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# Identification and Control of Dynamical System by One Neural Network

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**Abstract–The present paper proposes a new neural control scheme that can perform identification and control for a dynamical system with linear and nonlinear uncertainties. This scheme uses a single neural network for both the identification and the control. By the Lyapunov stability technique, stability of the proposed scheme is analyzed and a sufficient condition of the local asymptotic stability is derived. Then, computer simulation is performed in order to illustrate the effectiveness and the applicability of the proposed scheme.**

## I. INTRODUCTION

In recent years, applications of the neural network to control problems have been intensively conducted [1]~[5]. Yabuta and Yamada [1] proposed the direct neural controller that replaces a feedback controller with a neural network. Carelli et al. [2] proposed a neural controller using the feedback error learning. In this method, the neural network can gradually modify the control input from the feedback controller and can finally take the place of the feedback controller. Khalid et al. [3] presented a self-tuning controller that uses a neural network for regulating the gains of the feedback controller in order to improve the performance of the control system. In these methods, even if the inverse model of the controlled plant can be obtained by neural network learning, the uncertainties in the controlled plant cannot be expressed explicitly.

Another approach to the neural control with multiple neural networks has been shown in [4], [5]. In this approach, one neural network is dedicated to the forward model for identifying the uncertainties of the controlled plant and other neural networks may compensate for the effect of the uncertainties based on the trained forward model. However, for real controlled plants, multiple neural networks must be trained for a long learning time and stability analysis is quite difficult.

In this paper, a new neural control scheme that can simultaneously perform identification and control using only one neural network is proposed. In the proposed scheme, an identification model is composed of a neural network and a linear nominal model which is approximated for the controlled plant. The neural network can identify the uncertainties and can adaptively modify the control input computed from a predesigned feedback controller at the same time.

## II. IDENTIFICATION AND CONTROL

### A. System Formulation

In this section, we consider a controlled plant with multiplicative uncertainties described by

$$y(k) = H(z^{-1})u(k), \tag{1}$$

$$H(z^{-1}) = H_n(z^{-1})[1+\Delta_H(z^{-1})], \tag{2}$$

$$H_n(z^{-1}) = \frac{B_n(z^{-1})}{A_n(z^{-1})}, \tag{3}$$

$$\Delta_H(z^{-1}) = \frac{\Delta_B(z^{-1})}{\Delta_A(z^{-1})}, \tag{4}$$

where $y(k)$, $u(k)$, $H(z^{-1})$ and $\Delta_H(z^{-1})$ are respectively the output, the input, the controlled plant model and the multiplicative uncertainties. $H_n(z^{-1})$ is the known, controllable nominal model and $H_n^2(z^{-1}) \in RH_\infty$ is proper and stable [6]. Also $z^{-1}$ is the delay operator, and the polynomials $A_n(z^{-1})$, $B_n(z^{-1})$, $\Delta_A(z^{-1})$, $\Delta_B(z^{-1})$ are given as

$$A_n(z^{-1}) = 1+\sum_{j=1}^{n} a_j z^{-j}, \tag{5}$$

$$B_n(z^{-1}) = \sum_{i=0}^{m} b_i z^{-i} \quad (n \geq m), \tag{6}$$

$$\Delta_A(z^{-1}) = 1+\sum_{j=1}^{h} \alpha_j z^{-j}, \tag{7}$$

$$\Delta_B(z^{-1}) = \sum_{i=0}^{l} \beta_i z^{-i} \quad (h \geq l). \tag{8}$$

Here, $\alpha_j$, $\beta_i$ are unknown coefficients and $l(\leq n)$, $h(\leq m)$ are unknown orders of the polynomials $\Delta_A(z^{-1})$, $\Delta_B(z^{-1})$.

The general block diagram of the feedback control system is shown in Fig. 1. First, let us consider a special case in which there is no uncertainty in the plant (1),

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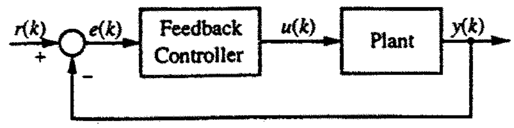


Fig. 1: Block diagram of feedback control system

that is $\Delta_H(z^{-1}) = 0$. The feedback controller $G_n(z^{-1})$ for $H_n(z^{-1})$ can be predesigned to produce a desirable response. The closed loop transfer function $F_n(z^{-1})$ is described by

$$F_n(z^{-1}) = \frac{y(k)}{r(k)} = \frac{G_n(z^{-1})H_n(z^{-1})}{1 + G_n(z^{-1})H_n(z^{-1})}. \tag{9}$$

Next, the general case of $\Delta_H(z^{-1}) \neq 0$ is considered with the controller $G(z^{-1})$ for $H(z^{-1})$ defined as

$$G(z^{-1}) = G_n(z^{-1})[1 + \Delta_G(z^{-1})], \tag{10}$$

where $\Delta_G(z^{-1})$ represents the modification of the controller $G(z^{-1})$. Thus, the closed loop transfer function $F(z^{-1})$ can be given as

$$F(z^{-1}) = \frac{G(z^{-1})H(z^{-1})}{1 + G(z^{-1})H(z^{-1})}. \tag{11}$$

If (9) and (11) are equal, the response of $F(z^{-1})$ can agree with the desirable response. Carrying out an operation using (4), (9), (10) and (11), we can obtain the following transformation as

$$\Delta_G(z^{-1}) = -\frac{\Delta_H(z^{-1})}{1 + \Delta_H(z^{-1})}. \tag{12}$$

However, since $\Delta_H(z^{-1})$ is unknown, $\Delta_G(z^{-1})$ cannot be computed by (12). When $\Delta_H(z^{-1})$ is over the admissible error range of the feedback controller $G_n(z^{-1})$, the control system performance yields a steady-state error, or even turns into unstable state. In order to solve this control problem, in the next subsection we propose a new scheme using one neural network.

## B. *Proposed Scheme*

Fig. 2 shows the block diagram of the proposed scheme in this paper. The output $\hat{y}(k)$ of the identification model is a sum of the output $y_n(k)$ of the nominal model and the identified output $y_{id}(k)$ that is the neural network output $y_{NN}(k)$ passed through $H_n(z^{-1})$. The neural network is trained using the identified error $\epsilon(k)$ that is

$$\epsilon(k) = \hat{y}(k) - y(k). \tag{13}$$

The neural network output $y_{NN}(k)$ modifies the control input as the regulated input $\Delta_u(k)$ given by

$$\Delta_u(k) = -y_{NN}(k). \tag{14}$$

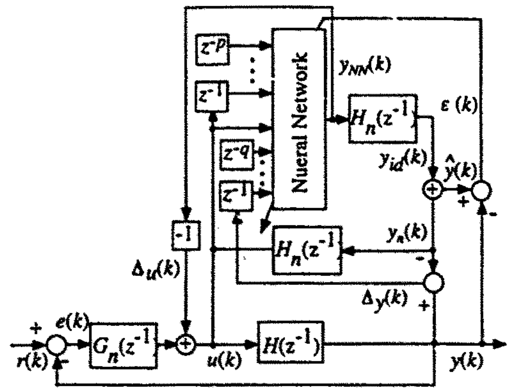


Fig. 2: Block diagram of the proposed scheme using one neural network

Next, the working principle of the proposed scheme is explained as follows. From Fig. 1 and (10), the control input $u(k)$ can be represented as

$$u(k) = u_n(k) + \Delta_u(k), \tag{15}$$

$$u_n(k) = G_n(z^{-1})e(k), \tag{16}$$

$$\Delta_u(k) = \Delta_G(z^{-1})u_n(k), \tag{17}$$

where $u_n(k)$ is the nominal control input.

Also, from (1) and (2) the output $y(k)$ becomes

$$y(k) = y_n(k) + H_n(z^{-1})\Delta_y(k), \tag{18}$$

$$y_n(k) = H_n(z^{-1})u(k), \tag{19}$$

$$\Delta_y(k) = \Delta_H(z^{-1})u(k), \tag{20}$$

where $\Delta_y(k)$ is the uncertain output via the uncertainties $\Delta_H(z^{-1})$.

Substituting (15), (17) into (20), we have

$$\Delta_y(k) = \Delta_H(z^{-1})[1 + \Delta_G(z^{-1})]u_n(k). \tag{21}$$

Using (17) and (21), $\Delta_u(k)$ can be rewritten as

$$\Delta_u(k) = \frac{\Delta_G(z^{-1})}{\Delta_H(z^{-1})[1 + \Delta_G(z^{-1})]}\Delta_y(k). \tag{22}$$

Substituting (12) into (22), we obtain the following relation:

$$\Delta_u(k) = -\Delta_y(k). \tag{23}$$

On the other hand, by Fig.2, (13) and (18), $\epsilon(k)$ can be rewritten as

$$\epsilon(k) = H_n(z^{-1})[y_{NN}(k) - \Delta_y(k)]. \tag{24}$$

If the neural network is well trained, we can expect $\epsilon(k) = 0$ in (24). Since $H_n(z^{-1})$ is the nominal model and is not identically zero, we can have

$$y_{NN}(k) = \Delta_y(k). \tag{25}$$

Consequently, we can see that the output of the plant under the proposed scheme can agree with the desirable response using (14). The next subsection will explain how the regulated input (14) can be realized using the neural network.

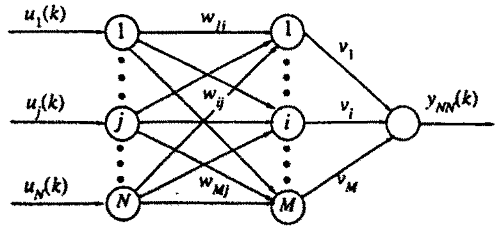


Fig. 3: Neural network used in the proposed scheme

### C. *Neural Network Model*

The multi-layer neural network used in this paper is shown in Fig.3. The numbers of units of the input layer and the hidden layer are $N$ and $M$, respectively. The number of units of the output layer is one. In Fig.3, $w_{ij}(k)$ represents the weight that connects the unit $j$ of the input layer and the unit $i$ of the hidden layer; $v_i(k)$ represents the weight that connects the unit $i$ of the hidden layer to the output unit; $\boldsymbol{W}(k) \in \Re^{M\times N}$ and $\boldsymbol{V}(k) \in \Re^{M\times 1}$ are the weight matrix of the hidden layer and the weight vector of the output layer, respectively. From Fig.2, the input vector to the neural network $\boldsymbol{U}_{IN}^T(k) = [u_1(k), u_2(k), \cdots, u_N(k)] \in \Re^{1\times N}$ is defined as

$$\begin{aligned} \boldsymbol{U}_{IN}^T(k) &= [u(k), u(k-1), \cdots, u(k-q), \\ &\quad \Delta_y(k-1), \cdots, \Delta_y(k-p)], \end{aligned} \tag{26}$$

where $p \geq h$, $q \geq l$, $N = p + q + 1$.

Let the unit $j$'s output of the input layer be denoted as $I_j = u_j(k)$ $(j = 1, \cdots, N)$, and the unit $i$'s output of the hidden layer be denoted as $H_i = \sigma(s_i)$, where $s_i = \sum_{j=1}^{N} w_{ij} I_j$ and $\sigma(x)$ is the sigmoid function defined as $\sigma(x) \equiv \frac{1}{\gamma} tanh(\gamma x)$, in which $\gamma$ is the positive parameter related with the shape of the sigmoid function. Moreover, the output of the output unit is denoted as $O_k = \sigma(\kappa)$, where $\kappa = \sum_{i=1}^{M} v_i H_i$.

Now, the energy function $J(k) = (1/2)\epsilon^2(k)$ is minimized by changing the weights $w_{ij}$ and $v_i$ in the training process. According to the error back-propagation algorithm, the weight updating rules at one sampling time can be described as

$$\boldsymbol{V}(k+1) = \boldsymbol{V}(k) - \eta H_n(z^{-1})\epsilon(k)\frac{\partial y_{NN}(k)}{\partial \boldsymbol{V}(k)}, \tag{27}$$

$$\boldsymbol{W}(k+1) = \boldsymbol{W}(k) - \eta H_n(z^{-1})\epsilon(k)\frac{\partial y_{NN}(k)}{\partial \boldsymbol{W}(k)}, \tag{28}$$

where $\eta > 0$ is the learning rate.

### D. *Stability Analysis*

This subsection will mainly deal with the local asymptotic stability of the proposed scheme for the plant (1) near the optimal set of the neural network weights. If the multi-layer neural network is used, there exists the optimal set of the weights that makes the identified error $\epsilon(k)$ zero [7].

Near the optimal set of the weights, $y_{NN}(k)$ is linearized by

$$y_{NN}(k) \approx \varrho \boldsymbol{V}^T(k)\boldsymbol{W}(k)\boldsymbol{U}_{IN}(k), \tag{29}$$

where $\varrho > 0$ is the gradient of the sigmoid function.

On the other hand, by Fig.2, (4), (7), and (8), the uncertain output $\Delta_y(k)$ can be written as

$$\begin{aligned} \Delta_y(k) &= [\sum_{i=0}^{l} \beta_i z^{-i}]u(k) - [\sum_{j=1}^{h} \alpha_j z^{-j}]\Delta_y(k) \\ &= \boldsymbol{\theta}^T \boldsymbol{U}_{IN}(k), \end{aligned} \tag{30}$$

$$\boldsymbol{\theta} = [\beta_0, \beta_1, \cdots, \beta_l, 0, \cdots, 0, -\alpha_1, \cdots, -\alpha_h, 0, \cdots, 0]^T,$$

where $\boldsymbol{\theta}$ is the parameter vector.

From (29), (30), $\epsilon(k)$ in (24) becomes

$$\epsilon(k) = H_n(z^{-1})\varphi^T(k)\boldsymbol{U}_{IN}(k), \tag{31}$$

where $\varphi^T(k) = \varrho \boldsymbol{V}^T(k)\boldsymbol{W}(k) - \boldsymbol{\theta}^T \in \Re^{1\times N}$ is defined as the parameter error.

From Fig.2 and (31), it can be seen that if the identified error $\epsilon(k)$ can be asymptotically stabilized, the asymptotic stability of the proposed scheme can be also guaranteed.

Since $H_n(z^{-1})$ is controllable and the control input $\boldsymbol{U}_{IN}(k)$ is bounded, the stability of the parameter error $\varphi(k)$ should be guaranteed in order to assure the stability of the identified error $\epsilon(k)$.

Now, let us consider a Lyapunov function $\Psi(k)$ of the following form:

$$\Psi(k) = \varphi^T(k)\varphi(k). \tag{32}$$

When the difference

$$\Delta\Psi = \Psi(k+1) - \Psi(k) < 0 \tag{33}$$

is held, the asymptotic stability of the parameter error $\varphi(k)$ can be guaranteed by the stipulations of the Lyapunov stability technique. If the neural network is trained until $\epsilon^2(k) \approx 0$, the sufficient condition of the local asymptotic stability is to choose the learning rate $\eta$ as

$$\frac{2}{\varrho\zeta\|\boldsymbol{Q}(k)\|_\infty} > \eta > 0, \tag{34}$$

$$\zeta = \sup_{0\leq\omega\leq\infty} |H_n^2(e^{-j\omega T})|, \tag{35}$$

$$\|\boldsymbol{Q}(k)\|_\infty = \sup_{0\leq k\leq k_L} \bar{\sigma}\{\boldsymbol{Q}(k)\}, \tag{36}$$

where $k_L$ is the learning time, $T$ is the sampling period, $\bar{\sigma}\{\boldsymbol{Q}(k)\}$ is the maximum singular value of the matrix $\boldsymbol{Q}(k) \in \Re^{N\times N}$ given by

$$\begin{aligned} \boldsymbol{Q}(k) &= \boldsymbol{U}_{IN}(k)[\boldsymbol{U}_{IN}^T(k)\boldsymbol{W}^T(k)\boldsymbol{\Omega}_1(k)\boldsymbol{W}(k) \\ &\quad + \boldsymbol{V}^T(k)\boldsymbol{\Omega}_2(k)\boldsymbol{V}(k)\boldsymbol{U}_{IN}^T(k)]. \end{aligned} \tag{37}$$

The diagonal elements $\omega_{1ii}(k)$, $\omega_{2ii}(k)$ of the diagonal matrices $\Omega_1(k)$, $\Omega_2(k)$ are given as

$$\omega_{1ii}(k) = \frac{\sigma'(\kappa)\sigma(s_i)}{s_i}, (\omega_{1ii}(k) = 0, \text{if } s_i = 0), \quad (38)$$

$$\omega_{2ii}(k) = \sigma'(\kappa)\sigma'(s_i), \quad (39)$$

where $\sigma'(\cdot)$ is the derivative of $\sigma(\cdot)$ (Proof see [8]). It can be easily seen that when the small positive learning rate is chosen, the condition (34) can be generally satisfied.

## III. COMPUTER SIMULATION

To illustrate the effectiveness of the proposed scheme, we will use the simulation plant with linear and nonlinear uncertainties.

The nominal model used in the computer simulation is

$$H_n(z^{-1}) = \frac{1}{1 + 4z^{-1} + 2.4z^{-2} + 0.448z^{-3} + 0.0256z^{-4}}, \quad (40)$$

and the following feedback controller $G_n(z^{-1})$ is design by using the pole-zero cancellation method [9], that is

$$G_n(z^{-1}) = \frac{1.889 + 7.131z^{-1} + 2.878z^{-2}}{z^{-1}} \quad (41)$$

For the reference signal $r(k)$ of the unit step function, the responses of the nominal model $H_n(z^{-1})$ of (40) under the control $G_n(z^{-1})$ of (41) are respectively shown in Fig. 6 as the desired response (DRE).

In the proposed scheme, $\gamma = 1$ in the sigmoid function is used, and the weight initial value of the neural network is chosen as the uniform random number in [-2.0, +2.0]. The learning rate is $\eta$=0.05 and the sampling time is 10 ms. Also, the order of $\Delta_H(z^{-1})$ is unknown, so it is set as the maximum order, that is $h = n(= p)$, $l = m(= q)$ to include the possible order range. This results $N = 5$ and $M = 5$ in Fig. 3.

Now, the results of the computer simulations are divided into two parts: a linear uncertainty and combined linear and nonlinear uncertainties.

### A. *Linear Uncertainties*

The simulation plant model

$$H^1(z^{-1}) = \frac{1.1}{1.2 + 1.1z^{-1} + 5.6z^{-2} + 0.48z^{-3} + 0.05z^{-4}} \quad (42)$$

is used with the reference signal of the unit step function. The simulation results under the proposed scheme is shown in Fig. 4. The response of the simulation plant model is converging on the desired response because of the learning of the neural network.

Fig. 5 shows the time history of $\bar{\sigma}(k)$ in (36) during the first iteration of the neural network learning. It should be noted that other singular values of $Q(k)$ are always nonnegative during the simulation. Since the matrix $Q(k)$ includes the control input $U_{IN}(k)$, the maximum singular

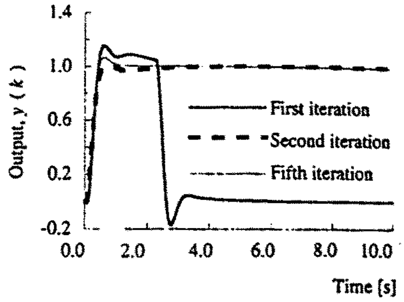


Fig. 4: Responses of the plant $H^1(z^{-1})$ by using the proposed method

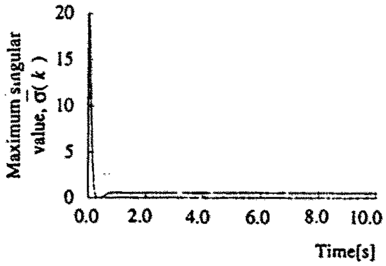


Fig. 5: Time history of the maximum singular value of $Q(k)$ .

value $\bar{\sigma}(k)$ archives the largest value $\bar{\sigma}(k) = 19.92$ at the beginning of the control time where the control input for the step-like reference signal is rapidly changed. As the result, $\eta$=0.05 satisfies the sufficient condition (34) and guarantees the local asymptotic stability of the proposed scheme.

On the other hand, Fig. 6 shows the desired response as the solid line, the response of the feedback control method as the dotted line (FBC), the response during the fifth learning iteration using the proposed scheme as the dashed line, respectively, where the proposed scheme is abbreviated as NBAC (Neuro-Based Adaptive Control). We can see that the response of the proposed scheme can almost achieve the desired response by only fifth learning iteration.

The control performance using the neural network is closely related with the initial values of the network weights. When the neural network is not sufficiently trained, the local asymptotic stability may not be always guaranteed as shown in II. *D*. Therefore, we examine the relationship between the control performance and the initial values of the weights in the proposed scheme. In the simulation experiment, 10 different sets of the initial values are chosen using the uniform random number within the range $[-\xi, \xi]$. For the unit step function as the reference signal, the mean square error between the desired response and the plant output, and the standard deviation are shown in Fig. 7 with the range $\xi$. As the range of the random number becomes large, the mean square error is increasing and the standard deviation is spreading. However, if smaller initial values are used, the mean square

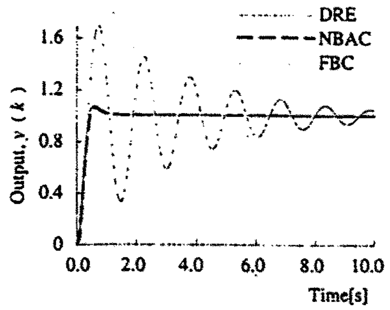


Fig. 6: Comparison of control results for the plant $H^1(z^{-1})$

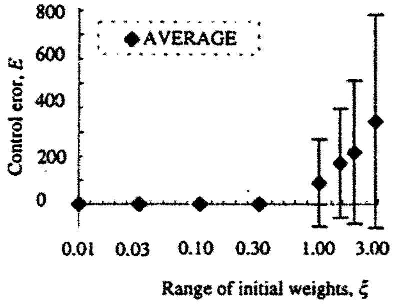


Fig. 7: Change of the control performance with the initial values of the neural networks weights

error converges almost zero.

*B. Linear and Nonlinear Uncertainties*

Consider the following simulation plant model with linear and nonlinear uncertainties,

$$y(t) = H^1(z^{-1})u(k) + \{1 - exp[\pi u(k)]\}, \quad (43)$$

where $\{1 - exp[\pi u(t)]\}$ is the nonlinear uncertainty. The parameter $\pi$ represents the index of nonlinear extent and is set as $\pi = 0.01$ in this simulation. It should be noted that $H^1(z^{-1})$ includes the parameter perturbation shown in (42).

The proposed control scheme is applied to the plant model (43) with the reference signal of the unit step function and the feedback controller as (41). Fig. 8 shows the simulation result. The response during the fifteen learning iteration almost agrees with the desired response shown in Fig. 6. Although the stability of the proposed scheme is proved to the plant with only linear uncertainties in Section II. *D*, it may be also effective to the class of plant with nonlinear uncertainties.

## IV. CONCLUSION

In this paper, the new neural control scheme that can regulate the control input and identify the controlled plant with uncertainties by using only one neural network is proposed. In future, we plan to extend the proposed scheme

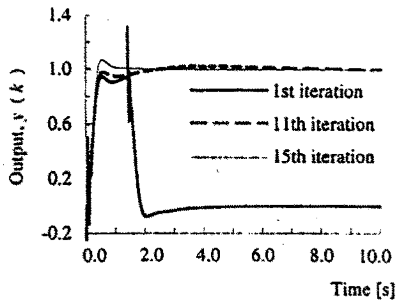


Fig. 8: Simulation results for the plant with linear and nonlinear uncertainties

to a general nonlinear plant and a multi-variable system, and to improve the learning speed of the neural network.

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