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Necessary and Sufficient Number of Fingers for Capturing Pyramidal-like Objects

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Abstract

This paper discusses how many fingers are necessary and sufficient for capturing a pyramidal-like object placed on a table under the gravitational field. Allowing that the contact friction is small enough to ensure that any direct grasp may fail in achieving an equilibrium grasp, we prove that a planar two-fingered hand are necessary and sufficient for achieving the task for an idealized 2D triangle object placed vertically on a table. We also consider 3D pyramidal-like objects, and prove that a spatial two-fingered hand are necessary and sufficient for achieving the task.

Key words: Multi-fingered Robot Hand, Equilibrium Grasp, Pyramidal-like Object, Grasp Algorithm.

1 Introduction

There are two grasp patterns, one is the finger tip grasp that emphasizes on dexterity and sensitivity, and the other is the power grasp that provides highly stable grasp due to a large number of distributed contacts on the grasped object. While there are many works discussing grasp issues for both grasp styles, most of them assume that the hand already grasps an object. On the other hand, we are particularly interested in the whole grasp procedure that includes, for example, the approaching, the detaching, and the grasping phases. The detaching as terminology means removing an object from a table in this paper. A robot hand first approaches an object placed on a table until the hand reaches the object. If the object has a shape such that the hand can directly grasp it, the hand can achieve a force-closure grasp by simply choosing an appropriate set of the grasp points, where the force-closure grasp means that the finger can generate a set of contact wrenches that can balance external object wrenches. In such a case, planning the detaching phase is not necessary, since the detaching motion is automatically completed by simply lifting up the grasped object. However, if the object is a polyhedra whose face is triangle, the direct grasp may often result in a failure due to a slip between the object and the finger tip. Under such a condition, the planning for the detaching phase is especially important for further steps. For a pyramidal-like object, we consider the strategy for achieving an equilibrium grasp by a multi-fingered robot hand under the gravitational field, as shown in Fig.1. In general, an equilibrium grasp under the gravitational field allows the object to move by an external wrench and, therefore, it provides a mild constraint compared with the force-closure grasp.

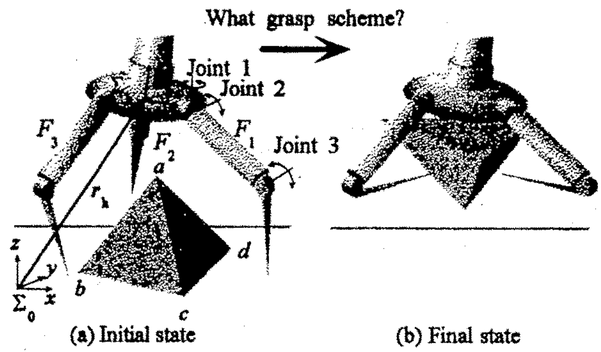

Fig.1: An overview of both initial and final states

An interesting question is that how many fingers are necessary and sufficient for finally achieving an equilibrium grasp for a pyramidal-like object placed on a table under the gravitational field. The goal of this paper is to answer the question. Assuming that each finger tip is modeled by a frictional point contact and the object is placed vertically on a table, we begin by considering a 2D problem, where the object is assumed to be an idealized 2D triangle object whose corner angles are less than rectangle. For the 2D problem, we show an algorithm leading to an equilibrium grasp by a planar two-fingered hand. With this algorithm, we prove that the necessary and sufficient number of finger is two for the 2D model.

Further, we discuss the same problem for a spatial pyramidal-like object, and prove that a spatial two-fingered hand can achieve the task given.

2 Related works

There are a number of papers discussing the manipulation of object for increasing the dexterity of robotic hands, while most of them assume that the hand already grasps the object. In [2]-[5], significant contributions are presented for the analysis and the control of rolling, sliding, and pivoting contacts. By utilizing these basic primitives, motion planning for re-grasps and re-orientation have been addressed [6]-[9]. On the other hand, there exist a couple of papers dealing with global motion planning that includes the approaching, the contacting, and the grasping phases. Fearing [10] has first discussed the whole procedure for approaching and grasping an object by using a two-fingered hand, and proposed the hand priority grasp, in which the finger closing motion automatically changes the orientation of an object placed on a table and finally leads to an equilibrium grasp. The idea is simple enough for applying to a planar object placed on a table. Kaneko, Tanaka, and Tsuji [11] observed how human envelope a cylindrical object placed on a table, and found that human changes his (or her) grasping strategy according to the size of objects, even though they have similar geometry (Scale-Dependent Grasp). Based on such human grasping they presented a way for achieving an enveloping grasp for cylindrical objects placed on a table [12]. This approach was further extended to column objects whose cross sections are polygon [13], [14]. Kleinmann, Henning, Ruhm and Tolle [15] focused on the transition grasp from a finger tip grasp to a power grasp, and showed four different strategies. Kaneko, Kessler, Weigl, and Tolle first discussed the global motion planning for achieving an equilibrium grasp for pyramidal-like objects [16]. For a two-fingered hand whose opening is controlled by a single parameter, Rimon and Blake [17] discussed a preshaping problem combined with the grasping phase. For an initial hand configuration, an object has some freedom to move but finally leads to the desired immobilizing grasp by simply closing the fingers.

3 In case of 2D problem

Suppose an idealized 2D triangle object whose corner angles are less than rectangle, as shown in Fig.2 (a), where ψ ($\pi/2 > \psi$) is the top corner angle, and a, b and c denote each corner, respectively, and s_1, s_2 and s_3, denote each edge, respectively. Before discussing in detail, we provide several assumptions.

- Assumption 1: Kinematic interference among fingers, table, and object are neglected.
- Assumption 2: Each contact between the finger tip and an object is modeled by point contact with friction.
- Assumption 3: The object and the finger tips are rigid, and the object shape, its position, orientation, and the center of gravity which never exists on the edges are known.
- Assumption 4: Each finger tip can produce an arbitrary force within the friction cone.
- Assumption 5: The effects of dynamics of the hand-object system are neglected and a quasi-static motion is assumed.
- Assumption 6: $\alpha > 0$, where α is the friction angle and $\tan\alpha = \mu$ exists for friction coefficient μ.
- Assumption 7: As shown in Fig.2 (a), we divide the triangle into six segments by a line between each corner and the middle point of the edge which does not include the corner. The center of gravity is assumed to be within one of the triangles making contact with the table.

Assumption 7 is for avoiding the explanation of a couple of preliminary procedures. Actually, a robot hand can easily produce such a situation by rotating the object around a corner.

For the 2D grasping problem, let us now consider how many fingers are necessary and sufficient for achieving an equilibrium grasp. We first show an algorithm enabling a planar two-fingered hand to achieve an equilibrium for an idealized 2D triangle object.

For our convenience, two fingers are called by F_1 and F_2, respectively. Without any loss of generality, we can focus on the case in which the center of gravity is located in the left part of the two triangles. This is because if the center of gravity exists within the right part, we can apply the following procedure by simply changing the rotating direction. *A necessary and*

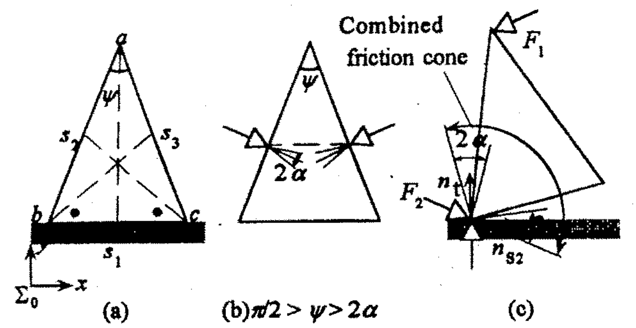

Fig.2: A triangle object

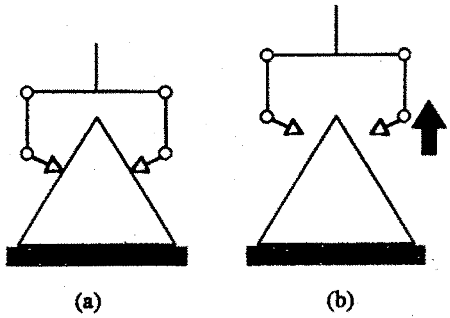

Fig.3: Any direct grasp results in failure under $\psi > 2\alpha$

sufficient condition for two contact points to form an equilibrium grasp is that the angle between the normals to contact edges lies in the interval $[\pi - 2\alpha, \pi + 2\alpha]$, *and the line joining the two points lies in their friction cones [17], [18].* This theorem ensures that a direct grasp can easily achieve an equilibrium grasp if $\psi \leq 2\alpha$. However, it finally fail in achieving an equilibrium grasp under $\psi > 2\alpha$, as shown in Fig.3. Hereafter, assuming $\psi > 2\alpha$, we focus on an alternative algorithm apart from the direct grasp. Due to the fact that any direct grasp results in failure under $\psi > 2\alpha$, the alternative algorithm begins by rotating the object around the corner b. Suppose that F_1 is assigned as a pushing finger at a, and F_2 as a supporting finger at b. We first introduce an idea of combined frictional cone. Suppose that a triangle object is rotating in the counterclockwise direction around b as shown in Fig.2 (c). During this phase, we can regard that the object is supported by three fingers, F_1, F_2, and a virtual finger equivalently produced by the table. Therefore, at the corner b, we can assign two friction cones whose normal vectors are expressed by $\boldsymbol{n}_{s2}$ and $\boldsymbol{n}_t$, respectively. We note that the two friction cones can be combined into a much larger cone which includes not only the two original cones but the cone between them [17]. We now consider whether the force-moment equilibrium (hereafter simply equilibrium) condition exists during the rotating motion or not. If we can see this condition, then a quasi-static based rotating motion is guaranteed. Otherwise, the object may either slide away from the fingers or fall down on the table. Now, we need a definition before introducing a sufficient condition for achieving an object rotating motion.

Definition 1: *A set of vectors positively span $\Re^n$ if any vector in $\Re^n$ can be written as a positive combination of the given vectors, $\Sigma\, \lambda_i \boldsymbol{v}_i$, where $\lambda_i > 0$ and $\boldsymbol{v}_i$ is a given vector.*

We recall an important theorem for achieving an equilibrium by three contact forces. *A necessary and sufficient condition for the existence of three nonzero contact forces, not all of them being parallel, which achieve equilibrium is that there exist three forces in the friction cones at the contact points which positively span the plane and whose lines of action intersect at some point [17].* This theorem brings to the following extension form. *A necessary and sufficient condition for achieving an equilibrium for a triangle in rotating motion around b is that $\boldsymbol{f}_g$, $\boldsymbol{f}_1$, and $\boldsymbol{f}_b$ positively span the plane and lines of action intersect at some point, where $\boldsymbol{f}_g$, $\boldsymbol{f}_1$, and $\boldsymbol{f}_b$ are the gravitational force, the pushing force by F_1, and the force from the table, respectively.* The condition for realizing a counterclockwise rotating motion is naturally given as follows. The condition for achieving a counterclockwise rotating motion is to impart a contact force fa satisfying $m_b = (\boldsymbol{r}_a - \boldsymbol{r}_b) \times \boldsymbol{f}_1 + (\boldsymbol{r}_g - \boldsymbol{r}_b) \times \boldsymbol{f}_g > 0$, where $\boldsymbol{r}_a$, $\boldsymbol{r}_b$ and $\boldsymbol{r}_g$ denote the position vectors expressing a, b, and the center of gravity, respectively, and $\times$ denotes the vector product performing $\boldsymbol{x} \times \boldsymbol{y} = x_1 y_2 - x_2 y_1$, for two vectors $\boldsymbol{x} = (x_1, x_2)^T$ and $\boldsymbol{y} = (y_1, y_2)^T$. Now, let us introduce a convenient theorem providing a sufficient condition for making rotation. *By imparting the normal directional force at a, we can always find $\boldsymbol{f}_1$ making $m_b > 0$ under $\pi/2 > \psi$, irrespective of the coefficient of friction at the contact point [16].* This theorem guarantees that by increasing the normal directional force, the object necessarily starts to rotate in the counterclockwise direction when $m_b > 0$. Let us now define, positive span margin and critical state as follows.

Definition 2: *The positive span margin γ is defined by the angle between $\boldsymbol{n}_t$ and $\boldsymbol{r}_g$. When $\gamma = 0$, we call the critical state.*

Under $\gamma = 0$, $\boldsymbol{f}_1$, $\boldsymbol{f}_b$ and $\boldsymbol{f}_g$ can no more positively span the plane. We note that both equilibrium and rotating conditions are achieved even under frictionless condition, if the pushing force is applied to the

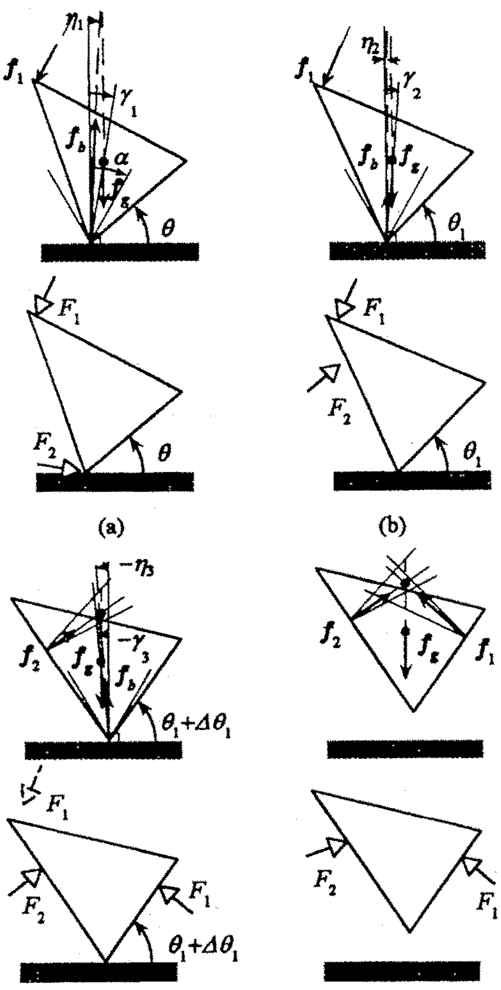

Fig.4: A procedure for caputuring a triangle

normal direction. Therefore, under $\pi/2 > \psi > 0$ and $\gamma > 0$, we can always achieve either rotating an object or keeping the orientation at an arbitrary rotating angle, irrespective of the contact friction. For an exact discussion, we rotate the object up to $\gamma = \varepsilon$ as shown in Fig.4 (a), where ε is a positive small value satisfying $\alpha > \varepsilon$. We note that since $\alpha > \varepsilon = \gamma_1 > \eta_1$, the table can produce the contact force resulting in the equilibrium without making any slipping motion at b as shown in Fig.4 (a) even though F_2 is removed. We also note that the object still starts to rotate in clockwise direction if F_1 is removed. Under such a condition, F_1 can be removed and placed in a position close to s_2. Then, F_1 pushes the object in anticlockwise direction until the center of gravity passes the line which is perpendicular to the table and passes b. At this moment, the object starts to rotate in the anticlockwise direction and is away from F_1, since the equilibrium condition can no more be satisfied. However, the rotating motion of object is blocked by F_2. Consequently, the object results in another equilibrium state if the position of F_2 is carefully chosen to keep $\varepsilon > |\gamma_3|$. In the next step, we place F_2 on s_1 as shown in Fig.4 (c) and, then, two fingers are simultaneously lifted up by keeping the distance between both finger tips. By such a lifting motion, the system will result in another equilibrium grasp as shown in Fig.4 (d). Thus, two planar fingers are sufficient for achieving an equilibrium grasp unless the contact friction is not zero.

[Theorem 1] *Suppose an idealized 2D triangle object whose corner angles are less than rectangle. Also suppose that each contact friction is not zero. A necessary and sufficient number of planar fingers for achieving an equilibrium is two.*

[Proof] Necessity: Under the assumption of finger tip contact, it is impossible to grasp an object by a single finger alone, which means that two fingers are at least necessary for achieving an equilibrium grasp irrespective of either a 2D or a 3D problem. Thus, the necessary number of fingers is two. Sufficiency: It is obvious from the discussions given so far. □

4 In case of 3D problem

In this chapter, we extend the discussion to a polyhedral convex cone. We define the plane Π_1 which is perpendicular to the table and includes the center of gravity, as shown in Fig.5 (a). For simplifying the discussions, we add the following assumptions.

Assumption 8: Finger tip can apply a pushing force at the edge as shown in Fig.5(a).

Assumption 9: Each edge contacting with the table can support a moment whose axis is perpendicular to the table. So, any rotational slip around the axis is avoided.

Assumption 8 does not make any barrier for practical application, because the rotating motion can be achieved by the finger tip unless the finger tip has a needle shape.

Now, suppose that a pushing force $\boldsymbol{f}_1$ is applied at a as shown in Fig.6 (a). Equivalently, this force produces moments m_1 and m_2, as shown in Fig.6 (b), in addition to the force $\boldsymbol{f}_1$. While m_1 contributes to rotating the object around the line bd, m_2 may bring a rotating slip around the axis perpendicular to the table. With assumption 9, we assume to avoid such a slip.

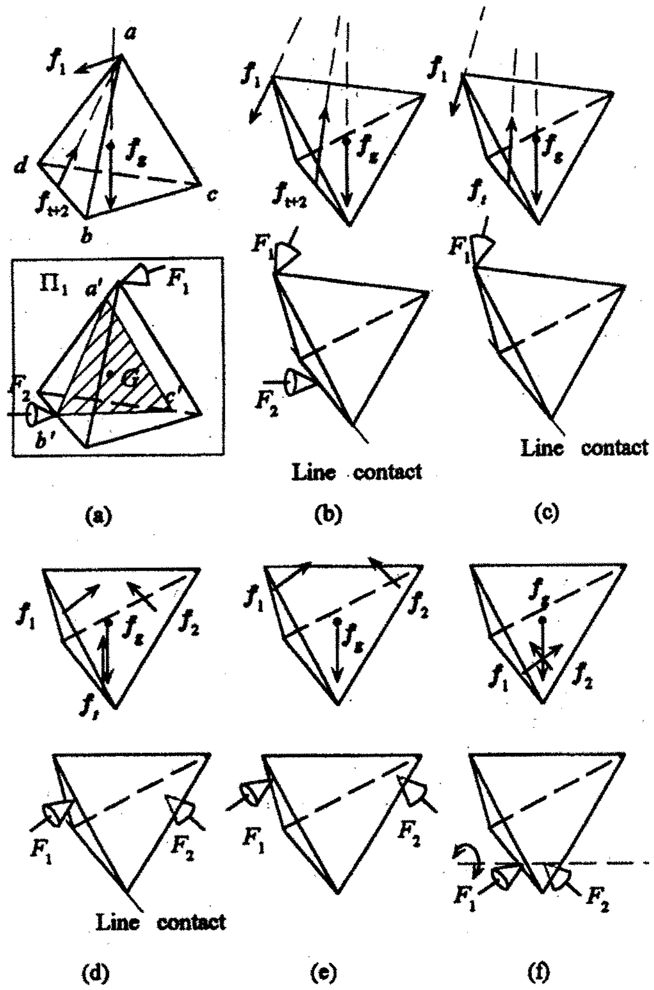

Fig.5: A procedure for the pendulum grasp

[Theorem 2] *Let $\Delta a'b'c'$ be the triangle constructed by the intersection between a polyhedral convex cone and the plane Π_1. Suppose that the top angle of the triangle is less than rectangle. The necessary and sufficient number of spatial fingers for achieving an equilibrium is two.*

[Proof] Since the necessity is obvious, we only discuss the sufficiency. Under these assumptions given, a rotating motion can be achieved by two spatial fingers as shown in Fig.5 (b). Since the center of gravity exists within the triangle $\Delta a'b'c'$, the problem finally results in the same one taken for a 2D triangular object if we replace $\Delta a'b'c'$ to Δabc in Fig.2. This means that we can achieve an equilibrium grasp by using two spatial fingers. This proves the theorem. □

A sufficient area for placing F_1 and F_2 is given by $\Pi_1 \cap S$, where S denotes the object surface where we place the finger. It is interesting to note that the object posture as shown in Fig.5 (d) is the one in which

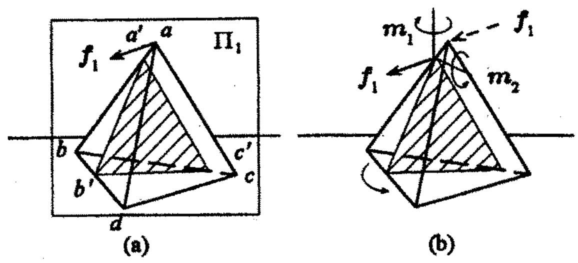

Fig.6: f_1 at a produces f_1 and two moment components at a'

an equilibrium grasp can be achieved by simply lifting up both F_1 and F_2 with keeping the distance between them. As a result, the grasps as shown in Fig.5 (e) and (f) will be obtained. If the center of gravity is lower than the contact points, as shown in Fig.5 (e), the grasp system becomes stable for a small rotation of object around the axis connecting two contact points. This is because the moment produced by the gravitational force in Fig.5 (e) always makes a restoring one for the disturbance. However, if the center of gravity is located upper than the contact points as shown in Fig.5 (f), the system becomes unstable just like an inverted pendulum. Therefore, a sufficient condition for achieving a stable behavior in the plane perpendicular to Π_1 is to place each finger tip at $\Pi_1 \cap S \cap H$, where H denotes the area upper than the center of gravity in z axis. We call this type of grasp the pendulum grasp in the sense that the object can swing in 2D plane.

5 Conclusion

We considered a necessary and sufficient number of fingers for achieving an equilibrium grasp for pyramidal-like objects. We proved that two planar fingers are necessary and sufficient for achieving an equilibrium grasp for a 2D triangle object. We also proved that two spatial fingers can grasp a pyramidal-like object in an equilibrium state.

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Appendix : Grasp experiment

Fig.7 shows the continuous photos in capturing a 3D pyramidal object, where two finger units of Hiroshima Hand [12] are utilized for this particular experiment.

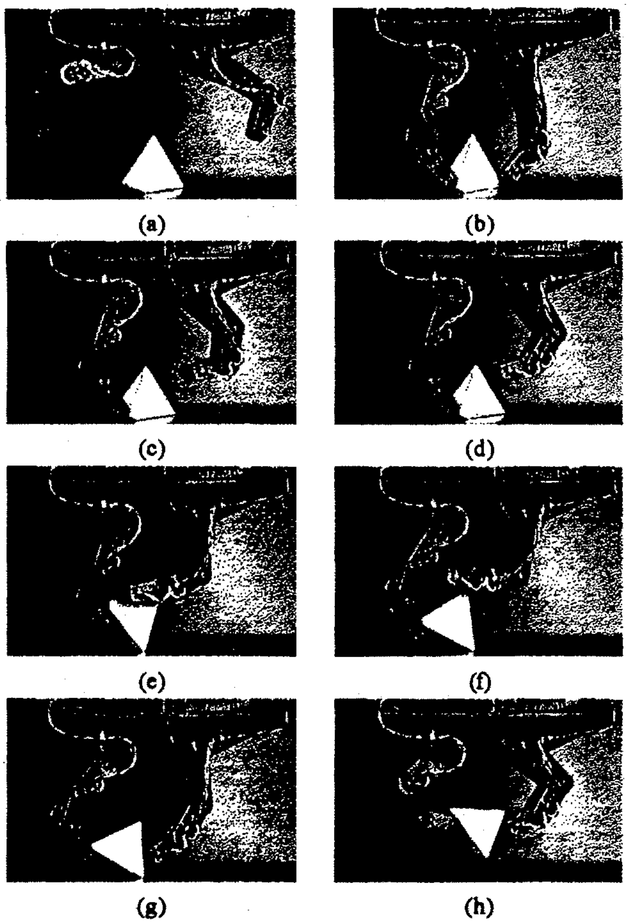

Fig.7: Continuous photos in capturing a pyramidal object

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