directly affect the other subsystems, the present method can easily deal with such a breakdown.

In addition, a comparison of Fig.5(a) with Fig.7(b) and (c) indicates that whereas in Fig.5(a) the angles between the end-points and the object are kept constant, they are very different in Fig.7(c). It is thus possible to generate the trajectories of the manipulators by taking into consideration various contact mechanisms between the end-points of the manipulators and the object.

In Fig.7(c), the arm 1 and the arm 2 operate on the control of the task point, and the end point of the arm 3 carries out a relative motion with respect to the object. The relative motion of the end point of the arm 3 was given, as a function of the time t, as follows:

$$ {}^{c}X^{3}(t) = \begin{cases} [0.1t^2 - 0.1(m), 0.1(m), \frac{4}{3}\pi(rad)]^T & \text{if } 0 \le t < 1 \\ [-0.1t^2 + 0.4t - 0.3(m), 0.1(m), \frac{4}{3}\pi(rad)]^T & \text{if } 1 \le t < 2 \\ [0.1(m), 0.1(m), \frac{4}{3}\pi(rad)]^T & \text{if } t \ge 2 \end{cases} \quad \cdots\cdots (32) $$

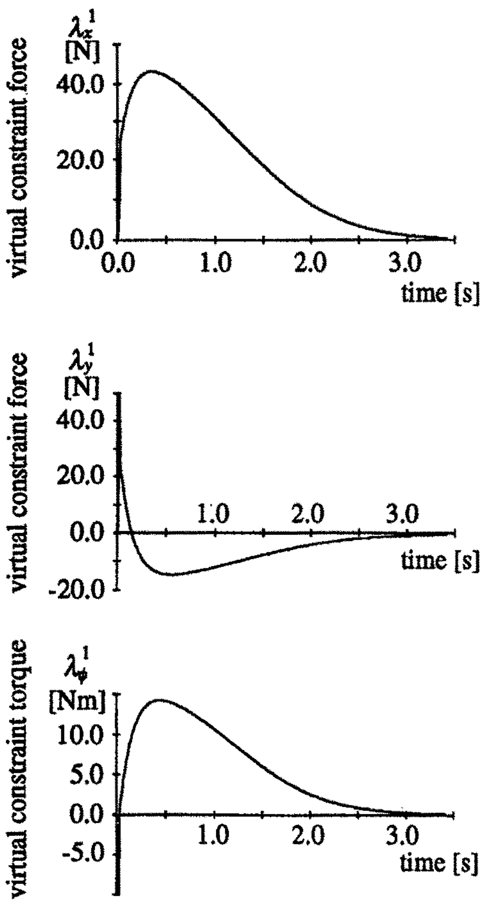

Fig. 6. Time variations of the virtual constraint force λ^1.

It is seen from the figure that even in the case of the end points carrying out relative motions, a cooperative task can be realized maintaining the closed link structure.

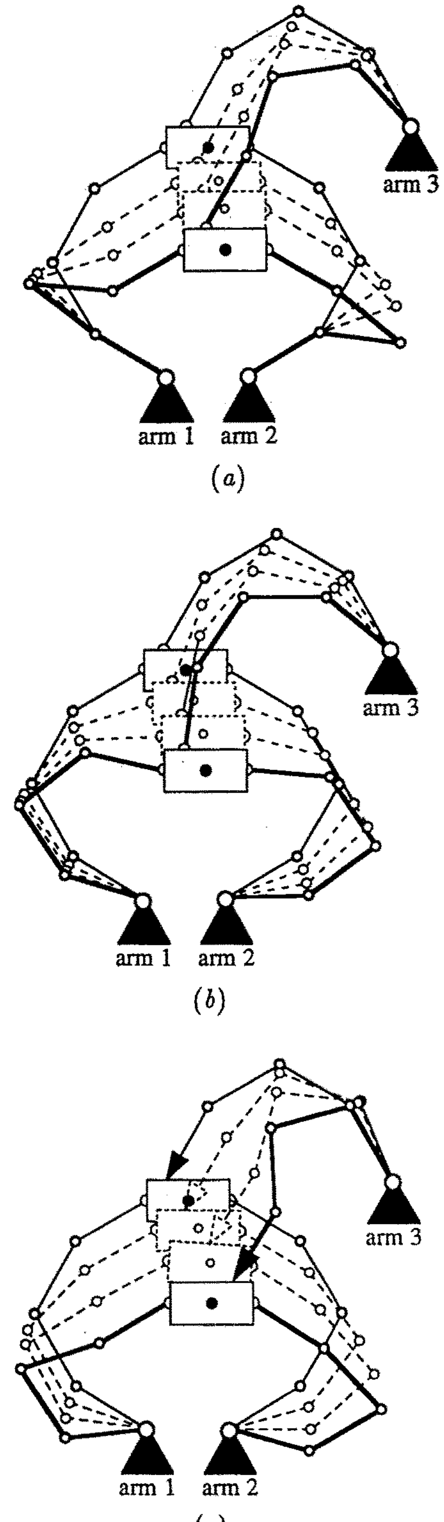

Fig. 7. Results of the trajectory generation.

5. Conclusions

In this paper, a method of generating in a decentralized way the trajectories of multiple manipulators by using the concept of virtual dynamics has been proposed. This method is based on an effort to express interactions among the manipulators by using the virtual force transmitted from each manipulator to the task point and the positional constraints arising from the fact of these manipulators being connected, and each manipulator is expressed as a subsystem containing virtual dynamics. In addition, the following facts, among others, have been made clear:

1) It is possible to generate in a parallel and decentralized way trajectories for multiple manipulators by the cooperation among the subsystems;

2) The relative motions between the end-points of the manipulators and the task point can be expressed as constraint conditions imposed on the end-points;

3) Each subsystem can function independently of the other subsystems, and the stability of the whole system can be assured.

For the future, the author is planning to examine ways to improve energy functions and consider the problem of deadlock avoidance based on sophisticated information communication, and also to look into the introduction of the concept of "subtask" by making positive use of redundant degrees of freedom and into methods of planning target trajectories for more complicated cooperative tasks.

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