Guaranteed Cost PI Control for Uncertain Discrete-Time Systems with Additive Gain

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Abstract— **This paper investigates a novel design method for robust proportional-integral (PI) control that is based on the guaranteed cost control (GCC) problem for a class of uncertain discrete-time systems with additive gain. On the basis of linear matrix inequality (LMI), a class of fixed PI controller parameters is obtained, and some sufficient conditions for the existence of the GCC are derived. A novel concept is that the fixed PI controller parameters are combined with additive gain such as that in neural networks (NNs) to enhance the transient stability. Although the additive gain is included in the feedback systems, both the stability of closed-loop systems and desired transient response are obtained. Finally, the experimental result demonstrates that the transient response can be appropriately changed by adjusting the additive gain.**

I. INTRODUCTION

It is well known that uncertainties such as those resulting from unmodelled dynamics occur in many dynamic systems and are frequently a source of instability and performance degradation of systems. In recent years, numerous results have been reported for the problem of designing robust controllers for linear systems. In all these researches, special attention has been devoted to find a controller that guarantees robust stability against uncertainties. However, it is also desirable to design a control system that realizes not only robust stability but also an adequate transient response. A design approach to this problem is the so-called guaranteed cost control (GCC) [1]. This approach has the advantage of providing an upper bound to a given quadratic cost function and thus the system performance degradation resulting from the uncertainties is guaranteed to be less than this bound.

It is well known that PID controller synthesis is the most widely used control algorithm in industrial applications. Over the past few decades, many methodologies have been proposed for tuning PID parameters. In past results, the Ziegler-Nichols step response method was generally used. These formulas were obtained empirically on the basis of extensive simulations of a large number of simple and stable plants. Therefore, it appears that exploiting a special class of stabilizing PID controller parameters results in a robust PID controller design that is not only robust with respect to plant uncertainty but also optimal nonfragile in the space of the controller coefficients. However, for arbitrary plants, these ad-hoc methods may not guarantee the closed-loop stability.

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In order to improve the unstable mode and easily implement PID tuning, a new self-tuning PID control scheme was proposed [7]. Subsequently, the auto-tuning PID control scheme that is based on neural networks for a nonlinear system was studied [8]. Moreover, fuzzy neural networks for tuning PID controller parameters for plants were proposed [9]. These approaches have the advantage that the controllers can be implemented even without the complete knowledge of the plant dynamics. However, stability may not be guaranteed because in these researches the stability of the original overall closed-loop system that includes an artificial intelligence controller is not explicitly considered.

On the other hand, the guaranteed cost PID control problem has been investigated by employing the linear matrix inequality (LMI) [11], [12], [13]. In these studies, the PID controller parameters were easily obtained by solving the LMI. However, these parameters cannot be varied while the control is being carried out. From the viewpoint of self-tuning or auto-tuning PID control synthesis, it is preferable for these parameters to be adaptively variable changing for the plant dynamics.

In this paper, a robust guaranteed cost PI controller design problem for the uncertain discrete-time system with additive gain is proposed to enhance the transient stability. It should be noted that the guaranteed cost PI control design methodology for the practical uncertain systems has not been examined as compared with the existing results [4], [5], [6]. Particularly, although the existing results are based on the theoretical aspect, the considered problem is based on the practical application such as PI control. As a result, the considered control systems can be easily implemented. In fact, it is shown that the experimental result demonstrates the efficiency of our control systems. The new contributions of our study are as follows. First, it is shown that the actual state feedback gain can be designed by adopting an additive control input such as a neurocontroller. Second, although the additive gain such as that in neurocontrollers is included in the uncertain discrete-time system, robust stability of the closed-loop system is realized. Moreover, the transient response can be appropriately changed by adopting the additive control gain or adjusting the weight matrices of the cost function. It should be noted that the proposed concept of additive gain is significantly different from the auto-tuning PID control [8], [9] in the sense that the stability of the overall system is guaranteed even if the additive gain cannot adapt the optimal gain completely. Furthermore, although robust nonfragile PID control has been examined in [10], our proposed PI control considers not the fragility but

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the additive gain as a part of the control gain. It is noteworthy that such a novel concept has not been reported thus far. Finally, a class of fixed PI control parameters of the uncertain discrete-time system with additive gain is newly established by using LMI. In order to demonstrate the efficiency of our design approach, an experimental result is provided. In particular, it is shown that the experimental system is asymptotically stable by adopting the additive gain and that it is unstable without additive gain.

Notation: The notations used in this paper are fairly standard. The superscript T denotes the matrix transpose. **block diag** denotes the block diagonal matrix. I_n denotes the $n \times n$ identity matrix. $\|\cdot\|$ denotes its Euclidean norm for a matrix. $\lambda_{\min}$ denotes the minimum eigenvalue for a matrix. **Tr** denotes the trace of matrix. E denotes the expectation.

II. PROBLEM FORMULATION AND PRELIMINARY

Consider the discrete-time system with both state and input uncertainties as follows.

$$x(k+1) = [A + \Delta A(k)]x(k) + [B + \Delta B(k)]u(k), \quad \text{(1a)}$$
$$u(k) = [K_P + \Delta K_P(k)]x(k) + [K_I + \Delta K_I(k)] \sum_{i=0}^{k-1} x(i), \quad \text{(1b)}$$
$$x(0) = x^0,\ x(-1) = 0,$$

where $x(k) \in \Re^n$ is the state, $u(k) \in \Re^m$ is the control input, respectively. It should be noted that the PI controller consists of the proportional and integral form [14]. Moreover, without loss of generality, it is assumed that the equilibrium point is zero because the regulator problem is considered. The matrices A and B are known real constant matrices of appropriate dimensions. $\Delta A(k)$ and $\Delta B(k)$ are unknown matrix representing time-varying parameter uncertainty. These parameter uncertainties are assumed to be of the following form

$$\left[\ \Delta A(k) \quad \Delta B(k)\ \right] = \left[\ D_a F_a(k) E_a \quad D_b F_b(k) E_b\ \right], \quad (2)$$

where D_a, D_b, E_a and E_b are known real constant matrices and $F_a(k) \in \Re^{p_a \times q_a}$ and $F_b(k) \in \Re^{p_b \times q_b}$ are unknown matrix satisfying $F_a^T(k)F_a(k) \le I_{q_a}$ and $F_b^T(k)F_b(k) \le I_{q_b}$. On the other hand, K_P and K_I are the fixed PI controller parameters, $\Delta K_P(k)$ and $\Delta K_I(k)$ represent the additive gain of the form

$$\left[\ \Delta K_P(k)\ \Delta K_I(k)\ \right] = \left[\ D_P N_P(k) E_P\ D_I N_I(k) E_I\ \right]. \quad (3)$$

Here D_P, D_I, E_P and E_I are known real constant matrices and $N_P(k) \in \Re^{p_P \times q_P}$ and $N_I(k) \in \Re^{p_I \times q_I}$ are the additive gains satisfying $N_P^T(k)N_P(k) \le I_{q_P}$, $N_I^T(k)N_I(k) \le I_{q_I}$. It should be noted that the PI parameters can be changed under the constraints of $K_P + \Delta K_P(k)$ and $K_I + \Delta K_I(k)$ while the control is being carried out. On the other hand, these additive gains can also be permitted the uncertainties such as A/D conversion, D/A conversion, finite word length and round-off errors.

Using (1a) and (1b), the state equations can be rewritten

$$\tilde{x}(k+1) = [\tilde{A} + \Delta\tilde{A}(k)]\tilde{x}(k) + [\tilde{B} + \Delta\tilde{B}(k)]u(k), \quad \text{(4a)}$$
$$u(k) = [\tilde{K} + \Delta\tilde{K}(k)]\tilde{x}(k), \quad \text{(4b)}$$

where

$$\tilde{x}(k) = \begin{bmatrix} x(k) \\ \sum_{i=0}^{k-1} x(i) \end{bmatrix},\ \tilde{A} = \begin{bmatrix} A & 0 \\ I_n & I_n \end{bmatrix},\ \tilde{B} = \begin{bmatrix} B \\ 0 \end{bmatrix},$$
$$\Delta\tilde{A}(k) = \begin{bmatrix} \Delta A(k) & 0 \\ 0 & 0 \end{bmatrix},\ \Delta\tilde{B}(k) = \begin{bmatrix} \Delta B(k) \\ 0 \end{bmatrix},$$
$$\tilde{K} = \left[\ K_P \quad K_I\ \right],\ \Delta\tilde{K}(k) = \left[\ \Delta K_P(k) \quad \Delta K_I(k)\ \right].$$

Associated with the system (4), the cost function is

$$J = \sum_{k=0}^{\infty} \left[\tilde{x}^T(k)Q\tilde{x}(k) + u^T(k)Ru(k)\right], \quad (5)$$

where Q and R are the positive definite symmetric matrices.

In this situation, the definition of the guaranteed cost control with the additive gain is given below.

Definition 1: For the discrete-time system (4) and the cost function (5), assume that there exist the gain matrix $\tilde{K}$ and the positive scalar J^* such that for the admissible uncertainties and the additive gain, the closed-loop system is asymptotically stable and the closed-loop value of the cost function (5) satisfies $J < J^*$. Then J^* and $\tilde{K}$ are said to be the guaranteed cost and the guaranteed cost control gain matrix, respectively.

The above definition is very popular for dealing with the time-varying uncertainties and is also used [1].

A. Sufficient Condition for Existence of Guaranteed Cost Control

In this subsection, we give the sufficient condition for existence of the guaranteed cost control with additive gain that is based on the PI control.

Theorem 1: Suppose that the following matrix inequality holds for the discrete-time system (4).

$$\left[\tilde{A} + \Delta\tilde{A}(k) + [\tilde{B} + \Delta\tilde{B}(k)][\tilde{K} + \Delta\tilde{K}(k)]\right]^T P \times \left[\tilde{A} + \Delta\tilde{A}(k) + [\tilde{B} + \Delta\tilde{B}(k)][\tilde{K} + \Delta\tilde{K}(k)]\right] - P + Q + [\tilde{K} + \Delta\tilde{K}(k)]^T R[\tilde{K} + \Delta\tilde{K}(k)] < 0. \quad (6)$$

If such condition is satisfied, the gain matrix $\tilde{K}$ of the controller (4b) is the guaranteed control gain matrix associated with the cost function (5). Then, the closed-loop uncertain system

$$\tilde{x}(k+1) = \left[\tilde{A} + \Delta\tilde{A}(k) + (\tilde{B} + \Delta\tilde{B}(k))(\tilde{K} + \Delta\tilde{K}(k))\right]\tilde{x}(k) \quad (7)$$

is asymptotically stable and achieves the following inequality.

$$J < J^* = \tilde{x}^T(k)P\tilde{x}(k). \quad (8)$$

Proof: Suppose now that there exist the symmetric positive definite matrices $P > 0$ such that the matrix inequality (6) holds for all admissible uncertainties and the additive gain (4b). Let us define the following Lyapunov function candidate.

$$V(\tilde{x}(k)) = \tilde{x}^T(k)P\tilde{x}(k), \tag{9}$$

where $V(\tilde{x}(k))$ satisfies $V(\tilde{x}(k)) > 0$ for all $\tilde{x}(k) \neq 0$. The corresponding difference along any trajectory of the closed-loop system is given by

$$\begin{aligned}\Delta V(\tilde{x}(k)) &= V(\tilde{x}(k+1)) - V(\tilde{x}(k)) \\ &= \tilde{x}^T(k)\Phi_1(P,\tilde{K})\tilde{x}(k),\end{aligned} \tag{10}$$

where

$$\begin{aligned}&\Phi_1(P,\tilde{K}) \\ &= \left[\tilde{A} + \Delta\tilde{A}(k) + [\tilde{B} + \Delta\tilde{B}(k)][\tilde{K} + \Delta\tilde{K}(k)]\right]^T P \\ &\times \left[\tilde{A} + \Delta\tilde{A}(k) + [\tilde{B} + \Delta\tilde{B}(k)][\tilde{K} + \Delta\tilde{K}(k)]\right] - P.\end{aligned} \tag{11}$$

From condition (6), we have

$$\begin{aligned}&\Delta V(\tilde{x}(k)) \\ &< -\tilde{x}^T(k)\left[Q + [\tilde{K} + \Delta\tilde{K}(k)]^T R[\tilde{K} + \Delta\tilde{K}(k)]\right]\tilde{x}(k) \\ &\leq -\lambda_{\min}\left[Q + [\tilde{K} + \Delta\tilde{K}(k)]^T R[\tilde{K} + \Delta\tilde{K}(k)]\right]\|\tilde{x}(k)\|^2.\end{aligned} \tag{12}$$

Using Lyapunov stability theory, it follows that the system (7) is asymptotically stable. Furthermore, from (12) we have

$$\begin{aligned}&-\Delta V(\tilde{x}(k)) \\ &> \tilde{x}^T(k)\left[Q + [\tilde{K} + \Delta\tilde{K}(k)]^T R[\tilde{K} + \Delta\tilde{K}(k)]\right]\tilde{x}(k).\end{aligned}$$

Summing both sides of the above inequality from 0 to ∞ and using the system stability yield

$$J \leq \tilde{x}^T(0)P\tilde{x}(0). \tag{13}$$

It follows from Definition 1 that the result of the theorem is true. This completes the proof of theorem. ■

B. LMI Design Approach for Fixed PI Controller Parameters

The objective of this subsection is to design the fixed guaranteed cost control gain matrix $\tilde{K}$ that consist of PI controller parameters for the discrete-time system (4) with the cost function (5) via the LMI design approach.

Before giving the result, in order to simplify the notation, the following matrices are introduced.

$$\begin{aligned}&\Delta\tilde{A}(k) := \tilde{D}_a F_a(k)\tilde{E}_a,\ \Delta\tilde{B}(k) := \tilde{D}_b F_b(k)E_b, \\ &\Delta\tilde{K}(k) := \tilde{D}_k N(k)\tilde{E}_k, \\ &\tilde{D}_a := \textbf{block diag}\begin{pmatrix} D_a & 0\end{pmatrix},\ \tilde{D}_b := \begin{bmatrix} D_b \\ 0\end{bmatrix}, \\ &\tilde{D}_k := \begin{bmatrix} D_P & D_I\end{bmatrix},\ \tilde{E}_a := \textbf{block diag}\begin{pmatrix} E_a & 0\end{pmatrix}, \\ &\tilde{E}_k := \textbf{block diag}\begin{pmatrix} E_P & E_I\end{pmatrix}, \\ &N(k) := \textbf{block diag}\begin{pmatrix} N_P(k) & N_I(k)\end{pmatrix}.\end{aligned}$$

Theorem 2: Consider the discrete-time system (4) and the cost function (5). Now we assume that there exist the matrices $X \in \Re^{2n\times 2n}$, $Y \in \Re^{m\times 2n}$ and the positive constants ϵ_a, ϵ_b and ϵ_k satisfying LMI (14) for all the arbitrary function $N(k)$ as the additive gain. Then fixed gain matrix $\tilde{K} = YX^{-1}$ is the guaranteed cost control gain matrix. Moreover, it is also called the fixed PI controller parameters.

In order to prove Theorem 2, the following Lemma will be used [2].

Lemma 1: [2] Consider the appropriate matrix $\mathcal{F}$ which is satisfying $\mathcal{F}\mathcal{F}^T \leq I_n$ and for any matrices $\mathcal{G}$ and $\mathcal{H}$. There exists the positive parameter $\lambda > 0$ such that the following inequality holds

$$\mathcal{G}\mathcal{F}\mathcal{H} + \mathcal{H}^T\mathcal{F}^T\mathcal{G}^T \leq \lambda\mathcal{G}\mathcal{G}^T + \lambda^{-1}\mathcal{H}^T\mathcal{H}. \tag{18}$$

Let us prove Theorem 2 by using the above Lemma 1.

Proof: Let us introduce matrices $X = P^{-1}$ and $Y = \tilde{K}P^{-1}$. Pre- and post-multiplying both sides of the inequality (14) by the positive definite matrix

$$\textbf{block diag}\begin{pmatrix} P & I_{2n} & I_n & I_{q_b} & I_{q_a} & I_{q_n} & I_{2n}\end{pmatrix} > 0, \tag{19}$$

with $q_n := q_P + q_I$ and applying Schur complement [3] gives (15). It should be noted that the LMI (14) is equivalent to the matrix inequality (15). Using the standard matrix inequality (18) of Lemma 1 for all admissible uncertainty and $L < 0$ of (15), the matrix inequality (16) holds. By applying Schur complement to (16) again, it is easy to verify that $\mathcal{T} < 0$ is equivalent to $\mathcal{U} < 0$. Finally, using the matrix inequality (17), it is concluded that the matrix inequality (6) is satisfied. Thus, $\tilde{K}$ is the guaranteed cost control matrix. On the other hand, since the results of the cost bound (8) can be proved by using the similar argument for the proof of Theorem 1, it is omitted. ■

Since the LMI (14) consists of a convex solution set of $(\epsilon_a, \epsilon_b, \epsilon_k, X, Y)$, various efficient convex optimization algorithm can be applied. Moreover, its solutions represent a set of the guaranteed cost control gain matrix $\tilde{K}$. This parameterized representation can be exploited to design the guaranteed cost control gain which minimizes the value of the guaranteed cost for the closed-loop uncertain system. Consequently, solving the following optimization problem allows us to determine the optimal bound.

$$J < J^* < \min_{(\epsilon_a,\epsilon_b,\epsilon_k,X,Y)} \alpha, \tag{20}$$

such that (14) and

$$\begin{bmatrix} -\alpha & \tilde{x}^T(0) \\ \tilde{x}(0) & -X\end{bmatrix} < 0. \tag{21}$$

The problem addressed in this section is defined as follows:

Problem 1 : "Find $\tilde{K} = YX^{-1}$ such that the LMIs (14) and (21) are satisfied, and the cost α becomes as small as possible."

Since the bound in Problem 1 depends on the initial conditions $\tilde{x}(0)$, in order to remove such condition, it is assumed that $\tilde{x}(0)$ is a zero mean random variable satisfying $E\left[\tilde{x}(0)\tilde{x}^T(0)\right] = I_{2n}$. Then the LMI (21) yields

$$\begin{bmatrix} -M & I_n \\ I_n & -X\end{bmatrix} < 0, \tag{22}$$

$$\begin{bmatrix} -X & (\tilde{A}X+\tilde{B}Y)^T & Y^T & Y^T E_b^T & X\tilde{E}_a^T & X\tilde{E}_k^T & X \\ \tilde{A}X+\tilde{B}Y & \Theta & \epsilon_k \tilde{B}\tilde{D}_k\tilde{D}_k^T & \epsilon_k \tilde{B}\tilde{D}_k\tilde{D}_k^T E_b^T & 0 & 0 & 0 \\ Y & \epsilon_k \tilde{D}_k\tilde{D}_k^T\tilde{B}^T & -R^{-1}+\epsilon_k\tilde{D}_k\tilde{D}_k^T & \epsilon_k\tilde{D}_k\tilde{D}_k^T E_b^T & 0 & 0 & 0 \\ E_bY & \epsilon_k E_b\tilde{D}_k\tilde{D}_k^T\tilde{B}^T & \epsilon_k E_b\tilde{D}_k\tilde{D}_k^T & -\epsilon_b I_{q_b}+\epsilon_k E_b\tilde{D}_k\tilde{D}_k^T E_b^T & 0 & 0 & 0 \\ \tilde{E}_aX & 0 & 0 & 0 & -\epsilon_a I_{q_a} & 0 & 0 \\ \tilde{E}_kX & 0 & 0 & 0 & 0 & -\epsilon_k I_{q_n} & 0 \\ X & 0 & 0 & 0 & 0 & 0 & -Q^{-1} \end{bmatrix} < 0, \quad (14)$$

where $\Theta = -X + \epsilon_a\tilde{D}_a\tilde{D}_a^T + \epsilon_b\tilde{D}_b\tilde{D}_b^T + \epsilon_k\tilde{B}\tilde{D}_k\tilde{D}_k^T\tilde{B}^T$.

$$L := \begin{bmatrix} -P+Q+\epsilon_a^{-1}\tilde{E}_a^T\tilde{E}_a+\epsilon_k^{-1}\tilde{E}_k^T\tilde{E}_k & (\tilde{A}+\tilde{B}\tilde{K})^T & \tilde{K}^T & \tilde{K}^T E_b^T \\ \tilde{A}+\tilde{B}\tilde{K} & \Theta & \epsilon_k\tilde{B}\tilde{D}_k\tilde{D}_k^T & \epsilon_k\tilde{B}\tilde{D}_k\tilde{D}_k^T E_b^T \\ \tilde{K} & \epsilon_k\tilde{D}_k\tilde{D}_k^T\tilde{B}^T & -R^{-1}+\epsilon_k\tilde{D}_k\tilde{D}_k^T & \epsilon_k\tilde{D}_k\tilde{D}_k^T E_b^T \\ E_b\tilde{K} & \epsilon_k E_b\tilde{D}_k\tilde{D}_k^T\tilde{B}^T & \epsilon_k E_b\tilde{D}_k\tilde{D}_k^T & -\epsilon_b I_{q_b}+\epsilon_k E_b\tilde{D}_k\tilde{D}_k^T E_b^T \end{bmatrix} < 0. \quad (15)$$

$$\begin{aligned} \mathcal{T} &= \begin{bmatrix} -P+Q+\epsilon_a^{-1}\tilde{E}_a^T\tilde{E}_a & (\tilde{A}+\tilde{B}\tilde{K})^T & \tilde{K}^T & \tilde{K}^T E_b^T \\ \tilde{A}+\tilde{B}\tilde{K} & -P^{-1}+\epsilon_a\tilde{D}_a\tilde{D}_a^T+\epsilon_b\tilde{D}_b\tilde{D}_b^T & 0 & 0 \\ \tilde{K} & 0 & -R^{-1} & 0 \\ E_b\tilde{K} & 0 & 0 & -\epsilon_b I_{q_b} \end{bmatrix} \\ &\quad + \begin{bmatrix} 0 \\ \tilde{B}\tilde{D}_k \\ \tilde{D}_k \\ E_b\tilde{D}_k \end{bmatrix} N(k) \begin{bmatrix} \tilde{E}_k & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} \tilde{E}_k^T \\ 0 \\ 0 \\ 0 \end{bmatrix} N^T(k) \begin{bmatrix} 0 & \tilde{D}_k^T\tilde{B}^T & \tilde{D}_k^T & \tilde{D}_k^T E_b^T \end{bmatrix} \le L < 0. \quad (16) \end{aligned}$$

$$\begin{aligned} \mathcal{V} &= \begin{bmatrix} -P+Q & \left[\tilde{A}+\Delta\tilde{A}(k)+(\tilde{B}+\Delta\tilde{B}(k))(K+\Delta\tilde{K}(k))\right]^T & \left[\tilde{K}+\Delta\tilde{K}(k)\right]^T \\ \tilde{A}+\Delta\tilde{A}(k)+(\tilde{B}+\Delta\tilde{B}(k))(\tilde{K}+\Delta\tilde{K}(k)) & -P^{-1} & 0 \\ \tilde{K}+\Delta\tilde{K}(k) & 0 & -R^{-1} \end{bmatrix} \\ &\le \begin{bmatrix} -P+Q & \left[\tilde{A}+\tilde{B}(\tilde{K}+\Delta\tilde{K}(k))\right]^T & \left[\tilde{K}+\Delta\tilde{K}(k)\right]^T \\ \tilde{A}+\tilde{B}(\tilde{K}+\Delta\tilde{K}(k)) & -P^{-1} & 0 \\ \tilde{K}+\Delta\tilde{K}(k) & 0 & -R^{-1} \end{bmatrix} \\ &\quad +\epsilon_a \begin{bmatrix} 0 \\ \tilde{D}_a \\ 0 \end{bmatrix} \begin{bmatrix} 0 & \tilde{D}_a^T & 0 \end{bmatrix} + \epsilon_a^{-1} \begin{bmatrix} \tilde{E}_a^T \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} \tilde{E}_a & 0 & 0 \end{bmatrix} \\ &\quad +\epsilon_b \begin{bmatrix} 0 \\ \tilde{D}_b \\ 0 \end{bmatrix} \begin{bmatrix} 0 & \tilde{D}_b^T & 0 \end{bmatrix} + \epsilon_b^{-1} \begin{bmatrix} (\tilde{K}+\Delta\tilde{K}(k))^T\tilde{E}_b^T \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} E_b(\tilde{K}+\Delta\tilde{K}(k)) & 0 & 0 \end{bmatrix} = \mathcal{U} < 0 \Leftrightarrow \mathcal{T} < 0. \quad (17) \end{aligned}$$

$$\begin{aligned} \mathcal{V} < 0 \quad \Leftrightarrow \quad & \left[\tilde{A}+\Delta\tilde{A}(k)+(\tilde{B}+\Delta\tilde{B}(k))(\tilde{K}+\Delta\tilde{K}(k))\right]^T P \left[\tilde{A}+\Delta\tilde{A}(k)+(\tilde{B}+\Delta\tilde{B}(k))(\tilde{K}+\Delta\tilde{K}(k))\right] \\ & -P+Q+(\tilde{K}+\Delta\tilde{K}(k))^T R(\tilde{K}+\Delta\tilde{K}(k)) < 0 \Rightarrow (6) \end{aligned}$$

In this paper, the condition (22) is used instead of (21) and $\mathbf{Tr}[M]$ should be optimized.

In the next section we will discuss about the additive gain as the input of NNs.

III. ADDITIVE GAIN USING NEURAL NETWORKS

In this section, the neural networks (NNs) will be adopted as the additive gain. It should be noted that the fuzzy controller can also be considered instead of NNs as the additive gain.

A. On-line Learning Algorithm of Neurocontroller

It is well known that NNs have found wide potential applications in system control because of their ability to perform nonlinear mapping. Therefore, since a sufficiently accurate model of the system is generally not available, using the nonlinear mapping provided by the neural output with the uncertainty determined will result in a better performance. Moreover, it is expected that the response can be improved if NNs can be managed such that the system dynamics including uncertainty converges to the equilibrium point as rapidly as possible. Thus, NNs are adopted to enhance the transient stability of system. Let us define the energy function to establish the learning algorithm.

$$E(k) := \frac{1}{2}[\tilde{x}^T(k+1)\tilde{x}(k+1) + u^T(k+1)u(k+1)]. \quad (23)$$

If $E(k)$ can be reduced as small as possible, the state $\tilde{x}(k)$ would be improved. That is, robot manipulator will get to the desired position more precisely.

In the learning phase of NNs, the weight updating rules can be described as

$$w_g^{ij}(k+1) = w_g^{ij}(k) + \Delta w_g^{ij}(k). \quad (24)$$

On the other hand, the modification of the weight coefficient

$w_g^{ij}(k)$ is given by

$$\Delta w_g^{ij}(k) = -\eta \frac{\partial E(k)}{\partial w_g^{ij}(k)}, \quad (25a)$$

$$\frac{\partial E(k)}{\partial w_g^{ij}(k)} = \frac{\partial E(k)}{\partial N(k)} \cdot \frac{\partial N(k)}{\partial w_g^{ij}(k)}, \quad (25b)$$

where η is the learning ratio.

The term $\frac{\partial E(k)}{\partial N(k)}$ of the equation (25b) can be calculated from the energy function (23) as follows:

$$\frac{\partial E(k)}{\partial N(k)} = \tilde{x}(k+1)[\tilde{B} + \Delta\tilde{B}(k)]\tilde{D}_k\tilde{E}_k\tilde{x}(k). \quad (26)$$

However, since there exists the function $\Delta\tilde{B}(k)$, it is difficult to decide the learning rule of NNs. To remove this problem, suppose there exists the parameter $\lambda(k)$ such that $\tilde{B} + \Delta\tilde{B}(k) \approx \lambda(k)\tilde{B}$, where $\lambda(k)$ is the matrix value function and its elements are positive scalars. Hence, the modification of the weight coefficient of the equation (25a) is newly defined as follows.

$$\Delta w_g^{ij}(k) \approx -\delta\tilde{x}(k+1)\tilde{B}\tilde{D}_k\tilde{E}_k\tilde{x}(k)\frac{\partial N(k)}{\partial w_g^{ij}(k)}, \quad (27)$$

where $\delta := \eta\lambda(k)$ is defined as a new learning ratio.

δ is used instead of deciding η according to $\lambda(k)$. Since $\frac{\partial N(k)}{\partial w_g^{ij}(k)}$ can be calculated using the chain rule on NNs, NNs can be trained.

The approximation of $\tilde{B} + \Delta\tilde{B}(k) \approx \lambda(k)\tilde{B}$ is a very particular case. Thus, this assumption may not satisfy the practical conditions. Even though this assumption seems to be conservative, it will be compensated by making use of the new learning ratio. In order to avoid such conservative assumptions, the fuzzy control can also be introduced. Finally, it is worth pointing out that even if NNs cannot result in the exact control gain, the robust stability is guaranteed and it is possible to attain the better transient response by adopting the other additive gain.

B. Multilayered Neural networks

The utilized NNs are of a three-layer feed-forward network as shown in Fig. 1. The linear function is utilized in the neurons of the input and the hidden layers, and a sigmoid function in the output layer. The inputs and outputs of each layer can be described as follows.

$$s_g^i(k) := \begin{cases} U_i(k) & \{g = 1 \text{ (input layer)}\} \\ \sum w_1^{(i,j)}(k)o_1^j(k) & \{g = 2 \text{ (hidden layer)}\} \\ \sum w_2^{(i,j)}(k)o_2^j(k) & \{g = 3 \text{ (output layer)}\}, \end{cases}$$

$$o_g^i(k) := \begin{cases} s_1^i(k) & \{g = 1 \text{ (input layer)}\} \\ s_2^i(k) + \theta_1^{(i)}(k) & \{g = 2 \text{ (hidden layer)}\} \\ \dfrac{1 - e^{(-s_3^i(k) + \theta_2^{(i)}(k))}}{1 + e^{(-s_3^i(k) + \theta_2^{(i)}(k))}} & \{g = 3 \text{ (output layer)}\}, \end{cases}$$

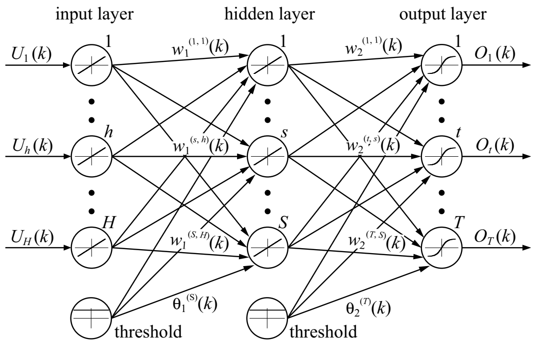

Fig. 1. Structure of the multilayered neural networks.

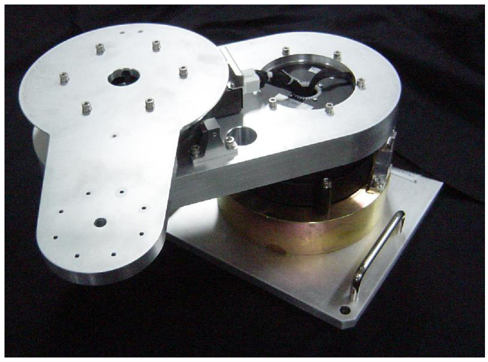

Fig. 2. An experimental system of two-link manipulator.

where $s_g^i(k)$ and $o_g^i(k)$ are the input and the output of the neuron i in the gth layer at the time t. $w_g^{ij}(k)$ indicates the weight coefficient from the neuron j in the gth layer to the neuron i in the $(g+1)$th layer. $U_i(k)$ is the input of NNs. $\theta_g^{(i)}(k)$ is a positive constant for the threshold of the neuron i in the $(g+1)$th layer. For the additive gain defined in the formula (4b), the outputs of NNs are set in the range of $[-1.0,\ 1.0]$.

It should be noted that it is easy to implement the considered controller because the above learning algorithm is very simple, while the output of NNs may not be appropriate signal due to the simple learning algorithm. However, it is worth pointing out that even if NNs cannot be trained exactly, the closed-loop stability is still guaranteed under the LMI conditions (14).

IV. EXPERIMENTAL RESULTS

In this section, in order to verify the effectiveness of the proposed method in a real system, a robot manipulator was used to examine the uncertainty and additive gain. The basic concept of the experiment can be stated as follows. The system comprises a personal computer that controls the system, a control box that connects the robot controller with the personal computer, a robot controller that contains the supply circuit, and the robot manipulator. The torque provided to the robot arm is generated as the product of the calculated gain and state. The two-link robot manipulator that has been used in the experiment is shown in Fig. 2. Each arm is equipped with a direct-drive motor and each joint angle

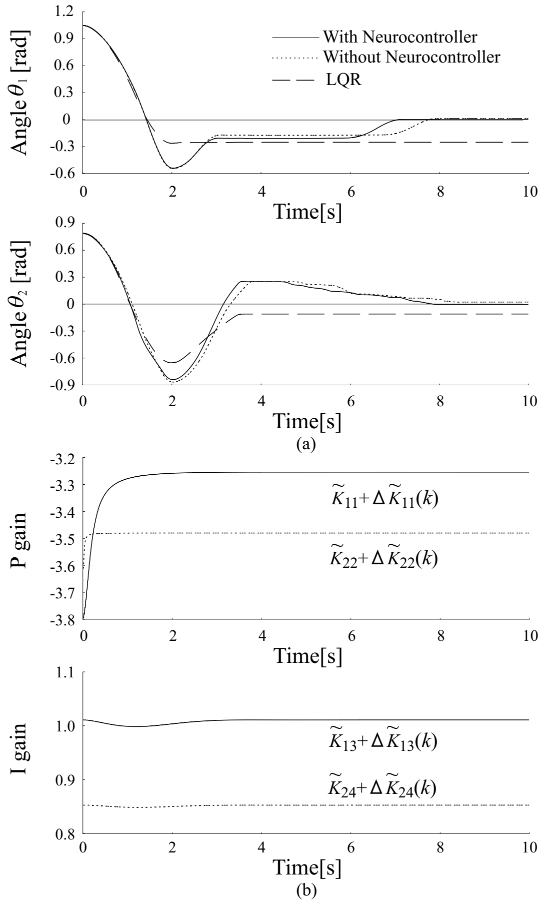

Fig. 3. Experimental results. (a) Angle θ_1, θ_2, (b) State feedback control with additive gain.

data is measured by an encoder. The manipulator is moved only in the horizontal direction. The system parameters are as follows: link mass $m_1 = 10$ [kg], $m_2 = 4$ [kg] and length $l_1 = l_2 = 0.2$ [m]. The observable states are the joint angular positions $\theta_1[\mathrm{rad}]$ and $\theta_2[\mathrm{rad}]$.

The experimental results are shown in Fig. 3. The response of the proposed system is stabilized faster than the system without a neurocontroller and an LQR system. In other words, it is verified that the proposed control strategy can be regulated shortly. However, it should be noted that the LQR controller has an offset (deviation from the equilibrium point). The reason is that a practical robot manipulator has a dead zone and the resulting gain is small. Another important observation is that when the additive gain is adopted, the states can be stabilized. On the other hand, there exists a trivial offset when NNs are not used. Thus, the validity of the use of the additive gain has been proved by the experimental results.

A consequence of the above-mentioned experimental observations is that even if the NNs are roughly trained such that the approximation in (27) is carried out, it can be verified that a better transient response is obtained. Finally, these results show that our proposed method is reliable not only in simulations but also in practical systems.

V. CONCLUSIONS

In this paper, we proposed a novel design method for a robust guaranteed cost PI control problem for a class of uncertain discrete-time systems with additive gain. By adopting the fixed PI controller parameters with additive gain by varying the weight matrices of the cost function, the transient stability can be enhanced. Moreover, even if the additive gain does not function appropriately, the robust stability and adequate cost bound are both realized. It is worth pointing out that the PI parameters can be varied while the control is being carried out. The experimental result demonstrates that the transient response can be changed appropriately by using a simple and roughly learning algorithm. In particular, it has been shown that the experimental system is asymptotically stable when additive gain is adopted, while it is unstable without additive gain. Hence, the usefulness of using additive gain has been demonstrated experimentally. This means that the proposed design method for a PI controller with additive gain will be very useful and reliable.

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