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Impedance Control for Redundant Manipulators and Its Application to Crank Rotation Tasks

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Abstract:

The present paper argues that kinematic redundancy of manipulators should be positively utilized in terms of impedance control, and proposes a new method called Redundant Impedance Control (RIC). The RIC can control not only end-effector impedance using one of conventional impedance control methods but also joint impedance which has no effect to end-effector motion of the manipulator. Firstly, joint impedance controller is incorporated to the end-effector impedance controller, and a sufficient condition about joint impedance for satisfying a given end-effector impedance is derived. Then, the optimal joint impedance corresponding to a given desired joint impedance is analytically derived using the least square method. Finally, omputer simulations using a four-joint planar manipulator for a crank rotation task are performed.

Keywords; Impedance Control, Kinematic Redundancy, End-Effector Impedance, Joint Impedance

1.INTRODUCTION

Impedance control [1, 2, 3] provides an unified approach to robot control in which the control variables are not positions or forces, but rather, the dynamic relation between positions and forces, and can control the end-effector impedance to follow the desired one which is specified depending on the given task.

On the other hand, kinematic redundancy in the manipulator structure yields increased dexterity and versatility which includes the ability to avoid singular configuration and collision with obstacles. Many investigators have focused on the kinematic redundancy, especially in connection with the inverse kinematic problem [4]. Up to the present, however, only a few number of study about utilizing kinematic redundancy in terms of impedance control has been reported: for example, the *Augmented Impedance Control* [5]. In the augmented impedance control, a vector of new task variables is defined, where its dimension is equal to the degree of redundancy of the system. This additional output vector is augmented to the end-effector position vector. Then, based on the augmented vector and the augmented Jacobian matrix, the impedance control was developed to achieve the desired end-effector impedance. This method, however, hasn't taken end-effector inertia into accounts, and so, in substantial, reduces to the stiffness control rather than impedance control.

The present paper argues that kinematic redundancy of manipulators should be positively utilized in the terms of impedance control, and proposes a new impedance control method for manipulators in accordance with the compliance control problem established in [6]. The method can control not only end-effector impedance using one of conventional impedance control methods but also joint impedance which has no effect to end-effector motion of the manipulator. Regulation of the joint impedance enables to specify dynamic behavior of joints for unknown external force beforehand. By setting impedance of specific joints very large, for example, it becomes possible to suppress large motion of the corresponding joints and reduce degrees of freedom of manipulator substantially.
In this paper, firstly, joint impedance controller is incorporated to the end-effector impedance control system, and a sufficient condition about joint impedance controller which has no effect to end-effector motion of the manipulator is given. Then, the optimal joint impedance corresponding to a given desired joint impedance is analytically derived using the least square method. Finally, in order to confirm of the validity and to show the advantages of the proposed method, computer simulations are performed using a four-joint

manipulator for a crank rotation task.

2. STRUCTURE OF REDUNDANT IMPEDANCE CONTROL

The motion equation of an m joint manipulator can be expressed as the following.

$$M(\theta)\,\ddot{\theta} + h(\theta,\dot{\theta}) = \tau - J^T(\theta)\,F_{ext} \; , \qquad (1)$$

where $-F_{ext} \in R^l$ is an external force exerted on the end-effector; $\theta \in R^m$ is the joint angle vector; $M(\theta) \in R^{m\times m}$ is a non-singular inertia matrix; $h(\theta,\dot{\theta}) \in R^m$ is a nonlinear term representing the torque vector due to centrifugal, coriolis, gravity and friction forces; $\tau \in R^m$ is the joint torque vector; $J(\theta) \in R^{l\times m}$ is the Jacobian matrix (hereafter denoted by J), and l is the dimension of the task space (m is larger than l). For this manipulator, the target impedance of the end-effector is described by

$$M_e d\ddot{X} + B_e d\dot{X} + K_e dX = -F_{ext} \; , \qquad (2)$$

where M_e, B_e, $K_e \in R^{l\times l}$ are the desired inertia, viscosity and stiffness matrices of the end-effector, and $dX=X-X_d \in R^l$ is the deviation vector of the end-effector position.

In the present paper, we adopt the end-effector impedance control law without calculation of inverse Jacobian matrix presented in [2]:

$$\tau = \tau_{effector} + \tau_{comp} \; , \qquad (3)$$

$$\tau_{effector} = J^T[W^{-1}(\theta)\,\{M_e^{-1}(-K_e dX - B_e d\dot{X}) + \ddot{X}_d - \dot{J}\,\dot{\theta}\} + \{I - W^{-1}(\theta)M_e^{-1}\}F_{ext} \; , \qquad (4)$$

$$\tau_{comp} = J^T W^{-1}(\theta) J M^{-1}(\theta)\tilde{h}(q,\dot{\theta}) \; , \qquad (5)$$

where $\tau_{effector} \in R^m$ is the joint torque vector needed to regulate the desired end-effector impedance and $\tau_{comp} \in R^m$ is the joint torque vector for the compensation. It is assumed that $\tilde{h}(\theta,\dot{\theta}) = h(\theta,\dot{\theta})$ and manipulator's configuration is not in singular posture, that is, $W(\theta) = JM^{-1}(\theta)J^T$ always invertible.

The control law given by equations. (3), (4) and (5) can be applied for redundant manipulator directly, but it cannot positively utilizes kinematic redundancy of the manipulator the same as other impedance control methods [1,3]. So, in order to utilize kinematic redundancy in the joint torque level, the joint impedance controller is incorporated to the end-effector impedance controller in a parallel way (see **Fig. 1**):

$$\tau = \tau_{joint} + \tau_{effector} + \tau_{comp} \; , \qquad (6)$$

$$\tau_{joint} = -M_j\, d\ddot{\theta} - B_j d\dot{\theta} - K_j d\theta \; , \qquad (7)$$

where M_j, B_j, $K_j \in R^{m\times m}$ are matrices of the desired joint inertia, viscosity and stiffness, respectively; and $d\theta=\theta-\theta_d \in R^m$ is the deviation vector between the present joint angle, θ, and the desired joint trajectory, θ_d ; and $\tau_{joint} \in R^m$ is the joint torque vector for the regulation of the joint impedance. If τ_{joint} is selected as satisfying the condition

$$(J^T)^+\tau_{joint} = 0 \; , \qquad (8)$$

then τ_{joint} has no effect to the end-effector motion and the end-effector impedance is equal to the target one given in (2). Not that $(\cdot)^+$ denotes the pseudo-inverse matrix. In the following section, we will derive the joint impedance matrices which satisfy the condition (8).

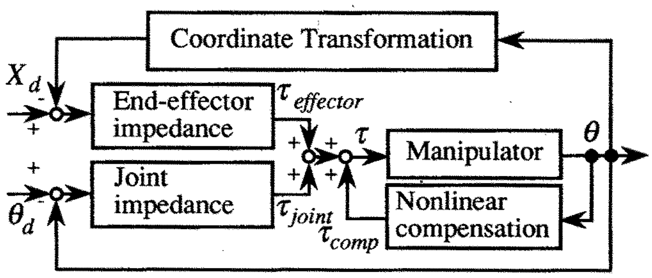


Fig.1 Block diagram of Redundant Impedance control (RIC). The RIC can control joint impedance as well as end-effector impedance of the redundant manipulator.

3. JOINT IMPEDANCE CONTROL LAW

From equation (7), the sufficient conditions for the joint torque τ_{joint} of (8) is that the following equations are satisfied at the same time for all joint impedance parameter matrices, M_j, B_j and K_j .

$$(J^T)^+M_j = 0 \; , \qquad (9)$$

$$(J^T)^+B_j = 0 \ , \quad (10)$$

$$(J^T)^+K_j = 0 \ . \quad (11)$$

Firstly, we will explain the optimal way to obtain the joint inertia matrix M_j that satisfies (9).

The general solution of the matrix equation (9) is given by [7]

$$M_j = (I - J^+J)Z_1 \ , \quad (12)$$

where $Z_1 \in R^{m\times m}$ is an arbitrary constant matrix. The matrix M_j in (12) always satisfy the sufficient condition (9), and so the problem becomes how to find the arbitrary constant matrix Z_1 in (12) to minimize the cost function $G(M_j)$:

$$G(M_j) = \|W(M_j^*-M_j)\| \ , \quad (13)$$

where $W \in R^{m\times m}$ is a diagonal positive definite weighting matrix, each diagonal component of which assign order of priority to each column of M_j^* . Also $\|A\|$ stands for matrix norm defined by

$$\|A\| = [tr(A^TA]^{0.5} = [\sum_{i=1}^{m}\sum_{j=1}^{m} a_{ij}^2]^{0.5} \ , \quad (14)$$

where $A = [a_{ij}] \in R^{m\times m}$ and tr $[\cdot]$ denotes trace of matrix. Using the least square method, we can find the optimal solution as given by (*see appendix*)

$$M_j = \Gamma M_j^* \ , \quad (15)$$

$$\Gamma = I - \{J^+(W^{-1}J^+)^+W^{-1}\}^T \ , \quad (16)$$

where I is an $m\times m$ unit matrix. Using the same way as the above, we can also find the optimal solutions for the joint viscosity matrix, B_j, and the joint stiffness matrix K_j, given by

$$B_j = \Gamma B_j^* \ , \quad (17)$$

$$K_j = \Gamma K_j^* \ , \quad (18)$$

where B_j^*, $K_j^* \in R^{m\times m}$ are the desired matrices of the joint viscosity and stiffness, respectively. Utilizing kinematic redundancy, we can control not only end-effector impedance but also joint impedance without any effect to end-effector motion of the manipulator.

4. APPLICATION TO CRANK ROTATION TASK

A four-joint planar manipulator (**Fig.2**) was used in the simulation experiments for a crank rotation task, and the link parameters of the manipulator are shown in **Table 1.** Two kinds of coordinate systems are chosen as follows: (i) the world coordinate system, $X(x,y)$; and (ii) the polar coordinate system, $\Phi(\phi,r)$, with its origin at the center of the crank where ϕ is the rotational angle of the crank and r is the distance from the center of rotation to the end-effector, that is, the radius of the crank. Dynamic computations of the manipulator and crank movements were performed by the Appel's method [8] under the closed link structure with the crank radius of 0.15 m and viscous friction of 10.0 Nms/rad for each manipulator's joint.

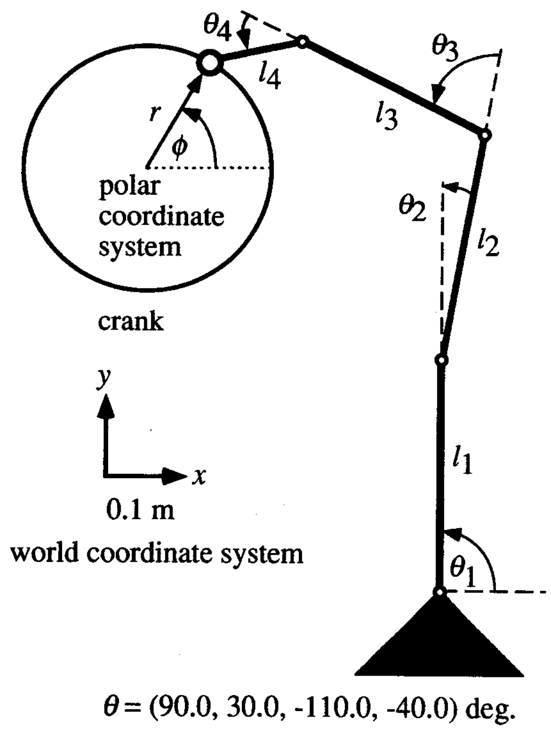


Fig. 2 A four-joint planar manipulator performing a crank rotation task.

Table 1 Link parameters of the four-joint planar manipulator

	link 1, 2	link 3	link 4
length (m)	0.28	0.28	0.25
mass (kg)	3.392	3.392	1.92
center of mass (m)	0.128	0.128	0.1025
moment of inertia (kgm^2)	0.29312	0.29312	0.011017

The target end-effector impedance (2) is expressed in the polar coordinate system, and the target inertia, viscosity and stiffness matrices are given as M_e = diag. [2.25×10^{-3} kgm^2, 0.1 kg], B_e = diag. [0.45 Nms/rad, 2 Ns/m], and K_e = diag. [22.5 Nm/rad, 10 N/m], respectively. Also the desired end-effector trajectory (equilibrium trajectory) is defined as the following:

$$\begin{bmatrix} \phi_d(t) \\ r_d(t) \end{bmatrix} = \begin{bmatrix} 20\pi t^3/t_f^3 - 30\pi t^4/t_f^4 + 12\pi t^5/t_f^5 \\ r \end{bmatrix}, \quad (19)$$

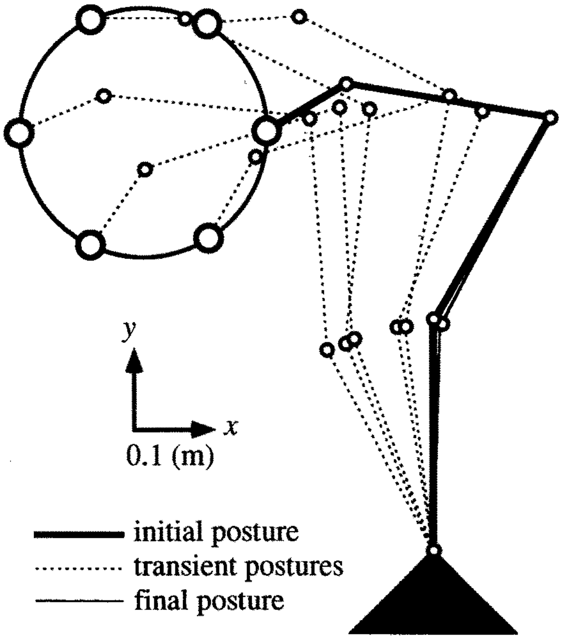
where the time duration t_f is set to 2.0 sec. The desired velocity and acceleration of the end-effector can be obtained from (19) using differentiation.

Fig. 3 shows simulation results performed under conventional impedance control (equations (3), (4), (5)), where fig. 3(a) and 3(b) indicate the change of manipulator's posture and the time profiles of joint angles, respectively. On the other hand, **Fig. 4** shows a simulation result performed under the redundant impedance control proposed in this paper (equations (4)-(7) and (15)-(18)), where the desired joint impedance matrices in (7) are given as M_j^* = diag. [0.001, 0.0001, 0.0001, 0.001] kgm^2, B_j^* = diag. [2, 0.2, 0.2, 2] Nms/rad, K_j^* = diag. [1000, 100, 100, 1000] Nm/rad, the weighting matrix is set to W = diag. [50, 1, 1, 50], and the desired joint trajectory is given as $\theta_d(t) = \theta(0)$.

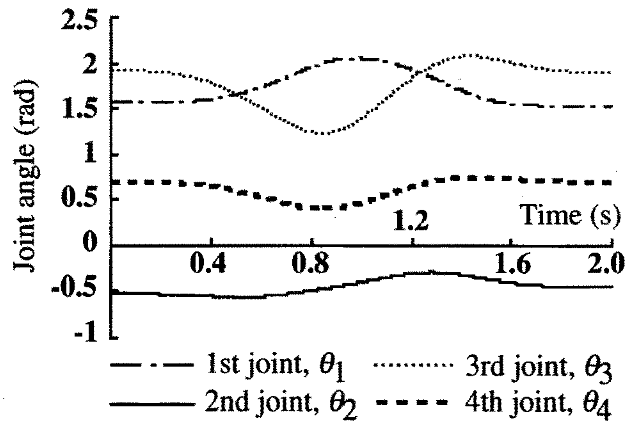
From these figures, it can be seen that both of the manipulator can rotate the crank finely. However, the effectiveness of the proposed method appears clearly in the level of joint motion of the manipulator. Since the impedance of the 1st and the 4th joints were set to large values, they almost didn't move during the task, and so the manipulator became non-redundant and end-effector motion was realized by the 2nd and the 3rd joints.

5. CONCLUSION

In this paper, the new method named *Redundant Impedance Control* (RIC) has been proposed. The RIC can control not only the end-effector impedance but also joint impedance which has no effect to end-effector motion of the manipulator. For a given desired joint impedance, the RIC gives the optimal joint impedance in the least squared sense while satisfying the required end-effector impedance. It was shown by computer simulations for the crank rotation task, that by setting impedance of specific joints very large, it becomes possible to suppress the motion of the corresponding joints and reduce joint degrees of freedom of the manipulator substantially.



(a) Stick pictures.



(b) Joint angles.

Fig.3 Motion profile of four-joint manipulator during crank rotation under the conventional impedance control method.

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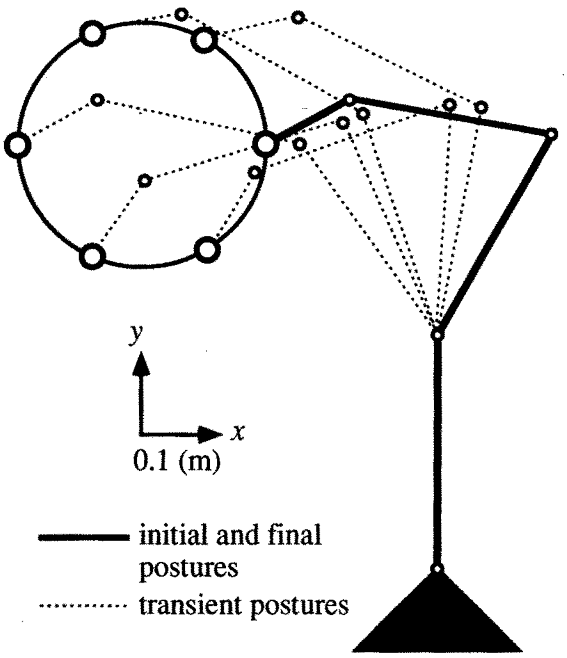
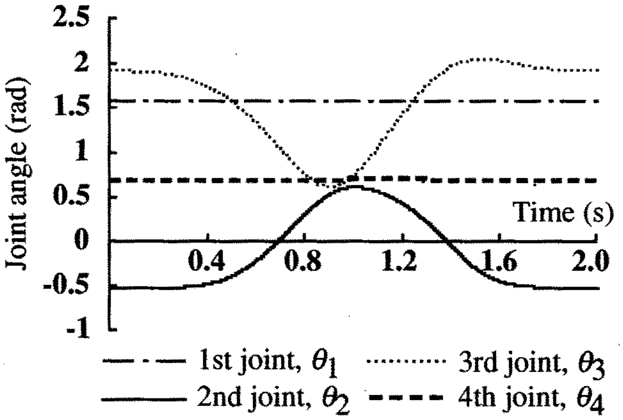


(a) Stick pictures.



(b) Joint angles.

Fig.4 Motion profile of four-joint manipulator during crank rotation under redundant impedance control. The 1st joint and the 4th joint impedance are set very large.

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APPENDIX

Firstly, let's consider the case of $W = I$. In this case, the cost function (13) becomes

$$G(M_j) = \|M_j^* - M_j\| \cdot \qquad \text{(A.1)}$$

Substituting (12) into (A.1), we find

$$G(Z_1) = [tr\,\{(M_j^* - (I - J^+J)Z_1)^T (M_j^* - (I - J^+J)Z_1)\}]^{0.5} \cdot \qquad \text{(A.2)}$$

Now, the problem is how to obtain the matrix Z_1 minimizing $G(Z_1)$. It is well known that the necessary condition regarding the optimal solution of the above problem is given by

$$\partial G(Z_1)/\partial Z_1 = 0 \cdot \qquad \text{(A.3)}$$

Substituting (A.2) into (A.3) and expanding it, we have

$$(I - J^+J)Z_1 = (I - J^+J)M_j^* , \qquad \text{(A.4)}$$

using the partial differential formulas about trace of matrices [7]

$$\partial tr\,[AZ_1B] / \partial Z_1 = A^TB^T , \qquad \text{(A.5)}$$

$$\partial tr\,[AZ_1^T BZ_1 C\,] / \partial Z_1 = BZ_1 CA + B^T Z_1 A^T C^T \ , \tag{A.6}$$

and the property of A^+ , $(A^+A)^T(A^+A) = (A^+A)^T = A^+A$. Note that A,B and C are matrices with appropriate dimensions.

Finally, substituting (A.4) into (12), we obtain

$$M_j = (I - J^+ J\,)M_j^* \ . \tag{A.7}$$

The above equation is the least squared solution of matrix equation (9) with the cost function given by (A.1).

Next, we will derive the optimal solution for the general case, where the weighting matrix, W, not equal to unit matrix, I . Firstly, we rewrite the matrix equation (9) in the form as the following.

$$(J^+)^T W^{-1} WM_j = (W^{-1}J^+)^T WM_j = 0\ . \tag{A.8}$$

The general solution of (A.8) is given by

$$WM_j = [I - (W^{-1}J^+)(W^{-1}J^+)^+]Z_2 \ , \tag{A.9}$$

where $Z_2 \in R^{m\times m}$ is an arbitrary constant matrix.

Substituting (A.9) into (13), and finding the optimal matrix Z_2 minimizing the cost function, we have

$$[I - (W^{-1}J^+)(W^{-1}J^+)^+]Z_2 = [I - (W^{-1}J^+)(W^{-1}J^+)^+]WM_j^* \ . \tag{A.10}$$

Finally, substituting (A.10) into (A.9) and expanding it, we have the optimal solution:

$$\begin{aligned} M_j &= W^{-1}[I - (W^{-1}J^+)(W^{-1}J^+)^+]WM_j^* \\ &= [I - (W^{-1}J^+)(W^{-1}J^+)^+]M_j^* \\ &= \Gamma M_j^* \ . \end{aligned} \tag{A.11}$$