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Adaptive Neural Controller for a Class of Plant with Nonlinear Uncertainties

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Abstract: This paper presents a Neural-Based Adaptive Control (NBAC) for the torque control of a flexible beam with structural uncertainties. In the NBAC, a neural network (NN) is connected in parallel with a linearized plant model, so that the NN is expected to identify the uncertainties included in the plant. At the same time the NN works as an adaptive controller that can compensate for the unknown system dynamics.

At first, stability of the NBAC system including the nonlinear NN is analysed and a sufficient condition of the local asymptotical stability is derived by the Lyapunov stability technique. Then, the NBAC is applied to the torque control of a flexible beam that includes linear and nonlinear uncertainties caused by a contact force and by not exactly known shape and material of the beam. Experimental results illustrate effectiveness and applicability of the NBAC.

1 Introduction

In recent years, application of the neural networks (NNs) to adaptive control has been intensively conducted [1]-[5], since NNs have excellent capabilities of nonlinear mapping, learning ability and parallel computations. For example, Yabuta and Yamada [1] proposed direct adaptive control that replaces a feedback controller with NN. Their method can be applied to various feedback control systems, but stability property is not guaranteed. Carelli et al.[2] proposed an adaptive controller that applies NN as a feedforward controller in order to modify input to the plant computed from the conventional feedback controller. This method may maintain stability of the system by the feedback controller, but uncertainties in the controlled plant can not be expressed explicitly, even if the inverse model of the controlled plant can be obtained by learning. When the forward model is necessary, the controlled plant must be identified again. On the other hand, Narendra and Parthasarathy [3], Iguchi and Sakai [4], Ku and Lee [5] have proposed another type of adaptive control with two NNs, where one NN makes up the forward model for uncertainties in the controlled plant and the other NN may compose the inverse model by the NN's training in order to control uncertainties. However, these two NNs must be trained for a long time and stability problems still exist in these methods.

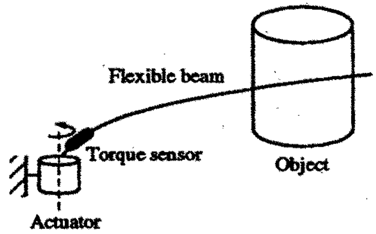

Fig. 1: Flexible beam in contact with an object

In this paper, we propose a Neural-Based Adaptive Control (NBAC) that makes use of a NN for a plant with uncertainties. In our approach, the NBAC uses only one NN in order to identify multiplicative uncertainties in the plant and modify the control input to the plant. When a linear NN is used in the NBAC for a plant with linear uncertainties, local asymptotical stability can be guaranteed [6].

This paper is organized as follows. At first, stability of the NBAC system including the nonlinear NN is analysed and a sufficient condition of the local asymptotical stability is derived by the Lyapunov stability technique. Then, the NBAC is applied to the torque control of a flexible beam in contact with an object.

As shown in Fig.1, the joint torque of the flexibel beam is controlled in accordance with a reference signal. The contact point between the flexible beam and the object can be detected by active motion of the joint[7]- [9]. So, if the joint torque can be controlled, the force applied to the object can be also controlled. Dynamics of the system under consideration nonlinearly depends on the material and the shape of the beam, the external contact force and on the contact friction [7]- [9]. Since the beam's stiffness at the joint is varied by the distance between the joint and the contact point on the object, it is not possible to obtain the exact dynamic model of the system beforehand. Note that the precise torque control is very difficult and often conventional control schemes do not work well due to nonlinear uncertainties and parameter's perturbation. Finally, applicability of the NBAC to the torque control of the flexible beam is tested under experiments. The experimental results demonstrate effectiveness of the control scheme.

2 NBAC Approach

2.1 Plant model

Consider a nonlinear discrete-time plant described by the following single-input-single-output form

$$y(k) = H_L(z^{-1})u(k) + f(u(k)), \tag{1}$$

where $u(k)$ and $y(k)$ are the input and the output of the plant, respectively; $f(u(k))$ represents the nonlinear part; and $H_L(z^{-1})u(k)$ represents the linear part of the plant. The linearized model $H_L(z^{-1})$ is further supposed to be divided into the nominal model $H_{La}(z^{-1})$ and uncertainties $\Delta_{La}(z^{-1})$:

$$H_L(z^{-1}) = H_{La}(z^{-1}) + \Delta_{La}(z^{-1}), \tag{2}$$

where z^{-1} stands for the delay operator.

As to the nonlinear part $f(u(k))$, it is assumed that its linear approximation is given by

$$f(u(k)) \approx H^*(z^{-1})u(k) \tag{3}$$

By eqs. (2), (3), the plant dynamics (1) can be represented as

$$\begin{aligned} y(k) &\approx H_L(z^{-1})u(k) + H^*(z^{-1})u(k) \\ &= H(z^{-1})u(k), & (4) \\ H(z^{-1}) &= H_n(z^{-1})[1 + \Delta_H(z^{-1})], & (5) \\ H_n(z^{-1}) &= H_{La}(z^{-1}) + H^*(z^{-1}), & (6) \\ \Delta_H(z^{-1}) &= \frac{\Delta_{La}(z^{-1})}{H_n(z^{-1})}, & (7) \end{aligned}$$

where $H_n(z^{-1})$, $\Delta_H(z^{-1})$ are respectively the known nominal model and the uncertainty in the plant (1). Let us assume that the structure of the $H_n(z^{-1})$ and $\Delta_H(z^{-1})$ can be described by the following expressions:

$$\begin{aligned} H_n(z^{-1}) &= \frac{B_n(z^{-1})}{A_n(z^{-1})}, & (8) \\ A_n(z^{-1}) &= 1 + \sum_{j=1}^{n} a_j z^{-j}, & (9) \\ B_n(z^{-1}) &= \sum_{i=0}^{m} b_i z^{-i} \quad (n \geq m), & (10) \\ \Delta_H(z^{-1}) &= \frac{\Delta_B(z^{-1})}{\Delta_A(z^{-1})}, & (11) \\ \Delta_A(z^{-1}) &= 1 + \sum_{j=1}^{h} \alpha_j z^{-j}, & (12) \\ \Delta_B(z^{-1}) &= \sum_{i=0}^{l} \beta_i z^{-i} \quad (h \geq l), & (13) \end{aligned}$$

where $H_n(z^{-1})$ is controllable; $\Delta_A(z^{-1})$, $\Delta_B(z^{-1})$ include unknown coefficients α_j, β_i; and $l(\leq n)$, $h(\leq m)$ are unknown orders of the polynomials $\Delta_A(z^{-1})$, $\Delta_B(z^{-1})$. Note that appearance of the uncertainties in the plant dynamics is changed from the additive form (4) to the multiplicative one (5).

2.2 Adaptive Control Scheme

The block diagram of the NBAC of the discrete-time plant with uncertainties (1) is shown in Fig.2, where $r(k)$ is the reference value and $e(k) = r(k) - y(k)$ is the error between the reference value and the output. The output $\hat{y}(k)$ of the identification model is a sum of the nominal model output $y_n(k)$ and the identified output $y_{ID}(k)$. $y_{ID}(k)$ is the NN's output $y_{NN}(k)$ passed through the model $H_n(z^{-1})$. The NN is trained using the identified error $\epsilon(k)$. Thus, the input $u(k)$ is regulated by

$$u(k) = u_n(k) - y_{NN}(k). \tag{14}$$

Now, the performance of the NBAC system can be explained as follows.

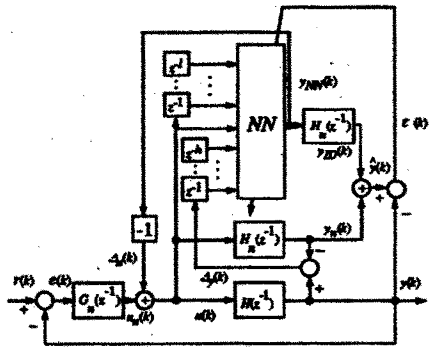

Fig. 2: Block diagram of the NBAC

First, consider a special case when there are no uncertainties in the plant (1) ($\Delta_u(k) = 0$). The controller $G_n(z^{-1})$ for the model $H_n(z^{-1})$ is designed to produce a desirable response. The closed loop transfer function $F_n(z^{-1})$ is described as

$$F_n(z^{-1}) = \frac{y(k)}{r(k)} = \frac{G_n(z^{-1})H_n(z^{-1})}{1 + G_n(z^{-1})H_n(z^{-1})}. \tag{15}$$

Next, consider the general case of $\Delta_u(k) \neq 0$. In Fig.2, the controller $\Delta_G(z^{-1})$ is defined as

$$G(z^{-1}) = G_n(z^{-1})[1 + \Delta_G(z^{-1})], \tag{16}$$

where $\Delta_G(z^{-1})$ represents the modified value of the controller $G(z^{-1})$. Thus, the closed loop transfer function $F(z^{-1})$ that consists of $H(z^{-1})$ and $G(z^{-1})$ can be given as

$$F(z^{-1}) = \frac{G(z^{-1})H(z^{-1})}{1 + G(z^{-1})H(z^{-1})}. \tag{17}$$

If (15) and (17) are equal, the response by $H(z^{-1})$ and $G(z^{-1})$ can accord with the desired response [6]. In this case we have

$$\Delta_G(z^{-1}) = -\frac{\Delta_H(z^{-1})}{1 + \Delta_H(z^{-1})}. \tag{18}$$

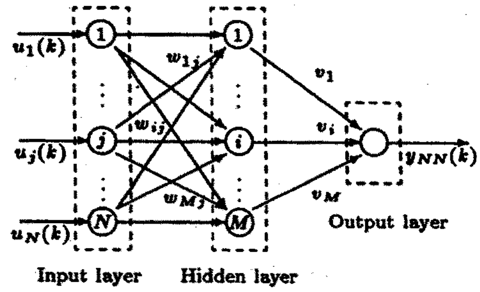

Fig. 3: Neural network used in the NBAC

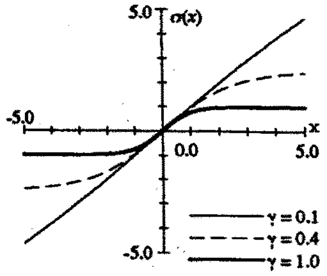

Fig. 4: Sigmoid function used in the NN

On the other hand, by eq. (16) the input $u(k) = G(z^{-1})e(k)$ can be represented as

$$u(k) = G_n(z^{-1})e(k) + \Delta_u(k), \quad (19)$$
$$\Delta_u(k) = \Delta_G(z^{-1})G_n(z^{-1})e(k). \quad (20)$$

By eq. (5), the output $y(k)$ becomes

$$y(k) = H_n(z^{-1})u(k) + H_n(z^{-1})\Delta_y(k), \quad (21)$$
$$\Delta_y(k) = \Delta_H(z^{-1})u(k), \quad (22)$$

where $\Delta_u(k)$, $\Delta_y(k)$ are respectively the modified value of the input and the output of the uncertainties.

Substituting eqs. (19), (20), (22) into (18) yields

$$\Delta_u(k) = -\Delta_y(k). \quad (23)$$

In the NN's training process, the identified error $\epsilon(k) = \hat{y}(k) - y(k)$ can be described by

$$\epsilon(k) = H_n(z^{-1})\{y_{NN}(k) - \Delta_y(k)\}. \quad (24)$$

If the NN is well trained, we can expect $y_{NN}(k) \approx \Delta_y(k)$. By eqs. (23), (24), we know that the response of the control scheme shown in Fig.2 agrees with the desired response when $\Delta_u(k) = -y_{NN}(k)$ is used (see eq.(14)).

2.3 NN Scheme

The multi-layer NN scheme in the NBAC is shown in Fig.3. The number of units of the input layer and the hidden layer are N and M, respectively. The number of units of the output layer is one. In Fig.3, $w_{ij}(k)$ represents the weight's element that connects the unit j of the input layer and the unit i of the hidden layer; $v_i(k)$ represents the weight's element that connects the unit i of the hidden layer and the output layer's unit; $W(k) \in R^{M\times N}$, $V(k) \in R^{M\times 1}$ are the weight matrices of the hidden layer and the weight vector of the output layer, respectively. By Fig.2, the NN's input vector $U_{IN}^T(k) = [u_1(k), u_2(k), \cdots, u_N(k)] \in R^{1\times N}$ is defined as

$$U_{IN}^T(k) = [u(k), u(k-1), \cdots, u(k-l), \Delta_y(k-1), \cdots, \Delta_y(k-h)], \quad (25)$$

where $N = l + h + 1$.

Let the unit j's output of the input layer be $I_j = u_j(k)$ $(j = 1, \cdots, N)$, the unit i's output of the hidden layer be $H_i = \sigma(s_i)$, $s_i = \sum_{j=1}^N w_{ij} I_j$, and the sigmoid function be $\sigma(x) \equiv \frac{1}{\gamma} tanh(\gamma x)$. Here γ is the positive parameter related with the shape of the sigmoid function. Fig.4 shows the input-output relation of the sigmoid function. When $\gamma \leq 0.1$ $\sigma(x)$ can be approximated by the linear function and when $\gamma \geq 1$ $\sigma(x)$ is the form of the $tanh$ function. In the same way, let the unit's output of the output layer be $O_k = \sigma(\kappa)$, $\kappa = \sum_{i=1}^M v_i H_i$.

Let the energy function be $J(k) = \frac{1}{2}\epsilon^2(k)$. In the NN's training process, the energy function is minimized by changing the weights w_{ij} and v_i. According to the error back propagation algorithm [10], the weight updating rules at one sampling time can be described as

$$V(k+1) = V(k) - \eta H_n(z^{-1})\epsilon(k)\frac{\partial y_{NN}(k)}{\partial V(k)}, \quad (26)$$
$$W(k+1) = W(k) - \eta H_n(z^{-1})\epsilon(k)\frac{\partial y_{NN}(k)}{\partial W(k)}, \quad (27)$$

where $\eta > 0$ is the learning rate.

2.4 Stability Analysis

The NBAC scheme shown in Fig.2 can guarantee the local asymptotical stability when the parameter γ of the sigmoid function is relatively small and the plant includes linear uncertainties only. In this case the nonlinear NN can be reduced to the linear one [6]. This section will mainly deal with the local asymptotical stability for the plant (5) near the optimal value of the weights of the nonlinear NN with the sigmoid function. If the multi-layer NN is used, there exists the optimal set of the weights that makes the identified error $\epsilon(k)$ zero [11].

Near the optimal values of the weights, the NN's output $y_{NN}(k)$ can be linearized by

$$y_{NN}(k) \approx \varrho V^T(k) W(k) U_{IN}(k), \quad (28)$$

where $\varrho > 0$ is the gradient of the sigmoid function.

On the other hand, by eqs. (11), (22), the output $\Delta_y(k)$ can be written as

$$\Delta_y(k) = [\sum_{i=0}^{l} \beta_i z^{-i}]u(k) - [\sum_{j=1}^{h} \alpha_j z^{-j}]\Delta_y(k)$$

$$= \theta^T U_{IN}(k), \tag{29}$$

where $\theta = [\beta_0, \beta_1, \cdots, \beta_l, -\alpha_1, \cdots, -\alpha_h]^T \in R^{N\times 1}$ is the parameter vector. Thus, the identified error $\epsilon(k)$ in eq. (24) becomes

$$\epsilon(k) = H_n(z^{-1})\varphi^T(k)U_{IN}(k), \tag{30}$$

where

$$\varphi^T(k) = \varrho V^T(k)W(k) - \theta^T \quad \in R^{1\times N}, \tag{31}$$

is defined as the parameter error.

From Fig.2 and eq. (30), it follows that if the identified error $\epsilon(k)$ can be asymptoticaly stabilized, then the asymptotical stability of the NBAC system can be guaranteed. In order to assure stability of the identified error, stability of the parameter error $\varphi(k)$ should be guaranteed.

Consider a Lyapunov function $\Psi(k)$ of the following form

$$\Psi(k) = \varphi^T(k)\varphi(k). \tag{32}$$

When the difference $\Delta\Psi < 0$, the asymptotical stability of the parameter error $\varphi(k)$ can be guaranteed by the stipulations of the Lyapunov's method. If the NN is trained until $\epsilon^2(k) \approx 0$, the sufficient condition of the local asymptotical stability is to choose the learning rate η as

$$\frac{2}{\varrho\zeta\|Q(k)\|_\infty} > \eta > 0, \tag{33}$$

$$\zeta = \sup_{0\le\omega\le\infty} |H_n^2(e^{-j\omega T})|, \tag{34}$$

$$\|Q(k)\|_\infty = \sup_{0\le k\le k_L} \bar{\sigma}\{Q(k)\}, \tag{35}$$

(Proof: see Appendix)
where k_L is the learning time, T is the sampling period, $\bar{\sigma}$ is the maximum singular value of the matrix $Q(k) \in R^{N\times N}$ given by

$$\begin{aligned} Q(k) &= U_{IN}(k)[U_{IN}^T(k)W^T(k)\Omega_1(k)W(k) \\ &+ V^T(k)\Omega_2(k)V(k)U_{IN}^T(k)]. \end{aligned} \tag{36}$$

The diagonal elements $\omega_{1ii}(k)$, $\omega_{2ii}(k)$ of the diagonal matrices $\Omega_1(k)$, $\Omega_2(k)$ are given as

$$\omega_{1ii}(k) = \frac{\sigma'(\kappa)\sigma(s_i)}{s_i}, (\omega_{1ii}(k) = 0, \text{if } s_i = 0), \tag{37}$$

$$\omega_{2ii}(k) = \sigma'(\kappa)\sigma'(s_i), \tag{38}$$

where $\sigma'(\cdot)$ is the derivative of $\sigma(\cdot)$. Now, the condition (33) can be satisfied when the small positive learning rate is chosen.

3 Torque Control of a Flexible Beam

3.1 Experimental Device

An experimental device for the torque control of a flexible beam is shown in Fig.5 [9]. The beam is plastic, 0.25 m length, 2 mm across in diameter. The torque sensor is made of a semiconductor gauge glued on an aluminum sheet. When the beam contacts with a fixed object, the torque τ at the joint of the beam can be measured by the torque sensor. The actuator is velocity-controlled with the reference angular velocity of the joint being assigned by computer. It should be noted that driving torque of the actuator can not be controlled directly. Since the experimental device includes various nonlinear and unknown uncertainties, it is very difficult to obtain the exact dynamic model of the beam in contact with the object. The rotational stiffness of the beam is largely changed depending on the position of the contact point. When the distance from the joint to the contact point is small, the rotational stiffness is increased. When the contact point goes away from the joint, the joint becomes less stiffer. Thus, the parameters in the experimental device are largely varied by the position of the contact point.

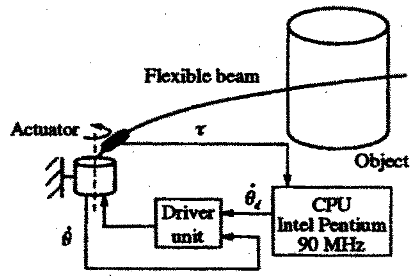

Fig. 5: Experimental setup for torque control of a flexible beam

In this section, first the nominal model being used in the NBAC is identified. Let the reference angular velocity $\dot{\theta}_d$ be the input to the flexible beam, so the transfer function from $\dot{\theta}_d$ to the torque τ at the joint can be approximately described by

$$H_n(s) = \frac{K_s K_b}{s(T_f s + 1)}, \tag{39}$$

where K_s is the gain; K_b is the elastic constant of the beam; and T_f is the time constant of the velocity-controlled system. Sampling eq. (39) yields

$$H_n(z^{-1}) = \frac{b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_1 z^{-2}}. \tag{40}$$

In order to identify the parameters of eq. (40), the contact point $L=$ 0.18m is specified and fixed to the environment.

Let the amplitude of the input signal $\dot{\theta}_d$ be 2.0×10^{-4} rad/s and the period be 0.5s. The joint torque is measured on-line with the sampling frequency 100 Hz. The identified values of the model parameters are $\hat{a}_1$=-0.5024, $\hat{a}_2$=-0.4982, $\hat{b}_1$=0.0178, $\hat{b}_2$=0.0191. The response of the nominal model with the identified parameters is shown in Fig.6 by the thick line. From Fig.6, we see that the error between the response of $H_n(z^{-1})$ and the response of the flexible beam is increased in time.

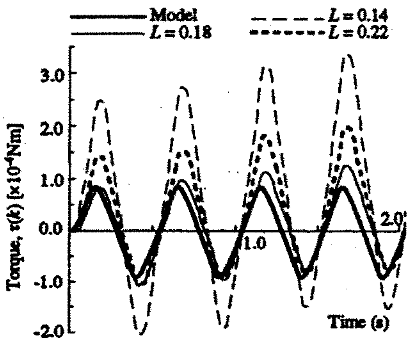

Fig. 6: Responses of the nominal model and the flexible beam

For the case when only the fixed position L of the beam is changed, the result of measurement is shown in Fig.6. Here, the dashed line represents the result with L= 0.14m and the dotted line represents the result with L= 0.22m. From Fig.6 we see that if the position L is changed, the joint torque is varied larger than the response of the nominal model with L= 0.18m.

3.2 Experimental Result

The NN used in the experiments consist of 4 units in the input layer and 4 units in the hidden layer. The initial value of the weight is the uniform random number in $[-1.0 \times 10^{-3}, +1.0 \times 10^{-3}]$. The learning rate is η=0.05 and the parameter γ of the sigmoid function is γ=1.

In the control system, let the desired joint torque τ_d be of a rectangular form with amplitude 2.0×10^{-5}Nm and the period 5s. The controller $G_n(z^{-1})$ is $G_n(z^{-1})$=3, and the control duration is 100s. The NBAC is applied to 5 different contact points L =0.14, 0.16, 0.18, 0.20, 0.22m.

Fig.7 shows the experimental results, where Fig.7(a) corresponds to the case of L=0.18m, Fig.7(b) does to the case of L=0.14m and Fig.7(c) to L=0.22m. In Fig.7, the fine lines represent the results corresponding to the use of the P control only. The thick lines represent the results obtained with the use of the NBAC. The dotted lines represent the reference value $r(k)$. In Fig.7 (a), due to the fact that the same value of L is used for identifying the nominal model, the experimental results obtained with the use of the P control and the NBAC are not different.

However, when L is varied (Fig.7(b) and (c)), the P control produces significant overshoot and undershoot. As can be seen from the experimental results, the NBAC designed for the nominal model with L=0.18m always

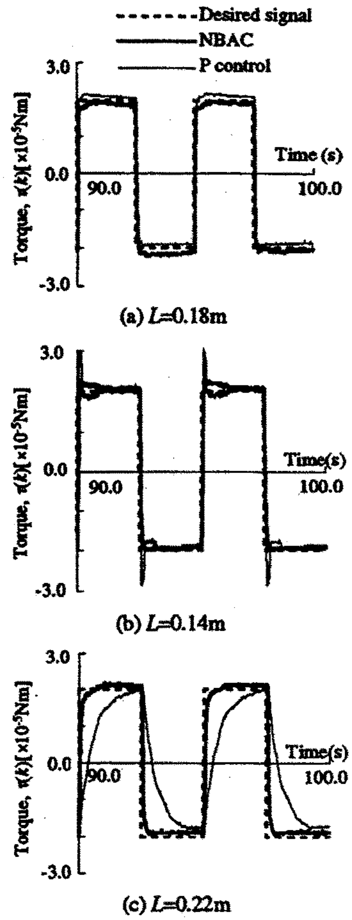

Fig. 7: Experimental results of the torque control

produces stable response. It can be concluded that even there is a large error between the nominal model and real parameters of the flexible beam, the stable response is always obtained.

4 Conclusions

In this paper, a Neural-Based Adaptive Control (NBAC) of discrete-time plant with uncertainties is proposed. The governing equation of the NBAC are derived, and the sufficient condition of the local asymptotical stability near the optimal weight is obtained. The NBAC with only one NN can identify and control the plant at the same time. The NBAC is applied to the torque control of a flexible beam in contact with the external environment. Even though the parameters of the flexible beam are largely varied, the precise control can be realized. Experimental results illustrate effectiveness and applicability of the NBAC. In future, we plan to extend the NBAC method to a general nonlinear plant by represented as a multi-variable system.

Acknowledgment

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Appendix

Near the optimal value of the weight, the updating weight rules (26), (27) with the sigmoid function can be approximated as follows

$$
\begin{aligned}
V(k+1) &\approx V(k) - \eta H_n(z^{-1}) \\
&\quad \epsilon(k)\Omega_1(k)W(k)U_{IN}(k), \qquad (41)\\
W(k+1) &\approx W(k) - \eta H_n(z^{-1}) \\
&\quad \epsilon(k)\Omega_2(k)V(k)U_{IN}^T(k), \qquad (42)
\end{aligned}
$$

where the diagonal elements are $|\omega_{1ii}(k)| \leq M_1 = constant$, $|\omega_{2ii}(k)| \leq M_2 = constant$. The sigmoid is of the *tanh* function, so the $\Omega_1(k)$, $\Omega_2(k)$ are bounded matrices.

By eqs. (41), (42), assuming the identified error to be sufficiently small, we get

$$
\begin{aligned}
&V^T(k+1)W(k+1) \\
&= [V^T(k) - \eta H_n(z^{-1})\epsilon(k)U_{IN}^T(k)W^T(k)\Omega_1(k)] \\
&\quad [W(k) - \eta H_n(z^{-1})\epsilon(k)\Omega_2(k)V(k)U_{IN}^T(k)] \\
&\approx V^T(k)W(k) - \eta H_n(z^{-1})\epsilon(k) \\
&\quad [U_{IN}^T(k)W^T(k)\Omega_1(k)W(k) \\
&\quad + V^T(k)\Omega_2(k)V(k)U_{IN}^T(k)]. \qquad (43)
\end{aligned}
$$

Substituting eq. (30) into eq.(43) yields

$$
\begin{aligned}
&V^T(k+1)W(k+1) \\
&\approx V^T(k)W(k) - \eta H_n^2(z^{-1})\varphi^T(k)U_{IN}(k) \\
&\quad [U_{IN}^T(k)W^T(k)\Omega_1(k)W(k) \\
&\quad + V^T(k)\Omega_2(k)V(k)U_{IN}^T(k)] \\
&= V^T(k)W(k) - \eta H_n^2(z^{-1})\varphi^T(k)Q(k). \qquad (44)
\end{aligned}
$$

Then substituting eq.(44) into (31) yields

$$
\begin{aligned}
\varphi^T(k+1) &= \varrho V^T(k+1)W(k+1) - \theta^T \\
&= \varphi^T(k)[\Gamma - \eta\varrho H_n^2(z^{-1})Q(k)], \qquad (45)
\end{aligned}
$$

where $\Gamma \in R^{N\times N}$ is the unit matrix. So, $\Delta\Psi$ can be written as

$$
\begin{aligned}
\Delta\Psi &= \Psi(k+1) - \Psi(k) \\
&= \varphi^T(k)[\Gamma - \eta\varrho H_n^2(z^{-1})Q(k)] \\
&\quad [\Gamma - \eta\varrho H_n^2(z^{-1})Q^T(k)]\varphi(k) - \varphi^T(k)\varphi(k) \\
&= -\eta\varrho H_n^2(z^{-1})\varphi^T(k)[Q(k) + Q^T(k) \\
&\quad - \eta\varrho H_n^2(z^{-1})Q(k)Q^T(k)]\varphi(k). \qquad (46)
\end{aligned}
$$

When the learning rate η satisfies to the following expression

$$
Q(k) + Q^T(k) - \eta\varrho H_n^2(z^{-1})Q(k)Q^T(k) > 0, \qquad (47)
$$

The condition of $\Delta\Psi < 0$ can be guaranteed.

Next, we derive the condition that the learning rate η satisfies (47). Using the matrix norm, eq.(47) becomes [12]

$$
\|Q(k) + Q^T(k)\|_\infty > \eta\varrho\|H_n^2(z^{-1})Q(k)Q^T(k)\|_\infty. \qquad (48)
$$

Defining $Q(k)$ to be positive semi-definite matrix yields

$$
\begin{aligned}
\|Q(k) + Q^T(k)\|_\infty &= \|Q(k)\|_\infty + \|Q^T(k)\|_\infty \\
&= 2\|Q(k)\|_\infty. \qquad (49)
\end{aligned}
$$

Finally, eq.(48) can be represented as

$$
\begin{gathered}
2\|Q(k)\|_\infty > \eta\varrho\zeta\{\|Q(k)\|_\infty\}^2, \\
\frac{2}{\varrho\zeta\|Q(k)\|_\infty} > \eta > 0. \qquad (50)
\end{gathered}
$$