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HIERARCHICAL CONTROL OF END-EFFECTOR IMPEDANCE AND JOINT IMPEDANCE UTILIZING ARM REDUNDANCY

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Abstract. The present paper proposes a new method called *Hierarchical Impedance Control* (HIC) for redundant manipulators. The HIC can control not only end-effector impedance using one of the conventional impedance control methods but can also regulate additional arm impedance such as joint impedance or impedance of multiple points on the link of the manipulator. In the proposed method, controlling the end-effector impedance is considered as the highest priority task, and the regulating additional impedance as a sub task. That is why we call the proposed method the hierarchical impedance control. Firstly, the framework of the HIC is introduced by incorporating an additional controller to the end-effector impedance controller in a parallel way, and a sufficient condition of the additional controller for satisfying a given end-effector impedance is derived. Then, the optimal additional controller corresponding to the desired joint control torque is analytically derived using the least square method. Application of the HIC scheme for the regulation of joint impedance as a sub task is given, and finally, in order to confirm of the validity and to show the advantages of the proposed method, computer simulations using a four-joint planar manipulator are performed.

Key Words: Impedance control; compliance control; arm redundancy; end-effector impedance; joint impedance

1. INTRODUCTION

Impedance control, proposed by Hogan (1985) using the resolved acceleration control framework (Luh et al., 1980) provides a unified approach for position and force control in which control variables are not position or force, but rather, the dynamic relation between position and force. When the signals such as end-effector position, velocity and interaction force with the environment are measured, the method can control the end-effector impedance to follow the desired one which is specified depending on the task. Many papers which are addressed to the effectiveness and the robustness of the impedance control have been published (Hogan, 1987; Kazerooni, 1989; Colgate and Hogan, 1989; Luo and Ito, 1991). All of the researches made up an important framework to control the mechanical interaction between a manipulator and its environments.

On the other hand, a robotic manipulator is called kinematically redundant, if it possesses more degrees of freedom than the ones being necessary for performing a specified task. Redundancy increases dexterity and versatility which includes the ability to avoid singular configuration and collision with obstacles. Research activities focused on the kinematic redundancy of manipulators have been increased, especially in connection with the inverse kinematic problem (Klein and Huang, 1983). Up to the present, however, a few researches about using kinematic redundancy in terms of impedance control have been reported. For example, Newman and Dohring (1991) and Adachi et al. (1991) used the extended Jacobian scheme (Baillieul, 1986) to develop the impedance control for redundant manipulators.

In contrast, there are several researches of using kinematic redundancy in terms of the compliance control (a static version of the impedance control). For example, Kaneko et al. (1988) and Yokoi et al. (1990) proposed a method called *Direct Compliance Control* (DCC) which utilizes kinematic redundancy in order to make the joint compliance matrix diagonal. As a result, this method is relatively easy to be implemented, because the end-effector compliance can be achieved by regulating the compliance of each joint independently so that the collocation between the force sensor and the actuator (Eppinger and Seering, 1987) can always be assured. *Dynamic Direct Compliance Control* proposed by Tanie et al. (1990) is an extension of the DCC to dynamical case. However, since the method used approximated relationships between end-effector and joint accelerations, the accurate control of the end-effector inertia was impossible.

On the other hand, a method for controlling the desired joint compliance while satisfying the required end-effector compliance and a method called Multi-Point Compliance Control which is able to regulate not only the end-effector compliance but also the compliance of several points on the manipulator's links have been proposed by Tsuji et al. (1990, 1991a,b).

The present paper argues that kinematic redundancy of manipulators should be positively utilized in terms of impedance control, and proposes a new impedance control method for manipulators called *Hierarchical Impedance Control* (**HIC**). The method can control not only end-effector impedance as the highest priority task using one of conventional impedance control methods, but can also perform a sub task which dynamically has no effect to the end-effector motion of the manipulator. Firstly, an additional controller for realizing a sub task is incorporated to the end-effector impedance control system in a parallel way, and a sufficient condition for the additional controller which dynamically has no effect to end-effector motion of the manipulator is formulated. Then, the optimal additional controller corresponding to the desired joint control torque is analytically derived using the least square method. In the present paper, the HIC scheme is applied to regulate the joint impedance as a sub task according to the control problem addressed in (Tsuji et al, 1990). Regulating the joint impedance enables us to specify dynamic behavior of joints for unknown external forces beforehand. For example, by setting impedance of specific joints very large, it becomes possible to lock or to suppress the motion of the corresponding joints and reduce degrees of freedom of manipulator substantially. Moreover, by setting the desired joint positions, the joint impedance regulation method presented in this paper will also be useful for avoidance of singular configurations and maintaining the joint motions in the allowable joint angle limits. Finally, in order to confirm of the validity and to show the advantages of the proposed method, computer simulations are performed using a four-joint manipulator.

2. STRUCTURE OF HIERARCHICAL IMPEDANCE CONTROL

In general, the motion equation of an m-joint manipulator can be expressed as follows.

$$M(\theta)\ddot{\theta} + h(\theta,\dot{\theta}) = \tau + J^{T}(\theta)F_{ext} \qquad (1)$$

where $F_{ext} \in \Re^{l}$ is the external force exerted on the end-effector; $\theta \in \Re^{m}$ is the joint angle vector; $M(\theta) \in \Re^{m\times m}$ is a non-singular inertia matrix (hereafter denoted by M); $h(\theta,\dot{\theta}) \in \Re^{m}$ is the nonlinear term representing the torque vector due to the centrifugal, Coriolis, gravity and friction forces; $\tau \in \Re^{m}$ is the joint torque vector; $J(\theta) \in \Re^{l\times m}$ is the Jacobian matrix (hereafter denoted by J), and l is the dimension of the task space.

Now, the target impedance of the end-effector is described by

$$M_e d\ddot{X} + B_e d\dot{X} + K_e dX = F_{ext} \qquad (2)$$

Here, $M_e, B_e, K_e \in \Re^{l\times l}$ are the desired inertia, viscosity and stiffness matrices of the end-effector, and , $dX = X - X_d \in \Re^{l}$ is the deviation vector of the end-effector position from the desired trajectory X_d.

In the present paper, the impedance control law without calculation of inverse Jacobian matrix presented in (Hogan, 1987) is adopted:

$$\tau = \tau_{effector} + \tau_{comp} \qquad (3)$$

$$\tau_{effector} = J^{T}[\Lambda\{\ddot{X}_d - M_e^{-1}(B_e d\dot{X} + K_e dX) - \dot{J}\dot{\theta}\} - \{I_l - \Lambda M_e^{-1}\}F_{ext}] \qquad (4)$$

$$\tau_{comp} = (\bar{J}J)^{T}\hat{h}(\theta,\dot{\theta}) \qquad (5)$$

where $\Lambda = (JM^{-1}J^{T})^{-1}$; $\bar{J} = M^{-1}J^{T}\Lambda$; I_l is an $l\times l$ unit matrix; $\tau_{effector} \in \Re^{m}$ is the joint torque vector needed to produce the desired end-effector's impedance; and $\tau_{comp} \in \Re^{m}$ is the joint torque vector for the nonlinear compensation. It is assumed that $\hat{h}(\theta,\dot{\theta}) = h(\theta,\dot{\theta})$ and manipulator's configuration is not in a singular posture.

Although the control law given by eqs. (3), (4) and (5) can also be applied for a redundant manipulator directly, it cannot positively utilizes arm redundancy the same as other impedance control methods. So, in order to utilize arm redundancy in the joint torque level, an additional controller to perform a sub task is incorporated to the end-effector impedance control system in a parallel way. A new control law is given as follows:

$$\tau = \tau_{add} + \tau_{effector} + \tau_{comp} \qquad (6)$$

where $\tau_{add} \in \Re^{m}$ is the joint torque vector for the regulation of the additional arm impedance. Now, if the additional joint control torque, τ_{add}, satisfies the following condition

$$\bar{J}^{T}\tau_{add} = 0 \qquad (7)$$

then τ_{add} dynamically has no effect to end-effector motion of the manipulator, and the end-effector impedance remains equal to the target impedance given in (2) (*see Appendix A*). The investigation of this kind of force redundancy was firstly pointed out by Khatib (1990). Also, Kang and Freeman (1992) have been utilized for solving the stability problem encountered in local joint torque optimization techniques of the redundant manipulator. The present paper seeks other possibility to utilize this force redundancy in terms of impedance control in the sense that the manipulator can perform a sub task while controlling the end-effector impedance by a suitable selection of the additional controller, τ_{add}. In the following section, we will derive the optimal additional controller which satisfies the condition (7).

3. JOINT IMPEDANCE CONTROL LAW

Equation (7) describes the sufficient condition for the additional joint torque, τ_{add}, which dynamically has no effect to the end-effector motion. In this section, we will design the optimal controller to obtain such joint torque corresponding to the desired joint torque for performing the given sub task.

Firstly, the general solution of (7) is given by

$$\tau_{add} = (I_m - \bar{J}\bar{J}^{+})z_1 \qquad (8)$$

where I_m is an $m \times m$ unit matrix; $z_1 \in \Re^m$ is an arbitrary constant vector; and $(\cdot)^+$ denotes the pseudo inverse matrix. The joint torque τ_{add} in (8) always satisfies the sufficient condition (7), and now the problem becomes how to find the arbitrary constant vector z_1 in (8) to minimize the following cost function $G(\tau_{add})$:

$$G(\tau_{add}) = \|W(\tau^*_{add} - \tau_{add})\| \qquad (9)$$

where $\tau^*_{add} \in \Re^m$ is the desired additional joint torque which is needed to perform a given sub task. $W \in \Re^{m \times m}$ is a diagonal positive definite weighting matrix, each diagonal component of which assign order of priority to each element of τ^*_{add}, and $\|A\|$ stands for a metric norm defined by

$$\|A\| = [A^T A]^{0.5} = [\sum_{i=1}^{m} a_i^2]^{0.5} \qquad (10)$$

where a_i is the element of vector $A \in \Re^m$. Using the least square method, we can find the optimal solution (*see appendix B*) as given by

$$\tau_{add} = \Gamma \tau^*_{add} \qquad (11)$$

$$\Gamma = I_m - \{\bar{J}(W^{-1}\bar{J})^{+} W^{-1}\}^T \qquad (12)$$

It can be understood that using the hierarchical impedance control law composed by (4), (5), (6), (11) and (12), we can control not only the end-effector impedance but also the additional impedance as a sub task without any effect to the end-effector motion by positively utilizing force redundancy.

In the present paper, we will apply the HIC scheme for the regulation of joint impedance. Let us assume that the desired joint impedance is given according to the sub task, then the desired joint torque for the regulation of joint impedance can be computed by

$$\tau^*_{add} = -M^*_j d\ddot{\theta} - B^*_j d\dot{\theta} - K^*_j d\theta \qquad (13)$$

where $M^*_j, B^*_j, K^*_j \in \Re^{m \times m}$ are matrices of the desired joint inertia, viscosity and stiffness, respectively; and $d\theta = \theta - \theta_d \in \Re^m$ is the deviation vector between the present joint angle, θ, and the desired joint trajectory, θ_d. Using the additional impedance control law given by (11), (12) and (13), we can control the joint impedance as a sub task without any effect to end-effector motion of the manipulator (see **Fig.1**). This means that the arm redundancy can be utilized in terms of impedance control.

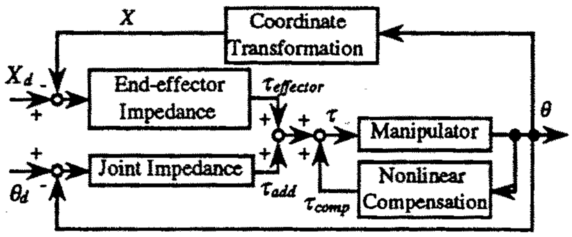
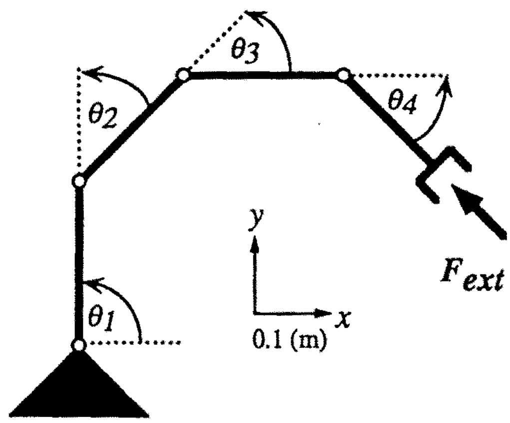


Fig. 1. Block diagram of Hierarchical Impedance Control (HIC). The HIC can control joint impedance as well as end-effector impedance of the redundant manipulator.

4. SIMULATION EXPERIMENTS

In order to show the effectiveness of the proposed method, computer simulations were performed using a four-joint planar manipulator for the case where the disturbance force, $F_{ext} = [-15, 15]^T$(N), exerted to the end-effector (see **Fig.2**). The link parameters of the manipulator are shown in **Table 1**.



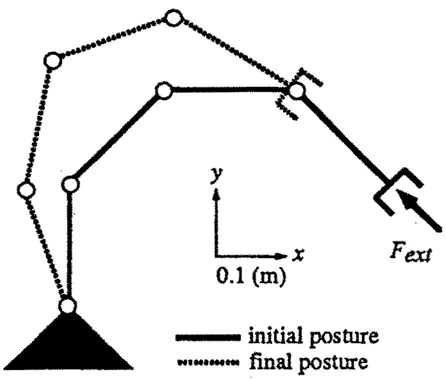
$$\theta = (\frac{\pi}{2}, -\frac{\pi}{4}, -\frac{\pi}{4}, -\frac{\pi}{4})^T \text{ (rad)}$$

Fig. 2. Model of the four-joint planar manipulator used in computer simulations

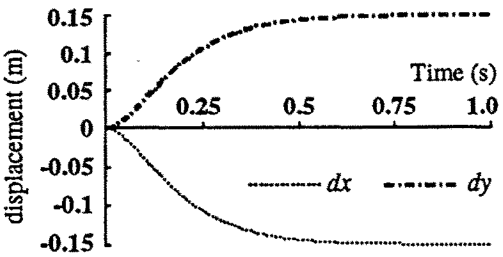
Table 1 Link parameters of the four-joint planar manipulators

	link i (i = 1,2,3,4)
length (m)	0.2
mass (kg)	1.57
center of mass (m)	0.1
moment of inertia (kgm^2)	0.8

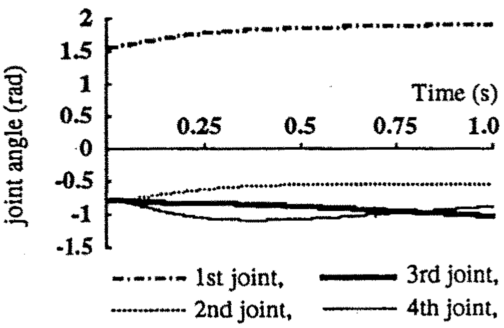
Figure 3 shows the simulation results of the manipulator under the conventional impedance control (equations (3), (4) and (5)), where Fig.3 (a), (b) and (c) indicate the change of manipulator's posture, the time profiles of the end-effector displacements and the joint angles, respectively. On the other hand, **Fig.4** shows the motion profiles of the manipulator under the HIC proposed in this paper (equations (4)~(6) and (11)~(13)). The simulation experiments were performed, where the desired end-effector impedance matrices are set as M_e = diag. [1, 1] (kg), B_e = diag. [20, 20] (N/(m/s)), K_e = diag. [100, 100] (N/m). In Fig.4, the desired joint impedance matrices are set as M_j^* = diag. [0.1, 0.01, 0.1, 0.01] (kgm^2), B_j^* = diag. [20, 2, 20, 2] (Nm/(rad/s)), K_j^* = diag. [1000, 100, 1000, 100] (Nm/rad), and the weighting matrix is set to W = diag. [50, 1, 50, 1]. Under these impedance parameters, the damping ratio of the desired dynamic behavior of the end-effector and each joint become equal to 1.0. Also, the desired end-effector trajectory and the desired joint trajectory are given as $X_d(t) = X(0)$ and $\theta_d(t) = \theta(0)$, respectively. The



(a) stick picture

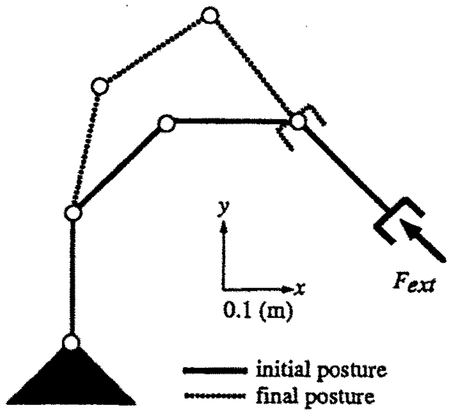


(b) end-effector displacements

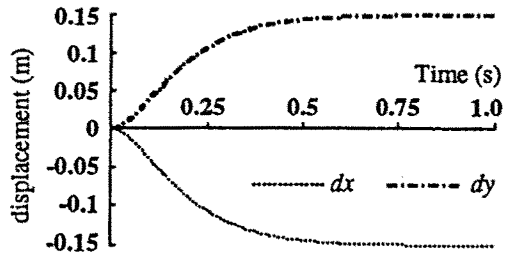


(c) joint angles

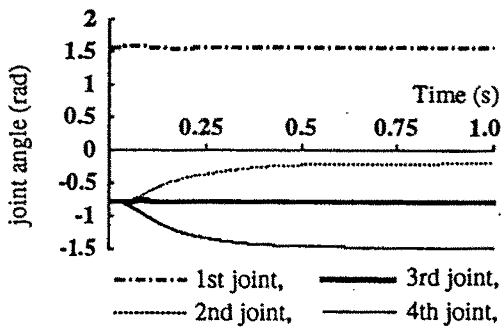
Fig.3. Motion profile of the four-joint planar manipulator for external force under the conventional impedance control



(a) stick picture



(b) end-effector displacements



(c) joint angles

Fig.4. Motion profile of the four-joint planar manipulator for external force under the hierarchical impedance control

computations of the manipulator dynamics were performed by the Appel's method (Potkonjak and Vukobratovic, 1987).

From the simulation results, it can be seen that the HIC proposed in this paper results the exactly same end-effector trajectory as the one resulted under the conventional impedance control, that is, both methods can control the end-effector impedance properly. However, the effectiveness of the proposed method appears clearly in the level of joint motion of the manipulator. Since the joint impedance of the 1st and 3rd joints were set very large, they almost didn't move during manipulator's motion, and the end-effector motion was realized by the 2nd and 4th joints. So, it can be understood that by the HIC, the arm redundancy has been positively utilized in terms of impedance control.

5. CONCLUSION

In the present paper, we have proposed a new method called Hierarchical Impedance Control (HIC). Using the HIC, the arm redundancy can be positively utilized in term of impedance control by incorporating an additional controller to the end-effector impedance control system. An important feature of the HIC is the ability to control not only the end-effector impedance but also the additional arm impedance without any effect to the end-effector motion of the manipulator.

Application of the HIC to the joint impedance control problem as the sub task was developed. When the desired joint impedance is given according to the sub task, the proposed method can realize the optimal joint impedance in the least squared sense while satisfying the required end-effector impedance. Control of the joint impedance enables to regulate dynamic behavior of joints for unknown external forces beforehand. It was shown that by setting impedance of specific joints very large, for example, it becomes possible to suppress the motion of the corresponding joints and reduce joint degrees of freedom of the manipulator substantially.

Future research will be directed to develop a method which also based on the HIC framework for another sub task, such as the regulation of the impedance of multiple points on the manipulator's links.

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APPENDIX A

Applying the control law given in eqs. (5) and (6) to the motion equation of the manipulator (1), we can find the following equation:

$$M(\theta)\ddot{\theta} + h(\theta,\dot{\theta}) = \tau_{add}+\tau_{effector}+(\mathcal{J}J)^{T}\ddot{h}(\theta,\dot{\theta}) + J^{T}F_{ext} \tag{A.1}$$

Now, using equations (2), (A.1), the kinematic relationships of the end-effector and applying the following relationship:

$$J^{T}\mathcal{J}^{T}J^{T}\mathcal{J}^{T} = J^{T}\mathcal{J}^{T} \tag{A.2}$$

we finally have

$$\begin{aligned}\tau_{effector} &= J^{T}[\Lambda\{\ddot{X}_d - M_e^{-1}(B_e d\dot{X} + K_e dX) - \dot{J}\dot{\theta}\,\} \\ &\quad - \{\,I_l - \Lambda M_e^{-1}\,\}F_{ext}] - J^{T}F_{add}\end{aligned} \tag{A.3}$$

$$F_{add} = \mathcal{J}^{T}\tau_{add} \tag{A.4}$$

where $F_{add} \in \Re^{l}$ is the additional force on the end-effector produced by the additional joint torque, τ_{add}.

In order for the additional joint torque, τ_{add}, to not produce any effect to the end-effector motion and the end-effector impedance remains equal to the target impedance given in (2), the additional force, F_{add}, in (A.3) must be equal to zero. This yields

$$\mathcal{J}^{T}\tau_{add} = 0 \tag{A.5}$$

APPENDIX B

Firstly, let us consider the case of $W = I_m$. For this case, the objective function (9) becomes

$$G(\tau_{add}) = \|\tau^{*}_{add} - \tau_{add}\| \tag{B.1}$$

Substituting (8) into (B.1), we find

$$G(z_1) = [\{\tau^{*}_{add} - (I_m - \mathcal{J}\mathcal{J}^{+})z_1\}^{T}[\{\tau^{*}_{add} - (I_m - \mathcal{J}\mathcal{J}^{+})z_1\}]^{0.5} \tag{B.2}$$

Now, the problem is how to obtain the vector z_1 in such a way that the objective function, $G(z_1)$, will be minimized. It is well known that the necessary condition regarding the optimal solution of the above problem is given by

$$\partial G(z_1) / \partial z_1 = 0 \tag{B.3}$$

Substituting (B.2) into (B.3) and expanding it, we have

$$(I_m - \mathcal{J}\mathcal{J}^{+})z_1 = (I_m - \mathcal{J}\mathcal{J}^{+})\tau^{*}_{add} \tag{B.4}$$

using the property of A^{+}, $(AA^{+})^{T}(AA^{+}) = (AA^{+}) = (AA^{+})^{T}$. Then substituting (B.4) into (8), we obtain

$$\tau_{add} = (I_m - \mathcal{J}\mathcal{J}^{+})\tau^{*}_{add} \tag{B.5}$$

The above equation is the least squared solution of matrix equation (7) under the objective function (B.1).

Next, we will derive the optimal solution for the general case, where the weighting matrix W is not equal to a unit matrix I_m. Firstly, we rewrite the matrix equation (7) in the form as the following:

$$\mathcal{J}^{T}W^{-1}W\tau_{add} = (W^{-1}\mathcal{J})^{T}W\tau_{add} = 0 \tag{B.6}$$

The general solution of (B.6) is given by

$$W\tau_{add} = \{I_m - (W^{-1}\mathcal{J})(W^{-1}\mathcal{J})^{+}\}z_2 \tag{B.7}$$

where $z_2 \in \Re^{m}$ is an arbitrary constant vector.

Substituting (B.7) into (9), and finding the optimal vector z_2 minimizing the cost function, we have

$$\{I_m - (W^{-1}\mathcal{J})(W^{-1}\mathcal{J})^{+}\}z_2 = \{I_m - (W^{-1}\mathcal{J})(W^{-1}\mathcal{J})^{+}\}W\tau^{*}_{add} \tag{B.8}$$

Finally, substituting (B.8) into (B.7) and expanding it, we have

$$\begin{aligned}\tau_{add} &= W^{-1}\{I_m - (W^{-1}\mathcal{J})(W^{-1}\mathcal{J})^{+}\}W\tau^{*}_{add} \\ &= [I_m - \{\mathcal{J}(W^{-1}\mathcal{J})^{+}W^{-1}\}^{T}]\tau^{*}_{add} \\ &\equiv \Gamma\tau^{*}_{add}\end{aligned} \tag{B.9}$$

It can be seen that the objective function (9) will be minimized by the above τ_{add}.