# Multi-Point Compliance Control of Dual-Arm Robots

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Abstract— One of the basic characteristics of the dual-arm robots which manipulate a common object is the redundancy of joint degrees of freedom. The present paper proposes a compliance control method utilizing kinematic redundancy of the dual-arm robot. The method presented here can regulate the compliance or stiffness of several points on the dual armrobot as well as the object compliance through regulation of the joint stiffness.

#### I. INTRODUCTION

There are many tasks that require cooperative manipulation by the dual arm-robots for industrial manufacturing, ranging from simple handling tasks to more complicated assembly tasks. Also, the dual-arm robots may be applied to robotics in unstructured environments such as the space and the ocean where auxiliary equipments are not available [1].

One of the most important features of the dual-arm robots which manipulate a common object is that the arm and the object form a closed kinematic chain. From this feature many interesting problems arise, and a number of interesting works have been appeared. Most of these works focused on modeling and analysis of the closed kinematic chain [2], [3] and [4], load or force distribution problem [5], [6], [7] and [8], and the dynamic control [9], [10]. However, less attention has been paid to the compliance control for the dual-arm robot.

The compliance control is one of the most effective control methods for manipulators with interactions to their environments. The compliance control provides a mechanism for controlling manipulator position or force and facilitates stable behavior during the transition between unconstrained motion and sudden contact with the environment. Compliance (stiffness) of the parallel link structure have been studied by several investigators. For example, Mason and Salisbury [11], Nguyen [12], and Cutkosky and Kao [13] analyzed the grasp compliance for multifingered robotic hand, and an interesting compliance control method for parallel manipulators utilizing their internal forces was proposed in [14].

As well known, one of the basic characteristics of the dual-arm robots is the redundancy of joint degrees of freedom. The manipulator with more joint degrees of freedom than the minimum number required to the given tasks can offer significant advantages: for instance, avoiding obstacles or singular configurations of the manipulator in positioning tasks. Although it is widely recognized that the kinematic redundancy represents a key towards more flexible and sophisticated manipulations, no previous investigations of the compliance control for the parallel link structure have positively utilized kinematic redundancy. Utilizing redundancy, the dual-arm robot can perform subtasks while controlling the object compliance.

Reference [15] proposed Direct Compliance Control (DCC) for parallel link arms which utilizes the kinematic redundancy of the robotic system in order to make the joint stifness matrix become a diagonal matrix. On the other hand, Tsuji, Takahashi and Ito proposed a method called the Multi-Point Compliance Control (MPCC) for a single redundant manipulator [16] and [17]. This method can regulate not only the end-effector compliance but also the compliance of several points on the manipulator's links utilizing kinematic redundancy.

In the present paper, the MPCC for dual-arm robots is developed. When a rigid object is manipulated by dual-arm robots, we may model the object as a virtual link depending on the size of the object and the contact type between the object and the end-effectors. Then the system can be viewed as a single manipulator with closed kinematic chain and the MPCC can be developed. The method presented here is effective for certain environment where some obstacles impose restrictions on the task space.

First of all, we formalize kinematic relationships between the joint compliance and the compliance of several virtual objects on the links including the object. We then derive the joint compliance (stiffness) to achieve the desired multiple objects compliance. The dual-arm robot may become over-constrained depending on the contact type and the number of the objects which we regulate their compliance. The method presented here can give the optimal joint compliance (stiffness) even for the over-constrained cases in the least squared sense. Finally, the effectiveness of the proposed method is shown by computer simulations.

# II. FORWARD KINEMATICS OF COMPLIANCE MATRICES FOR DUAL—ARM ROBOTS

We consider a dual arm robot manipulating a common object as shown in Fig.1. The robot is performing a task which requires the object's compliant motion. Since the robot is close to obstacles, the arm may collide with them due to some external force. Then, as shown in Fig.1, we locate a virtual object on the closest point of the arm to the obstacles. Using the virtual objects, the interaction between the robot and its environment can be considered within the framework of the compliance control. For example, to avoid any collision, the compliance of the virtual object should be as small (stiff) as possible in the direction of the obstacles [16].

### A. Forward Kinematics for Object Compliance

We define a set of Cartesian coordinate system as follows (see Fig.1): (i) the world coordinate system,  $\Sigma_w$ , is an immobile external coordinate system as a reference frame, (ii) the transmission coordinate system,  $\Sigma_{tr}$ , is a coordinate system on the object at the contact point where the z axis is normal to the object and the others are tangential to the object, and (iii) the object coordinate system,  $\Sigma_o$ , is a mobile coordinate system according to the motion of the object, but the direction of its axis  $x_o, y_o, z_o$  are constant i.e., parallel to the  $\Sigma_w$ .

Forward force/motion relationships of dual-arm robots grasping a common object can be summarized in

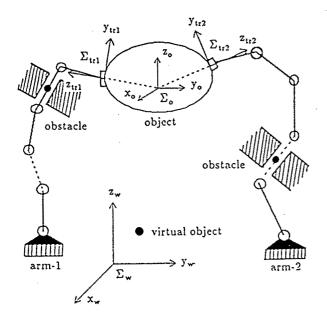


Fig.1. Dual-arm robot close to obstacles.

Fig.2. In the present paper, it is assumed that the contact points between the end-effectors and the object are constant, i.e., there is no slip motion between the end-effectors and the object. For derivations of the force/motion relationships, see [11], [13] and [18].

The matrix  $S \in \mathbb{R}^{l \times 2l}$  specifies the relationship between the object space and the contact space depending on the locations of the contact points and the reference point on the object such as the center of mass, where l is the dimension of the task space. The matrix  $H \in \mathbb{R}^{(l_{c1} + l_{c2}) \times 2l}$  expresses the filter characteristics which filter out some forces of the end-effectors and transmit other forces to the contact point space depending on the contact type, where  $l_{ci}$  (i=1,2) denotes the number of forces transmitted from the end-effector of the i-th arm to the contact point space. The matrix  $J_o \in \mathbb{R}^{2l \times (m_1 + m_2)}$  is a concatenated Jacobian matrix of both arms relating the joint displacements to the end-effector displacements, represented in the transmission coordinate system, where  $m_i$  denotes the number of joints of the i-th arm.

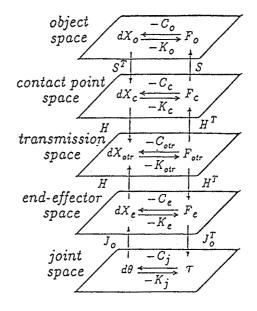


Fig.2. Force/motion relationships for the object.

Using the force/motion relationships shown in Fig.2, we can derive the forward kinematic relationships which relate a set of  $(m_1+m_2)$  joint coordinates to a set of l object coordinates. The forward kinematic equations of the object are given by:

$$dX_{otr} = G_o dX_o = \beta_o d\theta, \tag{1}$$

$$F_o = G_o^T F_{otr}, (2)$$

$$\tau = \beta_0^T F_{otr},\tag{3}$$

where

$$G_o = HS^T \in \mathbb{R}^{(l_{e1} + l_{e2}) \times l},$$
 (4)

$$\beta_o = HJ_o \in \mathbb{R}^{(l_{e1} + l_{e2}) \times (m_1 + m_2)}.$$
 (5)

To derive the forward kinematics for the object compliance, we define the compliance matrices of each space as follows:

Joint Space:

$$d\theta = -C_j \tau, \tag{6}$$

 $C_i \in \mathbb{R}^{(m_1+m_2)\times(m_1+m_2)}$ : joint compliance

End-effector Space:

$$dX_e = -C_e F_e, (7)$$

 $C \in \mathbb{R}^{2l \times 2l}$ : end-effector compliance

Transmission Space:

$$dX_{otr} = -C_{otr}F_{otr}, (8)$$

 $C_{ste} \in R^{(l_{e1}+l_{e2}) \times (l_{e1}+l_{e2})}$ : transmission compliance

Contact Point Space:

$$dX_e = -C_e F_e, (9)$$

 $C_c \in \mathbb{R}^{2l \times 2l}$ : contact point compliance

Object Space:

$$dX_{\alpha} = -C_{\alpha}F_{\alpha}. (10)$$

 $C_{\circ} \in \mathbb{R}^{l \times l}$ : object compliance

Using (1), (2), (3) and the compliance matrices defined above, we can obtain the forward kinematic relationship from the joint compliance to the object compliance via the transmission compliance, such as given by

$$C_{clo} = G_c C_c G_c^T \tag{11}$$

$$C_{otr} = \beta_o C_i \beta_o^T, \tag{12}$$

where  $G_o$  and  $\beta_o$  are given in (4) and (5).

#### B. Forward Kinematics for Virtual Object Compliance

To derive the kinematic relationship for the virtual object compliance, we introduce the virtual space shown in Fig.3 with the same kind of the definition of stiffness/compliance matrices as (6)-(10).

Furthermore, for the i-th virtual object, we define only two kinds of coordinate systems: (i) the virtual object coordinate system  $\Sigma_{vi}$ , and (ii) the world coordinate system,  $\Sigma_w$ .

For the virtual object, the contact type between the virtual end-effectors and the virtual object is always like rigid grasping, i.e., all forces of the virtual end-effector can be transmitted to the virtual contact point. Moreover, the virtual object is always like a point object. As a result, we can see

$$S_{vi} = [I_l \ I_l], \tag{13}$$

$$H_{vi} = I_{2l}, \tag{14}$$

where subscript i stands for the i-th virtual object and  $I_l$  is a  $l \times l$  unit matrix.

The forward kinematic equations of the i-th virtual object are given by

$$dX_{viri} = G_{vi}dX_{voi} = \beta_{vi}d\theta, \qquad (15)$$

$$F_{voi} = G_{vi}^T F_{vtri}, \tag{16}$$

$$\tau = \beta_{ri}^T F_{riri},\tag{17}$$

where

$$G_{ni} = H_{ni}S_{ni}^T \in \mathbb{R}^{2l \times l}, \tag{18}$$

$$G_{vi} = H_{vi}S_{vi}^{T} \in \mathbb{R}^{2l \times l}, \qquad (18)$$

$$\beta_{vi} = H_{vi}J_{vi} \in \mathbb{R}^{2l \times (m_1 + m_2)}, \qquad (19)$$

and  $J_{vi}$  is the concatenated Jacobian matrix of both arms relating the joint displacements to the i-th virtual end-point displacements in the virtual object coordinate system.

On the other hand, the forward kinematic equation from the joint compliance  $C_i$  to the *i*-th virtual object compliance  $C_{\text{toii}} \in R^{l \times l}$  can be derived in the same way as the previous section:

$$C_{vtrii} = G_{vi}C_{voii}G_{vi}^{T}, (20)$$

$$C_{vtrii} = \beta_{vi} C_i \beta_{vi}^T. \tag{21}$$

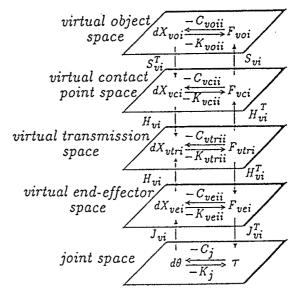


Fig.3. Force/motion relationships for the i-th virtual object.

#### C. Concatenated Forward Kinematics

In order to express the forward kinematics for the object and the virtual object simultaneously, we concatenate those forward kinematics:

$$dX_{tr} = GdX = \beta d\theta, \tag{22}$$

$$F = G^T F_{tr}, (23)$$

$$\tau = \beta^T F_{tr},\tag{24}$$

where F is the concatenated vector of the resultant forces/moments on the multi objects at the reference point, and  $F_{tr}$  is the concatenated transmitted forces/moments on the multi objects at the contact-point from the end-effector. Also, dX and  $dX_{tr}$  are the concatenated displacements of the multi objects and the concatenated transmission displacements, respectively, and

$$\beta = [\beta_{v1}^T \cdots \beta_{vn-1}^T \beta_o^T]^T, \tag{25}$$

$$G = block-diag. [G_{v1} \cdots G_{vn-1} G_{o}], \qquad (26)$$

n is the number of the objects which we want to regulate their compliance. block-diag.[] denotes a block diagonal matrix.

Consequently, the concatenated form of the forward kinematic from the joint compliance to the object and the virtual object compliances can be expressed as

$$C_{tr} = GCG^T, (27)$$

$$C_{tr} = \beta C_i \beta^T, \tag{28}$$

where

$$C = \begin{bmatrix} C_{vo11} & C_{vo12} & \cdots & C_{vo1n} \\ C_{vo21} & C_{vo22} & \cdots & C_{vo2n} \\ & \cdots & & \\ C_{von1} & C_{von2} & \cdots & C_{vonn} \end{bmatrix}, \tag{29}$$

$$C_{tr} = \begin{bmatrix} C_{vtr11} & C_{vtr12} & \cdots & C_{vtr1n} \\ C_{vtr21} & C_{vtr22} & \cdots & C_{vtr2n} \\ & \cdots & & & & \\ C_{vtrn1} & C_{vtrn2} & \cdots & C_{vtrnn} \end{bmatrix}, \tag{30}$$

and  $C_{vonn} \equiv C_o$ ,  $C_{vtrnn} \equiv C_{otr}$ .  $C_{voij}$  represents the cross compliance between the *i*-th and *j*-th virtual objects and  $C_{vtrij}$  represents the cross compliance between the  $i^{th}$  and the  $j^{th}$  virtual transmission compliance.

III. MULTI-POINT COMPLIANCE CONTROL FOR

DUAL-ARM ROBOTS

Now, let's assume that the desired compliance matrix  $C^*$  is given according to the tasks. We should solve (27) as a first step and then to solve (28) for the joint compliance matrix  $C_i$ .

Equation (28) may be underconstrained, overconstrained or singular depending on the matrix  $\beta$ . To derive a general solution of (28), we use the maximum rank decomposition of the matrix  $\beta$ , such that the (28) can be divided into two equations [16]

$$C_{ib} = \beta_b C_i \beta_b^T \tag{31}$$

and

$$C_{tr} = \beta_a C_{ib} \beta_a^T, \tag{32}$$

where  $\beta_a$  and  $\beta_b$  form the maximum rank decomposition of  $\beta$ :  $\beta = \beta_a \beta_b$  (rank $\beta = rank \beta_a = rank \beta_b = s$ ).

We should solve (32) as a first step. In general, the exact solution  $C_{jb}$  which satisfies (32) does not exist, since the matrix  $\beta_a$  is a full column rank matrix. In this case, the goal is to find a matrix  $C_{jb}$  to minimize

$$G(C_{jb}) = ||W(C_{tr}^* - C_{tr})W^T||$$
 (33)

$$G(C_{ib}) = ||W(C_{ir}^* - \beta_a C_{ib} \beta_a^T) W^T||, \qquad (34)$$

where  $C_{tr}^*$  is a desired concatenated transmission compliance matrix, computed from (27) for a given desired concatenated compliance matrix  $C^*$ . ||A|| stands for matrix norm defined by

$$||A|| = [tr(A^T A)]^{0.5}$$
 (35)

and tr  $(A^TA)$  denotes trace of matrix  $A^TA$ .

Using the differential formulas about trace of matrix, we can find an optimal solution such as given by

$$C_{ib} = (W\beta_a)^{\#} W C_{tr}^* W^T \{ (W\beta_a)^{\#} \}^T$$
 (36)

$$(W\beta_a)^{\#} = \{(W\beta_a)^T W\beta_a\}^{-1} (W\beta_a)^T.$$
 (37)

The weighting matrix W is a positive definite matrix which can assign order of priority to each object according to the given task.

The second step is to solve matrix equation (31). Since the matrix  $\beta_b$  is of full row rank, the solution  $C_j$  which satisfy (31) always exist. The general solution is given by

$$C_j = \beta_b^+ C_{jb} (\beta_b^T)^+ + \left[ Z - \beta_b^+ \beta_b Z (\beta_b^+ \beta_b)^T \right], \quad (38)$$

where  $Z \in R^{(m_1+m_2)\times(m_1+m_2)}$  is an arbitrary constant matrix. The matrix Z can be utilized in the other kind of subtasks. Then, we define the joint stiffness matrix  $K_i$  as

$$K_i \equiv C_i^+. \tag{39}$$

Equations (36), (37) and (38) give the closest object compliance to the desired one in the least squared sense

for the cases which the desired stiffness cannot be realized. On the other hand, when the exact solution of  $C_j$  in (28) exists, and we choose the minimum norm solution of the joint compliance matrix in (38), the computation of the joint compliance matrix becomes simple:

$$C_i = \beta^+ C_{tr}^* (\beta^T)^+ , \qquad (40)$$

because  $\beta$  has a full row rank.

## IV. APPLICATIONS TO OBSTACLE AVOIDANCE

To demonstrate the effectiveness of the proposed method, simulation experiments were performed. In the simulation experiments, the MPCC is applied to an obstacle avoidance problem using a planar dual arm robot as shown in Fig.4 ( $m_1 = 6$ ,  $m_2 = 3$ , l = 3).

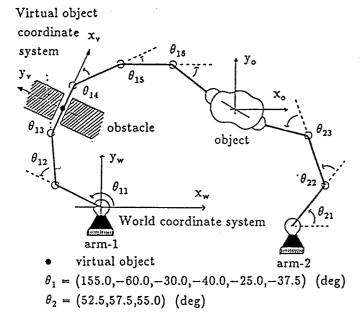


Fig.4. A planar dual-arm robot close to the obstacle.

The dual arm robot is needed to perform a task which requires the object to be soft in the direction of x axis and the rotation (0.005m/N) and 0.1rad/Nm, respectively) and to be stiff in the direction of y axis (0.001m/N) in the object coordinate system. And the third link of arm-1 lies between a couple of obstacles. A virtual object is located on the middle point of the third link. To avoid a collision with the obstacles, we wish to regulate the virtual object compliance to be stiff in the direction of  $y_x$  (0.001m/N) in the virtual object coordinate system, while satisfying the desired object compliance. As a result, the desired compliance matrix  $C^* \in \mathbb{R}^{6 \times 6}$  is given by

$$C^* = diag.[0.005(m/N), 0.001(m/N), 0.1(rad/Nm), 0.005(m/N), 0.001(m/N), 0.01(rad/Nm)],$$

where diag.[] denotes a diagonal matrix.

The simulation experiment was performed using the PD controller,

$$\tau = K_j d\theta + B_j \hat{\theta}, \tag{41}$$

where  $B_j \in \mathbb{R}^{(m_1+m_2)\times(m_1+m_2)}$  is a nonsingular feedback gain matrix. We used the Appel method [19] which extended for the dual arm robots grasping a common object, and the link parameters of each arm are shown in Table 1. The mass and the moment of inertia of the object are 5(kg) and  $0.5(kg-m^2)$  respectively.

TABLE I Link Parameters of the Dual-Arm Robot

	Link i (i=1,,9)
Length (m)	0.2
Mass (kg)	0.5
Center of mass (m)	0.1

Fig.5 shows the initial and final postures of the dual arm under the MPCC, where the external force,  $f_{ext} = [-15(N), -40(N), 0(Nm)]^T$  in terms of the object coordinate system, is exerted to the center of mass of the object. The joint stiffness  $K_j$  is calculated under the rigid grasping  $(l_{c1} = l_{c2} = 3)$ .

On the other hand, Fig.6 shows a simulation result where we consider to control only the object compliance. The displacements of the object for the two kinds of compliance control (using the MPCC and consider to control only the object compliance) are almost the same. In terms of the virtual end-point, however, the effect of the MPCC appears clearly. It can be seen that the MPCC can utilize effectively the redundant joint degrees of freedom.

#### V. CONCLUSION

We have proposed the Multi-Point Compliance Control method for dual arm robots utilizing kinematic redundancy. The method was able to regulate the compliance or stiffness of several points on the dual arm's links as well as the object stiffness.

In the present paper, we concentrated on the resultant forces of the object through the regulation of the joint stiffness according to a given task: for example, to restrain some external forces or to produce a desired force to generate a certain trajectory. In order to establish the compliance control of the parallel structure in

general, the control of the internal forces between the end-effectors and the object should be considered, because the internal forces have some effects on the object compliance [14].

Further research will be directed to develop the dual independent controller configuration algorithm. Also, it is interest to develop the Multi Point Impedance Control which enable to control the impedance (not only the compliance) of several points simultaneously.

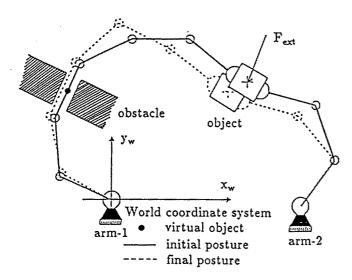


Fig.5. Arm configuration for a disturbance force with consideration of obstacle avoidance using MPCC.

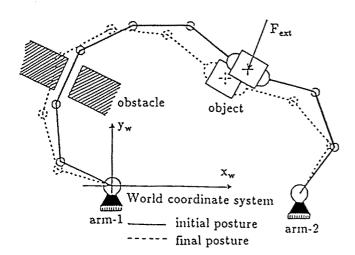


Fig. 6. Arm configuration for a disturbance force without consideration of obstacle avoidance.

#### REFERENCES

- [1] M. Uchiyama, "A unified approach to load sharing, motion decomposing, and for sensing of dual arm robots," In: Miura, H. and Arimoto, S. (Eds), Robotic research: The Fifth international symposium, Tokyo, pp. 225-232, 1990.
- [2] Y. F. Zheng, and J. Y. S. Luh, "Joint torques for control of two coordinated moving robots," Proc. 1986 IEEE Int. Conf on Robotics and Automation, San Fransisco, pp. 1375-1380, 1986.
- [3] V. Kumar, and J. F. Gardner, "Kinematics of redundantly actuated closed chains," *IEEE Trans. on Robotics and Automation*, vol. 6, no. 2, pp. 269-274, 1990.
- [4] C. Gosselin, and J. Angeles, "Singularity analysis of closed-loop kinematic chains," *IEEE Trans. on Robotics and Automation*, vol.6, no.3, pp. 281-290, 1990.
- [5] F. T. Cheng, and D. E. Orin, "Optimal force distribution in multiple-chain robotic systems," *IEEE Trans. on Syst. Man. and Cybern.*, vol.21, no.1, pp. 13-24, 1991.
- [6] V. R. Kumar, and K. J. Waldron, "Force distribution in closed kinematic chains," *IEEE J. of Robotics and Automation*, vol.4, no.6, pp. 657-664, 1988.
- [7] Y. F. Zheng, and J. Y. S. Luh, "Optimal load distribution for two industrial robots handling a single object," Proc. 1988 IEEE Int. Conf. on Robotics and Automation, Philadelphia, pp. 344-349, 1988.
- [8] T. Yoshikawa, and K. Nagai, "Analysis of multi-fingered grasping and manipulation," In: Venkataraman, S. T. and Iberall, T. (Eds), Dextrous Robot Hands, pp. 187-208, 1990.
- [9] I. D. Walker, and R. A. Freeman, "Internal object loading for multiple cooperating robot manipulators," Proc. 1990 IEEE Int. Conf. on Robotics and Automation, Arizona, pp. 606-611, 1990.
- [10]Y. R. Hu, and A. A. Goldenberg, "Dynamic control of multiple coordinate redundant manipulator with torque optimization," Proc. 1990 IEEE Int. Conf on Robotics and Automation, Ohio, pp. 1000-1005, 1990.
- [11]M. T. Mason, and J. K. Salisbury, Robot hands and the mechanics of manipulation. Cambridge, MA: MIT Press, 1985.
- [12]N. Nguyen, "Constructing force-closure grasps in 3-d," Proc. 1987 IEEE Int. Conf. on Robotics and Automation, pp. 240-245,1987.
- [13]M. R. Cutkosky, and I. Kao, "Computing and controlling the compliance of a robotic hand," IEEE Trans. on Robotics and Automation, vol.5, no.2, pp. 151-165,1989.
- [14]M. A. Adli, K. Nagai, K. Miyata, and H. Hanafusa, "Study on internal forces and their use in compliance control of parallel manipulators," Proc. of 29th SICE Annual Conference, pp. 853-856,1990.
- [15]K. Yokoi, M. Kaneko, K. Tanie, "Direct Compliance Control for a Parallel Link Arm," Trans. of the JSME, vol.54, no.505, pp. 2131-2139, 1988, in Japanese.
- [16]T. Tsuji, T. Takahashi, and K. İto, "Multi point compliance control for redundant manipulators," In: S. Stifter and J. Lenarcic (Eds), Advances in Robot Kinematics, 1991.
- [17]T. Tsuji, T. Takahashi, and K. Ito, "Multi point compliance control for manipulators constrained by task objects," Trans. of the Society of Instrument and Control Engineers, vol.27, no.1, pp. 85-92, 1991, in Japanese.
- [18]F. T. Cheng, and D. E. Orin, "Efficient formulation of force distribution equations for simple closed-chain robotic mechanisms," IEEE Tarns. on Syst. Man. and Cybern., vol.21, no.1, pp. 25-32, 1991.
- [19]V. Potknjak, and M. Vukobratovic, "Two new methods for computer forming of dynamic equation of active mechanisms," Mechanism and Machine Theory, vol.14, no.3,pp. 189-200, 1987.